M.Sc. 2nd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-201

(Fluid Dynamics)

Full Marks: 50

Time: 2 hours

Answer Q. No. 6 and any three questions from rest

The figures in the right hand margin indicate marks

1. (a) In the case of two-dimensional motion of a liquid past a fixed circular disc, the velocity at infinity is \( u \) in a fixed direction is \( u \), where \( u \) is variable. Show that the maximum value of the velocity at any point of the liquid is \( 2u \). Prove also that the force necessary to hold the disc is \( 2mu \), where \( m \) is the mass of the liquid displaced by the disc.
(b) Show that the complex potential for a liquid past a fixed elliptic cylinder with velocity $U$ parallel to the major axis of the section is given by

$$w = U (a + b) \cosh(\rho - \alpha),$$

where the symbols have their usual meaning. 6 + 6

2. (a) In the two dimensional irrotational motion of a liquid streaming part a fixed elliptic disc

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$  

the velocity at infinity being parallel to the major axis and equal to $V$, prove that if

$$x + iy = c \cosh(\xi + i\eta), \quad a^2 - b^2 = c^2$$

$a = c \cosh \alpha$, $b = c \sinh \alpha$, the velocity at any point is

$$\vec{q}^2 = V^2 \left( \frac{a + b}{a - b} \right) \frac{\sinh^2(\xi - \alpha)}{\sinh^2 \xi + \sin^2 \eta}$$

and that it has its maximum value $\frac{V(a + b)}{a}$ at the end of the minor axis.

(b) An infinite ocean of an incompressible liquid of density $\rho$ is streaming past a fixed spherical obstacle of radius '$a'$. The velocity is uniform and
equal to \( U \) except in so far as it is disturbed by the sphere, and the pressure in the liquid at a great distance from the obstacle is \( \Pi \). Show that the thrust on that half of the sphere on which the liquid impinges is

\[
\pi a^2 \left( \pi - \frac{1}{16} \rho U^2 \right).
\]

3. (a) Consider two parallel rows of vortices one below the other such that the upper vortices are located at \((0, 0), (\pm a, 0), (\pm 2a, 0)\) .... and the lower vortices are located at \((0, -b), (\pm a, -b), (\pm 2a, -b)\), ..... If each vortex in the upper row has strength \( k \) and each vortex in the lower row is of strength \(-k\), show that the vortex system moves with uniform velocity

\[
\frac{k}{2a} \coth \frac{\pi b}{a}.
\]

(b) The circle \(|z + a| = a\) is placed in an on coming wind of velocity \( U \) and there is a circulation \( k \). Find the complex potential and show that the moment about the origin is \( \rho kaU \), where \( \rho \) being the density of the fluid.
4. \( (a) \) Assuming the necessary stress-strain rate relation, deduce Navier-Stokes equations of motion (in cartesian coordinates) for the incompressible viscous fluid.

\( (b) \) Find the velocity distribution in an incompressible viscous fluid of infinite expanse adjacent to an infinite flat plate which is impulsively started from rest at time \( t = 0 \) and then moves in its own plane with a constant velocity \( U \). Find the thickness of the boundary layer at time \( t \).

5. \( (a) \) Determine the velocity distribution in the steady flow of uniform incompressible viscous fluid between two coaxial circular pipes under the action of a uniform pressure gradient along the common axis of the pipes.

\( (b) \) Deduce Prandtl boundary layer equations in two dimensional flow of a viscous liquid and the corresponding boundary conditions.

6. Answer any one question:

\( (a) \) A circular cylinder of radius \( a \) is fixed across a stream of velocity \( U \) with a circulation \( k \) round
the cylinder. Show that the maximum velocity in the liquid is \( 2U + \frac{k}{2\pi a} \).

(b) Consider the viscous in compressible flow between parallel planes when one plate is fixed and the other is moving with uniform velocity \( u_0 \) in its own plane. Find velocity.

[Internal Assessment : 10 Marks]