M.Sc. 2nd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Abstract Algebra/Linear Algebra*)

PAPER — MTM- 203

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

GROUP — A

(*Abstract Algebra*)

[*Marks* : 25]

Answer **Q.No.1** and any *two* questions from the rest

1. Answer any *two* questions : 2×2

   (*a*) Define quotient group G with one example. 2

   (*b*) What do you mean by principal ideal domain ? 2

(*c*) When an integral domain R is called unique factorisation domain ? 2

2. (*a*) State and prove Sylow's 1st theorem for a finite group G. 5

(*b*) If F is a field, then prove that $F[x]$ is an Euclidean domain. 3

3. (*a*) State and prove Cauchy's theorem for a finite group G. 5

(*b*) Let R be a commutative ring with 1. Show that an ideal M is a maximal ideal iff R/M is a field. 3

4. (*a*) Let R be a principal ideal domain. Then, prove that every $a \in R$ which is not a unit can be expressed as a product of irreducible elements. 3

(*b*) State and prove Ist Isomorphism theorem of a ring R. 5

[*Internal Assessment* – 5 Marks]

GROUP – B

(*Linear Algebra*)

[*Marks* : 25]

Answer **Q.No.5** and any *two* questions from the rest

5. Answer any *two* questions : 2 × 2

(*a*) Define the following : Sublattice, Semilattice.

(*b*) Justify with reason whether the following statement is *true* or *false* :
"Let T be a linear operator on a vector space V over a field such that T has n distinct eigen values where n = dim (v). Then the degree of the minimal polynomial of T equals n ."

(*c*) Let $T: U \to V$ be a subjective linear mapping and dim $U = 6$, dim $V = 3$. Find dim ker T.

6. (*a*) State and prove Sylvester law on linear transformation. 4

(*b*) A lattice L is distributive iff

$$(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \; \forall \, a, \; b \, , c \in L.$$ 4

7. (*a*) Let V and W be vector spaces over a field F. Prove that a linear mapping $T : V \to W$ is invertible if and only if T is one - to-one and onto. 4

(*b*) The matrix of a linear mapping $T : R^3 \to R^2$ relative to the ordered bases ((0,1,1), (1, 0, 1), (1, 1, 0)) of R^3 and ((1,0), (1, 1)) of R^2 is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$. Find the matrix of T relative to the ordered bases ((1,1,0), (1, 0, 1), (0, 1, 1)) of R^3 and ((1,1), (0, 1) of R^2. 4

8. (*a*) Find the minimal polynomial of the following matrix where α, β, γ are arbitary scalars. 4

$$A = \begin{bmatrix} 1 & \alpha & \beta \\ 0 & 1 & \gamma \\ 0 & 0 & 1 \end{bmatrix}$$

(*b*) A linear mapping $\phi : R^3 \to R^3$ maps the vectors (0,1,1), (1,0,1) and (1, 1, 0) to (2,1,1), (1,2, 1) and (1, 1, 2) respectively. Examine whether ϕ is an isomorphism or not. 4

[*Internal Assessment* – 5 Marks]