M.Sc 1st Semester Examination, 2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Classical Mechanics*)

PAPER—MA - 1105

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **two** questions from the rest

The figures in the right-hand margin indicate marks

1. Answer any *four* questions : 2x4

(*a*) What do you mean by inertial and non-inertial frames. Give examples of these frames.

(*b*) Define holonomic constraints with example.

(*c*) What is Coriolis force ? What is the cause of this force ?

(*d*) Find the expression for kinetic energy when a rigid body is rotating about a fixed point.

(*e*) State Hamilton principle.

(*f*) Deduce Hamilton's equations in terms of Poisson brackets.

2. (*a*) Deduce Lagrange's equation of motion for a conservative and unconnected holonomic system. 8

(*b*) If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, show by direct substitution that

$$L' = L + \frac{dF}{dt}(q_1, q_2, \ldots, q_n, t)$$

also satisfies Lagrange's equations where F is any arbitrary, but differentiable function of its arguments. 4

(*c*) Set up a Lagrangian in spherical polar coordinates for two-body problem under central force. 4

3. (*a*) A body moves about a point 0 under no force, the principal moments of inertia at 0 being $3A$, $5A$ and $6A$. Initially, the angular velocity has components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = n$ about the corresponding principal axes. Show that at any time t

$$\omega_2 = \frac{3n}{\sqrt{5}} \tan h \left(\frac{nt}{\sqrt{5}} \right)$$

and that the body ultimately rotates about the mean axis. 8

(*b*) Let $Q_j = Q_j(q_j, p_j, t)$ and $P_j = P_j(q_j, p_j, t)$ be the Canonical transformation and $G_1 = G_1(q_j, Q_j, t)$ be a generating function. Show that

$$p_j = \frac{\partial G_1}{\partial q_j}, \; P_j = -\frac{\partial G_1}{\partial Q_j} \text{ and}$$

$$\bar{H} = H + \frac{\partial G_1}{dt}$$

where H and $\bar{H}$ represent respectively Hamiltonians in old and new coordinates systems. 4

(*c*) Prove that the transformation

$$Q = \log\left(\frac{1}{q}\sin p\right),\ P = q\cot p$$

is Canonical. 4

4. (*a*) Prove that

$$J = \int_{x_0}^{x_1} F(y_1, y_2, \ldots, y_n, y_1', y_2', \ldots, y_n', x)\,dx$$

will be stationary only if

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y_j'}\right) - \frac{\partial F}{\partial y_j} = 0,\ j = 1, 2, \ldots, n$$

where $y_j' = \dfrac{\partial y_j}{\partial x}$, y_1, $y_2, \ldots,$ y_n are functions of x only. 8

(*b*) Show that in relativistic mechanics $E = mc^2$. 5

(*c*) Calculate the mean life-time of a particle moving with speed $2 \cdot 67 \times 10^{10}$ cm/sec whose proper life-time is $2 \cdot 8 \times 10^{-7}$ sec. 3

[*Internal Assessment*— 10 Marks]