M.Sc 1st Semester Examination, 2010

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Classical Mechanics*)

PAPER—MA - 1105

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **two** questions from the rest

The figures in the right-hand margin indicate marks

1. Answer any *four* questions : 2 x 4

(*a*) What do you mean by generalised coordinates and generalised momenta ?

(*b*) Coriolis force does not contribute to the energy equation — Justify.

(*c*) Write a brief note on moving frames of reference.

(*d*) The damped oscillator equation

$\ddot{q} = -q - \gamma \dot{q}$, $\gamma > 0$ is equivalent to the system $\dot{q} = p \exp(-\gamma t)$, $\dot{p} = -q \exp(\gamma t)$. Verify that these equations are Hamilton's equations with the Hamiltonian

$$H = \frac{1}{2}[p^2 \exp(-\gamma t) + q^2 \exp(\gamma t)].$$

(*e*) State the axioms of special theory of relativity.

(*f*) Write down the equation of motion of a one-dimensional harmonic oscillator. Deduce the equation of energy.

2. (*a*) Derive the differential equations of the lines of propagation of light in an optically non-homogeneous medium with speed of light $c(x, y, z)$. Also, discuss the case when c is constant. 8

(*b*) Obtain Hamilton's equations of motion of a system having n degrees of freedom. Obtain the equation of energy. 6 + 2

3. (*a*) Obtain the Lagrange's equations of motion for a symmetric top. 8

(*b*) Show that the following transformation is Canonical : 5

$$Q_1 = \frac{1}{\sqrt{2}}\left(q_1 + \frac{p^2}{mw}\right),\ P_1 = \frac{1}{\sqrt{2}}(p_1 - mw\, q_2)$$

$$Q_2 = \frac{1}{\sqrt{2}}\left(q_1 - \frac{p^2}{mw}\right),\ P_2 = \frac{1}{\sqrt{2}}(p_1 + mw\, q_2).$$

(*c*) If X and Y are constants of motion, show that $[X, Y]$ is also a constant of motion. 3

4. (*a*) Derive the Lorentz transformation equations in relativistic mechanics. 7

(*b*) Define cyclic coordinates. Show that a dynamical problem with n degree of freedom, which has k cyclic coordinates, can be reduced to a dynamical problem which has only $(n - k)$ degrees of freedom. 5

(*c*) If all the coordinates of a system are cyclic prove that the coordinates may be found by integration. 4

[*Internal Assessment* — 10 Marks]
