M.Sc. 2nd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM- 202

(*Numerical Analysis*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

The figures in the right hand margin indicate marks

1. Answer any *four* questions : 2 × 4

(*a*) Distinguish between implicit and explicit methods for solving an ODE using numerical method.

(*b*) Prove that :

$$\mu\delta f(x) = \frac{\Delta + \nabla}{2} f(x),$$

where the symbols have their usual meanings.

(*c*) Is power method be used for finding least eigen-value (in magnitude) of a matrix ? Explain.

(*d*) Find the weights w_1, w_2, w_3 so that the relation

$$\int_{-1}^{1} f(x)dx = w_1 f\left(-\sqrt{0\cdot 6}\right) + w_2 f(0) + w_3 f\left(\sqrt{0\cdot 6}\right)$$

is exact for the functions $f(x) = 1, x, x^2$.

(*e*) Show that the Chebyshev polynomials $T_n(x)$ is a polynomial in x of degree n. Also show that $T_n(x)$ even or odd according as n is even or odd.

(*f*) How can the improper integral

$$\int_0^1 \frac{dx}{x^2}$$

be integrated using numerical method ?

2. (*a*) Describe Seidal method for solving the non-linear system of equations $f(x, y, z) = 0$, $g(x, y, z) = 0$ and $h(x, y, z) = 0$ starting from an initial guess (x_0, y_0, z_0). Use this method for solving the following equations

$$x = \frac{8x - 4x^2 + y^2 + 1}{8} \text{ and}$$

$$4y = 2x - x^2 + 4y - y^2 + 3$$

starting with $(x_0, y_0) = (1\cdot 1, 2\cdot 0)$, correct upto four decimal places.

(*b*) Describe Birge-Vieta method for finding all roots of a polynomial equation. (2 + 6) + 8

3. (*a*) Describe finite difference method for solving the following boundary value problem :

$$y'' + p(x)\,y' + q(x)y = r(x),\ \ a < x < b$$

with boundary conditions $y(a) = \gamma_1$ and $y(b) = \gamma_2$.

(*b*) Describe Jacobi's method for finding all eigenvalues and eigenvectors of a real symmetric matrix. 8 + 8

4. (*a*) Describe LU-decomposition method for solving a system of linear equations. What are the merits and demerits of this method ?

(*b*) Deduce 4-point Gauss-Chebyshev quadrature formula. Use this method for finding the value of

$$\int_1^2 (x^3 + \log x)\,dx.$$ 8 + (5 + 3)

[*Internal Assessment* : 10 Marks]