2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Continuum Mechanics*)

PAPER—MA - 1204

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Answer **Q.No.1** and any *four* from the rest

1. Answer any *two* questions : 4x2

(*a*) What is the concept of stream function and complex potential ?

(*b*) The stress tensor at a point P are given by

$$(T_{ij}) = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 4 & 0 \\ 2 & 0 & 2 \end{pmatrix}.$$

Find the stress vector at P on the plane whose normal has direction ratios $1 : -3 : 2$.

(*c*) If the velocity of an incompressible fluid at the point (x, y, z) be

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5} \right),$$ then determine

the stream line where $r^2 = x^2 + y^2 + z^2$.

2. State and prove the Kelvin's Minimum Energy theorem. 8

3. Prove that the principal stress values are all real and the corresponding principal stress directions are mutually orthogonal. 8

4. Derive the Euler's equation of motion of a perfect fluid. 8

5. Calculate the strain invariants from strain tensor.

$$(E_{ij}) = \begin{pmatrix} 5 & -1 & -1 \\ -1 & 4 & 0 \\ -1 & 0 & 4 \end{pmatrix}.$$

Determine the principal strains. Obtain strain invariants from them. Show the equivalence of strain invariants. 8

6. Analyze the relative displacement in strain deformation and hence define small rotation vector and small rotation tensor. 8

7. (*a*) Differentiate between :

(*i*) Steady and Unsteady motion

(*ii*) Stream line and Path line.

(*b*) Find the condition for a given surface $F(x, y, z, t) = 0$ to be a boundary surface of a fluid motion. 3 + 5

[*Internal Assessment* : 10 Marks]