# M.Sc. 2nd Semester Examination, 2012

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM- 204

( *Continuum Mechanics* )

*Full Marks* : 50

*Time* : 2 hours

Answer **Q. No. 1** and any **four** from the rest

*The figures in the right hand margin indicate marks*

1. Answer any *two* questions : 4 × 2

(*a*) The stress tensor at a point $P$ are given by

$$(T_{ij}) = \begin{pmatrix} 3 & 2 & 5 \\ 2 & 1 & -4 \\ 5 & -4 & 1 \end{pmatrix}.$$

Find the stress vector at $P$ on the plane whose normal has direction ratios $1 : 2 : -3$.

( *Turn Over* )

(*b*) Find the image of a source with respect to a circle.

(*c*) If the strain tensor at a point is given by

$$e_{ij} = \begin{bmatrix} 1 & -3 & \sqrt{2} \\ -3 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{bmatrix}.$$

Calculate the principal strains and determine the shear between the directions $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

2. What is strain deformation ? Explain the geometrical interpretation of small strain components. 1 + 7

3. State and prove the Cauchy's first equation of motion. Deduce the equation of equilibrium when the continuum is in static equilibrium. 1 + 6 + 1

4. Define source and sink in two dimension. If the fluid fills the region of space on the positive side of $x$-axis, which is a rigid boundary and if there be a source +ve at the point $(0, a)$ and an equal sink at $(0, b)$, and if the

pressure on the negative side of the boundary be the same as the pressure of the fluid at infinity, show that the resultant pressure on the boundary is

$$\frac{\pi\rho m^2(a-b)^2}{ab(a+b)}$$

where $\rho$ is the density of the fluid. 1 + 7

5. What is the concept of stress vector ? Prove that the stress vector at a point on any arbitrary plane surface is a linear function of three stress vectors acting on any three mutually perpendicular planes through that point. 2 + 6

6. What do you mean by perfect fluid ? State and prove the Kelvin's minimum energy theorem for perfect fluid. 1 + 1 + 6

7. Derive the Euler's equation of motion of a perfect fluid. Hence obtain the Bernoulli's equation in its most general form. 8

[*Internal Assessment* : 10 Marks ]