PG/III S/MTM-305/12

M.Sc. 3rd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Dynamical Oceanology* - I/ *Advanced Optimization and Operation Research*)

PAPER—MTM-305

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Dynamical Oceanology* - I)

Answer any *five* questions

1. Establish Gibb's relation of thermodynamics in the form

$$Td\eta = d\in + pdv - \sum_{j=1}^{n} \mu_j dm_j$$

where the symbols have their usual meaning. Deduce Gibb's-Duhem relation. 8

2. Establish necessary condition of thermodynamic equilibrium of a finite volume of sea-water in macroscopic motion under stationary conservative external forces. 8

3. State Boussinesq approximation and under this approximation write down the field equations. Hence deduce the basic equation in spherical coordinate for the average fields. 8

4. What is Brunt-Vaisäla frequency ? What role does it play in a stratified medium ? Find out the expression within homogeneous layers of ocean where temperature and salinity very little with depth. 8

5. Supposing the sea-water to be viscous compressible heat-conducting fluid, derive the equation of conservation of energy. 8

6. Derive Fridman's equation for diffusion of absolute vorticity in a viscous flow in terms of motion relative to the earth. Define potential vorticity of a fluid particle. Deduce Ertel's formula for the evaluation of potential vorticity. 5 + 1 + 2

7. Show that the principle of conservation of mass is expressed by the pair of equation

$$\frac{D\rho}{Dt} = +\rho \operatorname{div} \vec{q} = 0$$

$$\rho \frac{Ds}{Dt} = -\operatorname{div} \vec{I}_s$$

in usual notations, assuming sea-water to be a two-component mixture of salt and pure water. 8

8. Write down the equation for small amplitude wave motion in the ocean and hence deduce energy conservation equation. 8

9. Show that the problem of free oscillation of the ocean reduces to be determination of eigen values of two distinct eigen value problem. 8

[*Internal Assessment* : 10 Marks]

(*Advanced Optimization and Operation Research*)

Answer **Q. No. 1** and any *two* from the rest

1. Answer any *one* question : 8 × 1

(*a*) Solve the following IPP by using Gomory's cutting plane method

$$\text{Maximize } Z = -4x_1 + 5x_2$$
$$\text{subject to } \quad -3x_1 + x_2 \le 6$$
$$2x_1 + 4x_2 \le 12$$
$$x_1, x_2 \ge 0 \text{ and integers.}$$

(*b*) Let X_{Br} be a negative basic variable in a dual simplex table and all net evaluations $Z_j - C_j$ be non-negative and the primal LPP is of maximization. If $y_{rj} > 0$ for all non-basic variables x_j, show that there does not exist any feasible solution to the primal LPP.

2. (*a*) Using Kuhn-Tucker conditions

$$\text{Maximize } f(x_1, x_2, x_3) = 13 + 24x_2 - 16x_2^2 - 4x_3^2$$
$$\text{subject to } \quad x_1 + 2x_2 \le 0$$
$$x_1 + 3x_2 \le 2$$

8

(*b*) Maximize $f(x) = \begin{cases} 2\sqrt{x}, & x \le 1 \\ 3 - x, & x > 1 \end{cases}$

in the interval [0, 5] by Fibonacci method taking six experiments. 8

3. (*a*) Solve by revised simplex method

$$\text{Maximize } Z = 2x_1 - 3x_2$$
$$\text{subject to } \quad 4x_1 + x_2 \le 8$$
$$x_1 + 4x_2 \le 8$$
$$x_1, \ x_2 \ge 0.$$ 8

(*b*) Discuss the effect of discrete change in the requirement vector b to the LPP

$$\text{Maximize } Z = CX$$
$$\text{subject to } \quad AX = b, \quad X \ge 0$$

where $C, \ X^T \in R^n, \ b^T \in R^m$ and A is an $m \times n$ matrix. 8

4. (*a*) Using Steepest Descent Method

Minimize $f(x_1, x_2) = x_1^2 + x_2^2 + 2gx_1 + 2fx_2 + c$

starting from the point (α, β). 8

(*b*) Solve the following GPP graphically

$$\text{Minimize } Z = P_1(d_1^+ + d_2^+) + P_2(30d_3^- + 40d_4^-) + P_3 d_5^-$$

subject to constraints

$$2x_1 + 4x_2 + d_1^- - d_1^+ = 80$$

$$3x_1 + 3x_2 + d_2^- - d_2^+ = 80$$

$$x_1 + d_3^- - d_3^+ = 10$$

$$x_2 + d_4^- - d_4^+ = 10$$

$$30x_1 + 40x_2 + d_5^- - d_5^+ = 1200$$

$$x_1, x_2, d_i^-, d_i^+ \geq 0, \; i = 1, 2, 3, 4, 5.$$ 8

[*Internal Assessment* : 10 Marks]