M.Sc. 2nd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM- 202

(Numerical Analysis)

Full Marks : 50

Time : 2 hours

Answer Q. No. 1 and any two from the rest

The figures in the right hand margin indicate marks

1. Answer any four questions : 2 × 4

(a) Distinguish between implicit and explicit methods for solving an ODE using numerical method.

(b) Prove that:

\[ \mu \delta f(x) = \frac{\Delta + \nabla}{2} f(x), \]

where the symbols have their usual meanings.

(c) Is power method be used for finding least eigenvalue (in magnitude) of a matrix? Explain.

(Turn Over)
(d) Find the weights \( w_1, w_2, w_3 \) so that the relation
\[
\int_{-1}^{1} f(x)dx = w_1 f(-\sqrt{0.6}) + w_2 f(0) + w_3 f(\sqrt{0.6})
\]
is exact for the functions \( f(x) = 1, x, x^2 \).

(e) Show that the Chebyshev polynomials \( T_n(x) \) is a polynomial in \( x \) of degree \( n \). Also show that \( T_n(x) \) even or odd according as \( n \) is even or odd.

(f) How can the improper integral
\[
\int_{0}^{1} \frac{dx}{x^2}
\]
be integrated using numerical method?

2. (a) Describe Seidal method for solving the non-linear system of equations \( f(x, y, z) = 0, g(x, y, z) = 0 \) and \( h(x, y, z) = 0 \) starting from an initial guess \((x_0, y_0, z_0)\). Use this method for solving the following equations
\[
x = \frac{8x - 4x^2 + y^2 + 1}{8} \quad \text{and} \quad 4y = 2x - x^2 + 4y - y^2 + 3
\]
starting with \((x_0, y_0) = (1.1, 2.0)\), correct up to four decimal places.
(b) Describe Birge-Vieta method for finding all roots of a polynomial equation.

3. (a) Describe finite difference method for solving the following boundary value problem:

\[ y'' + p(x) y' + q(x)y = r(x), \quad a < x < b \]

with boundary conditions \( y(a) = \gamma_1 \) and \( y(b) = \gamma_2 \).

(b) Describe Jacobi's method for finding all eigenvalues and eigenvectors of a real symmetric matrix.

4. (a) Describe LU-decomposition method for solving a system of linear equations. What are the merits and demerits of this method?

(b) Deduce 4-point Gauss-Chebyshev quadrature formula. Use this method for finding the value of

\[ \int_1^2 (x^3 + \log x) \, dx. \]

[Internal Assessment: 10 Marks]