M.Sc. 1st Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Ordinary Differential Equations and Special Functions*)

PAPER—MTM-103

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** from **Q. No. 2** to **Q. No. 5**

The figures in the right-hand margin indicate marks

The symbols have their usual meanings

1. Answer any *five* questions : 2×5

(*a*) Write the important features of Sturm-Liouville problems.

(*b*) Under suitable transformation to be considered by you, prove that Legendre differential equation can be reduced to hypergeometric equation.

(*c*) What is meant by singularity of a linear ordinary differential equation ?

(*d*) Define a self-adjoint differential equation.

(*e*) Find the nature of the singular point of the differential equation

$$z^3 \frac{d^2w}{dz^2} + z\frac{dw}{dz} - w = 0.$$

(*f*) Prove that :

$$F(-n; b, b; -z) = (1 + z)^n$$

where $F(a; b, c; z)$ denotes the hypergeometric function.

(*g*) Consider the boundary value problem

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad 0 \leq x \leq \pi$$

subject to $y(0) = 0$, $y(\pi) = 0$. Find the values of λ for which the boundary value problem is solvable.

2. (*a*) Let the Legendre equation

$$(1 - z^2)w''(z) - 2zw'(z) + n(n + 1)w(z) = 0$$

have nth degree polynomial solution $w_n(z)$ such that $w_n(1) = 3$. If

$$\int_{-1}^{1} [w_n^2(z) + w_{n-1}^2(z)]dz = \frac{144}{15}, \text{ then}$$

find the value of n. 5

(*b*) If α and β are the roots of the equation $J_n(z) = 0$ then show that 5

$$\int_0^1 z\, J_n(\alpha z)\, J_n(\beta z)\, dz = \begin{cases} 0\,, & \text{if } \alpha \neq \beta \\ \frac{1}{2}[J_n'(z)]^2, & \text{if } \alpha = \beta \end{cases}$$

3. (*a*) For the nonhomogeneous linear system

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}\vec{x} + \begin{pmatrix} -2 & \sin t \\ 6 & \cos t \end{pmatrix}$$

where $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, find the fundamental matrix of the corresponding homogeneous system and the solution of the given nonhomogeneous system. 6

(*b*) Solve the equation

$$\frac{d^2 y}{dx^2} = f(x), \ 0 \le x \le 1$$

subject to the boundary conditions $y(0) = \alpha$, $y'(1) = \beta$, by using Green's function method. 4

4. Show that if the solution of the differential equation

$$2z \frac{d^2 w}{dz^2} + (3-2z)\frac{dw}{dz} + 2w = 0$$

is expressed in the form

$$w(z) = z^{\rho} \sum_{n=0}^{\infty} a_n z^n,$$

then ρ can take two possible values. Find the relation between a_n and $a_{n+1} (n = 0, 1, ...)$, and show that one solution reduces to a polynomial. 10

5. (*a*) Prove that

$$\int_{-1}^{1} P_m(z) P_n(z) dz = \frac{2}{2n+1} \delta_{nm}$$

where δ_{nm} is the Kroneker delta symbol. 5

(*b*) Using the expression

$$J_\gamma(z) = \frac{1}{2\pi} \int_0^{2\pi} \cos(\gamma\theta - z\sin\theta)\, d\theta$$

show that

$$J_\gamma(0) = 0$$

for γ a non-zero integer, and

$$J_0(0) = 0.$$ 3 + 2

[*Internal Assessment* : 10 Marks]