# M.Sc 1st Semester Examination 2009

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

( *Ordinary Differential Equations and Special Functions* )

PAPER—MA - 1103

*Full Marks* : 50

*Time* : 2 hours

Answer **Q.No.1** and any **three** questions from the rest

*The figures in the right-hand margin indicate marks*

**1.** Answer any *five* of the following : 2×5

( *a* ) What are meant by regular and irregular singularities of the differential equation

$$\frac{d^2\omega}{dz^2} + p(z)\frac{d\omega}{dz} + q(z)\omega = 0.$$

( *b* ) What do you mean INDICIAL equation concerning ODE ?

(c) Show that infinity is not a regular singular point for the Bessel equation.

(d) Define orthogonal functions associated with Sturm-Liouville problem.

(e) Define Green's function involving ODE.

(f) Define fundamental set of solutions and fundamental matrix for system of differential equation.

2. (a) For the regular Strum–Liouville system

$$\frac{d^2u}{dx^2} + \lambda u = 0,$$

$$u(0) = \frac{du}{dx}(\Pi) = 0,\ 0 \leqslant x \leqslant \pi,$$

find the eigenvalues and eigenfunctions and obtain an expansion formula for a function $f \in C^1$ into a series of eigenfunctions.

(*b*) Let $\omega_1(z)$ and $\omega_2(z)$ be two solutions of $(1-z^2)\ \omega''(z)-2z\omega'(z)+(\sec z)\omega=0$ with Wronskian $W(z)$. If $\omega_1(0)=1$, $\omega_1'(0)=0$ and $W\left(\frac{1}{2}\right)=\frac{1}{3}$, then find the value of $\omega_2'(z)$ at $z=0$.

(*c*) Can the matrix

$$\begin{bmatrix} e^{4t} & 0 & 2e^{4t} \\ 2e^{4t} & 3e^{t} & 4e^{4t} \\ e^{4t} & e^{t} & 2e^{4t} \end{bmatrix}$$ be a fundamental

matrix of the system $A=\begin{bmatrix} -3 & 2 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$?

If not, find the fundamental matrix $x'=Ax$. 5 + 3 + 2

3. (*a*) Let $P_n(z)$ be the Legendre polynomial of degree $n$ such that $P_n(1)=1$, $n=1, 2, 3, \ldots$. If

$$\int_{-1}^{1}\left(\sum_{j=1}^{2}\sqrt{j(2j+1)}\,P_j(z)\right)^2 dz=20,$$

then find the value of $n$.

(*b*) Discuss the solution procedure for solving the homogeneous vector differential equation in the form $\frac{dx}{dt} = Ax$, where $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ and $A = (a_{ij})_{n \times n}$ matrix. 3 + 7

4. (*a*) Deduce Rodrigues formula for Legendre polynomial.

(*b*) Using Green's function method, solve the equation

$$\frac{d^2 u}{dx^2} = f(x), 0 \leqslant x \leqslant 1$$

subject to the boundary conditions $u(0) = u'(0)$ and $u(1) = -u'(1)$.

(*c*) Establish integral representation of confluent hypergeometric function. 3 + 5 + 2

5. ( *a* ) Find the series solution of the differential equation

$$z^2 \frac{d^2\omega}{dz^2} + z\frac{d\omega}{dz} + (z^2 - \gamma^2)\omega = 0,$$

where $\gamma$ is constant (real or complex).

( *b* ) Establish the orthogonality property for Legendre polynomials. 5 + 5

[ *Internal Assessment* — 10 Marks ]

---