M.Sc. 4th Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Nonlinear Optimization/Dynamical Oceanology*-II)

PAPER—MTM-404

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Nonlinear Optimization*)

Q. No. 1 is compulsory and answer any *three* from the rest

1. Answer any *five* of the following : 2 × 5

   (*a*) What do you mean by quadratic programming problem ? Give an example.

   (*b*) Define posynomial. Is there any difference between posynomial and polynomial ? Give justification.

(c) What is chance constrained programming technique ?

(d) What do you mean by decomposition principle of Dantzig and wolfe ? What is the advantage of decomposition principle ?

(e) What is the necessity of constraint qualification related with nonlinear programming ?

(f) Define : Nash equilibrium strategy and Nash equilibrium outcome.

2. (a) Solve the following problem by geometric programming :

Minimize

$$z = 5x_1x_2^{-1}x_3^2 + x_1^{-2}x_3^{-1} + 10x_2^3 + 2x_1^{-1}x_2x_3^{-3}$$

$$x_1, x_2, x_3 > 0.$$ 6

(b) State and prove wolfe's duality theorem. 4

3. (a) Prove that a pair $\{y^*, z^*\}$ constitutes a mixed -strategy Nash equilibrium solution to a bimatrix game (A, B) if and only if, there exists a pair (p^*, q^*) such that $\{y^*, z^*, p^*, q^*\}$ is a solution of the following bilinear programming problem :

$$\min_{y,\, z,\, p,\, q} \quad [\, y'Az + y'Bz + p + q \,]$$

Subject to $\quad Az \geq -pl_m$

$\quad B'y \geq -ql_n$

$\quad y \geq 0,\, z \geq 0,\, y'l_m = 1,\, z'l_n = 1.$ 6

(*b*) Define the following : 4

(*i*) Fritz John saddle point problem

(*ii*) Kuhn-Tucker saddle point problem

(*iii*) Minimization problem

(*iv*) Local minimization problem.

4. (*a*) State and prove Fritz-John saddle point necessary optimality theorem. 4

(*b*) Solve the following NLPP by Wolfe's method

Maximize $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

Subject to the constraints :

$x_1 + x_2 \leq 2$

$2x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0.$ 6

5. (*a*) Use the chance constrained programming technique to find an equivalent deterministic LPP to the following stochastic programming problem :

$$\text{Minimize } F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to } P\left[\sum_{j=1}^{n} a_{ij} x_j \le b_i\right] \ge p_i$$

$$x_j \ge 0,\ i, j = 1, 2, .., n$$

where b_i is random variable and p_i are specified probabilities. 7

(*b*) State and prove weak duality theorem. 3

6. (*a*) State and prove Kuhn-Tucker saddle point necessary optimality theorem. 3

(*b*) Let θ be a numerical differentiable function on an open convex set $\Gamma C R^n$. Prove that a necessary and sufficient condition that θ be convex on Γ is that for each

$$x^1, x^2 \in \Gamma, \left[\nabla\theta(x^2) - \nabla\theta(x^1)\right](x^2 - x^1) \ge 0.$$ 4

(c) Use the Kuhn-Tucker conditions to solve the NLPP

Minimize

$$z = f(x_1, x_2) = -x_1^2 - 2x_2^2 + 2x_1 + 3x_2$$

Subject to the conditions

$$x_1 + 3x_2 \le 6$$
$$5x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0.$$ 3

[*Internal Assessment* : 10 Marks]

(*Dynamical Oceanology*-II)

Answer any *four* questions

1. Show that shallow-water equation can be expressed in the form of

$$\frac{dH}{dt} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0. \text{ (usual symbolls)}$$

Hence deduce that the volume remains constant. 8 + 2

2. Deduce the equations of motion of thermal wind. Hence deduce Taylor-Proudman theorem. 6 + 4

3. Discuss Poincare' and Kelvin waves for

$$\alpha = \frac{n\pi}{L}, n = 1, 2, 3, \ldots$$

(α and L have their usual meanings). 10

4. Obtain the solution of the equation of motion for the pure drift currents in a finitely deep, plane, homogeneous layer of fluid which rotates uniformly about a vertical axis. Hence deduce the following : 10

(*i*) The surface current U_s is directed at an angle 45° to the right of the wind stress vector τ in the northern hemisphere.

(*ii*) At a certain depth, the current vector is opposite to U_s.

5. In two-dimensional model of ocean current, solve the problem of viscous boundary layer and show that weak back flow appears close to the external edge of the boundary layer. 10

6. Write down the vertical structure equation and hence show that the higher baro clinic mode will propagate its energy more slowly than the barotropic modes. 10

[*Internal Assessment* : 10 Marks]