2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Numerical Analysis*)

PAPER—MA - 1202

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Answer **Q.No.1** and any *two* from the rest

1. Answer any *four* questions : 2×4

(*a*) Find the values of $\left(\frac{\Delta^2}{E}\right) x^2$.

(*b*) What are the differences between Cotes' quadrature and Gaussian quadrature ?

(*c*) What do you mean by multistep method to solve a differential equation ?

(*d*) Explain ill condition system of linear equations with an example.

(*e*) Explain initial value and boundary value problems in connection with ordinary differential equation.

(*f*) Find the weights w_1, w_2, w_3 so that the relation

$$\int_{-1}^{1} f(x)\,dx = w_1 f(-\sqrt{0{\cdot}6}) + w_2 f(0) + w_3 f(\sqrt{0{\cdot}6})$$

is exact for the functions $f(x) = 1, x, x^2$.

2. (*a*) Describe Jacobi's method to find all eigenvalues and eigenvectors of a real symmetric matrix. 7

(*b*) (*i*) Deduce three points Gauss-Legendre quadrature formula. 6

(*ii*) Use this method to find the value of

$$\int_0^1 \frac{1}{1+x^2}\,dx.$$

3

3. (*a*) Describe fourth order Runge-Kutta method to solve the following initial value problem

$$a(x)\ y'' + b(x)\ y' + c(x)\ y = f(x),$$

with initial conditions

$$x = x_0,\ y(x_0) = y_0,\ y'(x_0) = z_0. \quad 5$$

(*b*) Describe a suitable finite difference method to solve the following parabolic equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

with initial condition $u(x, 0) = f(x)$ and boundary condition $u(0,\ t) = \phi(t)$, $u(1,\ t) = \psi(t)$. 8

(*c*) Prove that

$$\frac{\delta^2}{2} + \delta \sqrt{1 + \frac{\delta^2}{4}} \equiv E - 1,$$

the symbol have their usual meaning. 3

4. (*a*) Describe Cubic spline method to interpolate the set of observations (x_i, y_i), $i = 0, 1, 2, \ldots, n$. 8

(*b*) Define tri-diagonal system of linear equations. Describe an efficient method to solve such system of equations. Solve the following tri-diagonal system of equations :

$$x_1 + x_2 = 3,\ -x_1 + 2x_2 + x_3 = 6,$$
$$3x_2 + 2x_3 = 12.$$

5 + 3

[*Internal Assessment*: 10 Marks]
