PG/IS/MA-1101/10

M.Sc 1st Semester Examination, 2010

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Real Analysis*)

PAPER—MA-1101

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **three** from **Q.No.2** to **Q.No.6**

The figures in the right-hand margin indicate marks

1. Answer any *one* question : 1

(*a*) Find the total variation of the function

$f(x) = 3x + 2$ over the interval [2, 5].

(*b*) If $m^*(A) = 0$, find $m(A)$.

2. (*a*) Show that a function of bounded variation is expressible as a difference of two monotone increasing functions. 7

(*b*) Prove that product of two functions of bounded variation is also of bounded variation. 6

(*Turn Over*)

3. (*a*) If f is of bounded variation on $[a, b]$ then show that it is also of bounded variation on $[a, c]$ and on $[c, b]$, where $a < c < b$ and conversely. Also show that

$$V(f, a, b) = V(f, a, c) + V(f, c, b).$$ 8

(*b*) Show that

$$f(x) = \begin{cases} \sin\frac{\pi}{3x}, & 0 < x \leq 1 \\ 0 & , \quad x = 0 \end{cases}$$

is not of bounded variation over the interval $[0, 1]$. 5

4. (*a*) If $f(x)$ is Riemann–Stieltjes integrable with respect to $g(x)$ on $[a, b]$ and $c \in (a, b)$ then prove that $f(x)$ is Riemann–Stieltjes integrable with respect to $g(x)$ on $[a, c]$ as well as on $[c, b]$. Also prove that

$$\int_a^b f(x)\, dg(x) = \int_a^c f(x)\, dg(x) + \int_c^b f(x)\, dg(x).$$

Show also by an example that the converse of this theorem is not true. 9

(*b*) Evaluate the Riemann–Stieltjes integral

$$\int_{1}^{3} (2x^2+3)\, d([x]+2).$$ 4

5. (*a*) If $f(x)$ is Riemann–Stieltjes integrable with respect to $g(x)$ on $[a, b]$ and if $g(x)$ has a continuous derivative $g'(x)$ on $[a, b]$ then prove that the Riemann integral

$$\int_{a}^{b} f(x)\, g'(x)\, dx \text{ exists and}$$

$$\int_{a}^{b} f(x)\, dg(x) = \int_{a}^{b} f(x)\, g'(x)\, dx.$$ 7

(*b*) Prove that Cantor set is uncountable but has measure zero. 6

6. (*a*) Let $f(x)$ be bounded and Lebesgue integrable function on $[a, b]$ and $g(x)$ be a bounded function on $[a, b]$ such that

$$f(x) = g(x) \text{ a.e on } [a, b].$$

Prove that $g(x)$ is Lebesgue integrable on $[a, b]$ and

$$\int_a^b f(x)\,dx = \int_a^b g(x)\,dx.$$ 7

(*b*) Let $f(x)$ be defined as

$$f(x) = \begin{cases} \dfrac{1}{x^{4/5}}, & 0 < x \leqslant 1 \\ 0, & x = 0. \end{cases}$$

Show that $f(x)$ is Lebesgue integrable on $[0, 1]$ and find the value of the integral

$$\int_0^1 f(x)\,dx.$$ 6

[*Internal Assessment*: 10 Marks]