 PG/IIS/MA-1203/10

M.Sc. 2nd Semester Examination, 2010

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA-1203

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

GROUP—A

(*Abstract Algebra*)

[*Marks* : 25]

1. Answer any *two* questions : 2 × 2

   (*a*) Define external direct product of the groups $G_1, G_2, \ldots G_n$.

   (*b*) State Cauchy's theorem for finite groups.

(c) If a group G has only one p-Sylow group H, then H is normal subgroup in G.

2. Answer any *two* questions : 8 × 2

(a) (i) Define Conjugacy relation in a group G. Prove that two elements x, y of a group G give rise to the same conjugate of an element $a \in G$ if and only if they belong to the same right coset of the normalizer of a in G.

(ii) Define ideal of a ring R. Prove that a commutative ring with unity is without proper ideal, if it is a field. 4 + 4

(b) (i) Define Euclidean domain. Let D be a Euclidean domain. Then prove that any two elements a and b in D have greatest common divisor d which can be expressed in the form $d = \lambda a + \mu b$ for $\lambda, \mu \in D$.

(ii) Define automorphism of a group G. Prove that the set of all automorphisms of a group G themselves forms a group. 4 + 4 (*c*) (*i*) Define solvable group with examples.

(*ii*) Let R be a commutative ring with unity and H be an ideal of R. Prove that R/H is a field if and only if H is maximal. 3 + 5

[*Internal Assessment* : 5 Marks]

GROUP – B

(*Linear Algebra*)

[*Marks* : 25]

Answer **Q. No. 4** and any **two** questions from the rest

4. Answer any *two* questions : 2 × 2

(*a*) Define the following :

Null space and Range of a linear mapping.

(*b*) What do you mean by annihilate ? Define monic polynomial.

(*c*) Define Poset with an example.

5. (*a*) State and prove dimension theorem associated with linear transformation.

(*b*) Define median inequality for any lattice L and then prove it.

(*c*) Find the minimal polynomial of the following real matrix :

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}$$

4 + 2 + 2

6. (*a*) Let $p(t)$ be a minimal polynomial of a linear operator T on a finite dimensional vector space.

(*i*) For any polynomial $g(t)$, if $g(T) = T_0$, then prove that $p(t)$ divides $g(t)$ and in particular, $p(t)$ divides the characteristic polynomial of T.

(*ii*) Also prove that the minimal polynomial of T is unique.

(*b*) Let $T : P_3[0, 1] \rightarrow P_2[0, 1]$ be defined by $(Tp)(x) = p''(x) + p'(x)$. Then find the matrix represented by T wtih respect to the bases $\{1, x, x^2, x^3\}$ and $\{1, x, x^2\}$ of $P_3[0, 1]$ and $P_2[0, 1]$ respectively.

(*c*) Suppose that $T : R^2 \rightarrow R^2$ is linear, $T(1, 0) = (1, 4)$ and $T(1, 1) = (2, 5)$. What is $T[2, 3]$? Is T one-to-one ? 3 + 3 + 2

7. (*a*) Let V and W be vector spaces over a field F. Prove that a linear mapping $T : V \rightarrow W$ is invertible if and only if T is one-to-one and onto.

(*b*) Let $T : R^3 \rightarrow R^3$ be a linear transformation defined by $T(x, y, z) = (x + y, y + z, z + x)$, then, find an orthonormal basis for the range of T.

(*c*) Prove that every chain is a lattice. Examine that the poset $D_{12} = \{2, 3, 4, 6\}$ under divisibility 12 forms a lattice or not ? $4 + 2 + 2$

[*Internal Assessment* : 5 Marks]