M.Sc Third Semester Examination 2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA-2105

(*Dynamical Oceanology—I/ Advanced Optimization and Operations Research*)

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Dynamical Oceanology—I*)

Answer any *five* questions

1. Define salinity of sea-water. Derive the following relations :

(*i*) $C_v = C_p + T\left\{\left(\frac{\partial\tau}{\partial T}\right)^2 / \frac{\partial\tau}{\partial p}\right\}$

(Turn Over)

(*ii*) $K_\eta = K_T - \Gamma \cdot \alpha = K_T \left(\frac{C_v}{C_p} \right)$

where symbols have their usual meanings. 2 + 3 + 3

2. Find the condition of stability of equilibrium of a stratified fluid and hence obtain an expression of the Brünt - Väisälä frequency for the layers where temperature and salinity variations with depth, are large. 4 + 4

3. Obtain the boundary conditions at the free ocean surface $F(\vec{r}, t) = 0$ of pure water, assuming that the mass exchange across the free ocean surface amount to a flux b, in unit time per unit area. 8

4. Obtain Reynolds equations for ocean currents by averaging the Boussinesq's equations. Deduce the dynamic and kinematic boundary conditions for these equations. 5 + 3

5. Obtain the equation of motion of sea–water in the form

$$\frac{d\vec{q}}{dt} = \vec{F} + 2\vec{q} \times \vec{\Omega} - \frac{1}{\varrho}\nabla p + \frac{1}{\varrho}(\lambda + \mu)\nabla \textcircled{H} + \nu\nabla^2 \vec{q},$$

where the symbols have their usual meanings. 8

6. Explain β–plane approximation. Assuming the sea–water to be a non–viscous trarified fluid, deduce the β–plane equations and examine the range of validity of these equations. 1 + 6 + 1

7. Assuming the sea-water to be a viscous compressible heat conducting fluid, determine the energy equation in the form

$$\frac{\partial}{\partial t}(\zeta E_m) = -\text{div}\ \vec{I}_E .$$

where symbols have their usual meanings. 8

8. Derive the equations for small amplitude wave motion in the ocean. 8

[*Internal Assessment*— 10 Marks]

(*Advanced Optimization and Operations Research*)

Answer **Q. No. 1** and any *two* from the rest

1. Answer any *one* question: 8 × 1

(*a*) Solve the following LPP by using revised simplex method:

$$\text{Maximize } Z = 4x_1 + 7x_2 + 2x_3$$
$$\text{subject to } x_1 + 2x_2 + x_3 \leq 12$$
$$2x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0.$$

(*b*) Use Gomory's cutting plane method to find the optimal solution of the IPP

$$\text{Maximize } z = 4x_1 + 3x_2$$
$$\text{subject to } 3x_1 + 4x_2 \leq 12$$
$$4x_1 + 2x_2 \leq 9$$
$$x_1, x_2 \geq 0 \text{ and are integers.}$$

2. (*a*) The optimal basic feasible solution of a linear programming problem is x_B^*. If now one of the prices C_B, say c_k, is changed to a new value given by

$$\hat{c}_k = c_k + \delta_k$$

find the range in which δ_k will lie so that x_B^* remains optimal. 8

(*b*) Solve the following LPP using simplex method

$$\text{Maximize } z = 2x_1 + 5x_2$$
$$\text{subject to } 4x_1 + x_2 \leq 16$$
$$2x_1 + x_2 \leq 12$$
$$x_1, x_2 \geq 0$$

Using the optimal table find the optimal solution when the requirement vector is changed from $[16, \ 12]^T$ to $[10, \ 20]^T$. 8

3. (*a*) Solve the following problem using Kuhn – Tucker conditions: 8

$$\text{Maximize} \quad z = x_1^2 + 6x_1 + 5x_2$$
$$\text{subject to} \quad x_1 + 2x_2 \leqslant 10$$
$$x_1 + 3x_2 \leqslant 9.$$

(*b*) Using Golden Section method find the interval of uncertainty after four experiments for the problem:

$$\text{Maximize } f(x) = \begin{cases} 2x+3, & x \leqslant 4 \\ 15 - x, & x > 4 \end{cases}$$

if the initial interval of uncertainty is [1 , 6]. 8

4. (*a*) Using Davidon Fletcher Powell method

$$\text{Minimize } f(x_1, x_2) = 3x_1^2 + 4x_2^2 - 6x_1 + 24x_2 + 44$$

starting from the point $[2, 1]^T$. 8

(*b*) Solve graphically the following Goal programming problem : 8

A firm manufactures two products A and B. The pofits are Rs. 3 and Rs.4 respectively for each kg of products. The firm has two machines and the required processing time in hours for each machine on each product is given in the following table. Machines X and Y have 20 and 22 machine hours respectively.

		Products A	Products B
Machines	X	4	3
	Y	2	5

The management of the firm has established the following goal priorities :

Priority 1 : To meet production goal of 3 kg of A and 2 kg of B.

Priority 2 : To maximize profit.

[*Internal Assessment* — 10 Marks]