 PG/IS/MTM-105/11

M.Sc. 1st Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-105

(*Classical Mechanics*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** questions from the rest

The figures in the right-hand margin indicate marks

1. Answer any *four* questions : 2 × 4

(*a*) What do you mean by non-inertial frame ? Give an example of a non-inertial frame explaining why it is non-inertial ?

(*b*) What do you mean by generalised forces ? Find an expression of it in terms of generalised coordinates.

(*Turn Over*)

(*c*) Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H is the Hamiltonian function.

(*d*) What do you mean by canonical transformation ? Explain.

(*e*) Define poisson brackets. Show that it does not satisfy commutitive property.

(*f*) Write Hamilton-Jacobi's equation. What is the significance of the solution of this equation ?

2. (*a*) Deduce the Euler's dynamical equations when a rigid body is rotating about a fixed point. **8**

(*b*) In a dynamical system of two degrees of freedom, the kinetic energy

$$T = \frac{1}{2} \frac{\dot{q}_1^2}{a + b q_2^2} + \frac{1}{2} \dot{q}_2^2$$

and potential energy $V = c + dq_2^2$. Find q_1, q_2 where a, b, c, d are constants. **8**

3. (*a*) Derive the Hamilton's equations of motion from the variational principle. 5

(*b*) Find the least value of the integral

$$\int_A^B \frac{1}{y}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} dx$$

where A is $(-1, 1)$ and B is $(1, 1)$. 6

(*c*) Prove that the poisson bracket is invariant under canonical transformation. 5

4. (*a*) Derive the Lagrange's equation for conservative unconnected holonomic system. 8

(*b*) Show that with respect to a uniformly rotating reference of frame Newton's second law for a particle of mass m acted upon by real force $\overline{F}$ can be expressed as

$$\overline{F}_{\text{eff}} = \overline{F} - 2m\overline{w} \times \overline{V}_{\text{rot}} - m\overline{w}x\,(\overline{w} \times \overline{r}).$$

Assume that the origins of the inertial and non-inertial coordinates systems are coincident. $\overline{F}_{eff}$ and $\overline{V}_{rot}$ represent effective force and velocity with respect to rotating frames. 8

[*Internal Assessment* : 10 Marks]