PG/IIS/MTM-203/14

M.Sc. 2nd Semester Examination, 2014

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Abstract and Linear Algebra*)

PAPER – MTM - 203

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Notation have their usual meanings

GROUP – A

[NEW SYLLABUS]

(*Abstract Algebra*)

[*Marks* : 25]

Answer **Q.No.1** and any **two** from the rest

1. Answer any *two* questions : 2 × 2

(*a*) Define solvable group with example.

(*Turn Over*)

(b) Let $\phi : G \to G'$ be an epimorphism of groups. If H is a normal subgroup of G, then show that $\phi(H)$ is a normal subgroup of G'.

(c) Show that $\mathbb{Z}_2 \times \mathbb{Z}_{30}$ is isomorphic to $\mathbb{Z}_{10} \times \mathbb{Z}_6$.

2. (a) Show that if H be a subgroup of a cyclic group G, then the quotient group G/H is also cyclic. Give example to show that the converse of the above statement is not true.

(b) Show that the conjugacy relation in a group G is an equivalence relation. (5 + 1) + 2

3. (a) Let G be a group of order $p^n m$ where p is a prime integer and $gcd(p,m) = 1$. Show that for each $0 \leq k \leq n$, G has a subgroup of order p^k.

(b) Show that a group of order 200 is not simple.

(c) Show that D_8 and Q_8 are not isomorphic.

5 + 2 + 1

4. (a) Let R be the set of all real valued functions on the closed interval [0, 1].

Define $(f+g)$ and fg by

$(f+g)(x) = f(x) + g(x)$, $(fg)(x) = f(x)\,g(x)$, $x \in [0, 1]$.

Let $I = \left\{ f \in R : f\left(\frac{1}{5}\right) = 0 \right\}$.

Show that I is an ideal of R. Also show that it is a maximal ideal of R.

(*b*) What are the Prime ideals in the ring $\mathbb{Z}$.

(*c*) Give example of a principle ideal domain which is not an Euclidean domain. Also give an example of a unique Factorization domain which is not a principal ideal domain. 5 + 1 + 2

[*Internal Assessment* : 5 Marks]

GROUP – A

[OLD SYLLABUS]

(*Abstract Algebra*)

[*Marks* : 25]

Answer **Q.No.1** and any **two** from the rest

1. Answer any *two* questions : 2 × 2

(*a*) Define prime ideal in a ring R with two examples.

(*b*) Write down the groups of order 6 upto isomorphism.

(*c*) Define solvable group with examples.

2. (*a*) State and prove Cauchy's theorem for a finite abelian group.

(*b*) Define group isomorphism with example. Show that the multiplicative groups $\mathbb{R}-\{0\}$ and $\mathbb{C}-\{0\}$ are not isomorphic. 4 + 2 + 2

3. (*a*) Let R be a commutative ring with unity and I be an ideal of R. Prove that R/I is a field if and only if I is maximal.

(*b*) Show that a group of 255 is not simple. 5 + 3

4. (*a*) Let R be a commutative ring with unity. An element of R is said to be nilpotent if $a^n = 0$ for some natural number n. Show that the set

of all nilpotent elements of R forms an ideal of R.

(*b*) Define the following with examples principle ideal domain, unique factorisation domain.

4 + 2 + 2

[*Internal Assessment* : 5 Marks]

GROUP– B

[NEW & OLD SYLLABUS]

(*Linear Algebra*)

[*Marks* : 25]

Answer **Q.No.5** and any **two** from the rest

5. Answer any *two* questions : 2 × 2

(*a*) Let P_1 be the vector space of polynomials in t of degree 1 over the field of real numbers R. Define

$$T : P_1 \rightarrow P_1 \text{ such that } T(1+t) = t$$
$$T(1-t) = 1$$

Find $T[(2-3t)]$.

(*b*) Define minimal polynomial of a linear operater.

(*c*) Define poset with an example.

(*d*) Define Jordan canonical form with an example.

6. (*a*) Let $T : P_2(R) \to P_3(R)$ be a linear transformation defined by

$$T(ax^2 + bx + c) = (a - b)x^3 + (b - c)x^2 + (c - a)x + (c - a)$$

where $a, b, c \in R$ (set of real no.)

Find

(*i*) Ker (T)

(*ii*) A basis for ker (T)

(*iii*) Range (T)

(*iv*) A basis for Range (T).

(*b*) Let $T : P_3(R) \to P_2(R)$ be the linear transformation defined by $T[f(x)] = f'(x)$. Find $[T]_\beta^\gamma$ Where β and γ be the standard ordered bases for $P_3(R)$ and $P_2(R)$ respectively. $\left(1\frac{1}{2} \times 4\right) + 2$

7. (*a*) For the linear operator T on V, find the minimal polynomial of T when $V = P_2(R)$ and $T(f(x)) = -xf''(x) + f'(x) + 2f(x)$.

(*b*) Let T be the operator on R^3 defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}. 4 + 4

8. (*a*) Let $T : V \rightarrow W$ be a linear transformation and dim $V = n$. Then prove that the following statements are equivalent :

(*i*) T is injective

(*ii*) Rank of $T = n$

(*iii*) $\beta = \{ v_1, v_2, \dots v_n \}$ is a basis of V

$T(\beta) = \{T(v_1), T(v_2), \dots, T(v_n)\}$ is a basis of image of T i.e, dim $V =$ dim T.

(*b*) For any $n \in N$ (set of natural no.), let P_n denotes the vector space of all polynomials with real coefficient and of degree at most n. Define $T : P_n = P_{n+1}$ by

$$T(p(x)) = p'(x) - \int_0^x p(t)dt.$$

Find the dimension of the null space of T.

(*c*) **Write down the postulates of lattice.** 3 + 2 + 3

[*Internal Assessment* : 5 Marks]
