

Total Pages—9

PG/I/MATH/III/07

2007

**APPLIED MATHEMATICS WITH OCEANOLOGY
AND COMPUTER PROGRAMMING**

PAPER-III

Full Marks : 100

Time : 4 hours

The figures in the right-hand margin indicate marks

*Candidates are required to give their answers in their
own words as far as practicable*

Illustrate the answers wherever necessary

**Write the answers to Questions of each Group
in separate books**

GROUP-A

(Probability and Statistics)

[Marks : 30]

Answer any two questions

(Turn Over)

1. (a) What do you mean by Markov chain and order of a Markov chain in stochastic process ? 3+2
- (b) State and prove First Entrance theorem. 3
- (c) Prove that in an irreducible chain, all the states are of the same type. They are either all transient, all persistent null, or all persistent non-null. All the states are aperiodic and in the latter case they all have the same period. 7
2. (a) What do you mean by Galton-Watson Branching Process ? Prove that

$$P_n(s) = P_{n-1}(P(s))$$

$$\text{and } P_0(s) = P(P_{n-1}(s)),$$

the symbols have their usual meanings. 2+5

- (b) Let $(X_n, n \geq 0)$ be a Markov chain having state space $S = \{1, 2, 3, 4\}$ and corresponding transition probability matrix

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix} \end{matrix}$$

Show that. the state three and four are transient, the state two is persistent, and the state one is ergodic. 6

(c) State Chapman-Kolmogorov equation. 2

(a) Describe the Pure Birth process and deduce the corresponding Yule-Furry process. What will be the probability generating function for this process? 8

(b) What do you mean by partial correlation and multiple correlation coefficient of the three variables? 4

(c) Define the terms :

(i) Transition Probability

(ii) Doubly Stochastic Matrix. 3

GROUP-B

(Numerical Analysis)

[Marks : 40]

Answer Q. No. 4 and any *three* from the rest

4. Prove the following relations where the operators have their usual meanings : 4

(ii) $\mu = \cosh 12J$ where $u = hD$.

- S. (a) Describe power method to obtain numerically the largest eigenvalue and the corresponding eigenvector of a matrix. When does the method fail? 8
- (b) Define orthogonal polynomials and its sequences with example. 4
6. (a) Describe Milne's method for numerical solution of the equation $\frac{dy}{dx} = f(x, y)$ in the interval $[a, b]$, subject to the initial condition $y = y_0$ at $x = x_0$. Explain how you obtain the starting values. 6
- (b) Deduce the three-point Gauss-Legendre quadrature formula. Show that the formula gives exact result for $f(x)$ xs. 6
7. Describe LU-decomposition method to solve a system of linear equations $Ax = b$. Write the limitations of this method. Also give an idea to find the value of A^{-1} - 12

- (a) Describe a finite difference method to solve the wave equation defined below :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, t > 0, 0 < x < l$$

where initial conditions $u(x, 0) = f(x)$ and

$$u_t(x, 0) = g(x), 0 < x < l$$

and boundary conditions

$$u(0, t) = p(t); \text{ and } u(l, t) = q(t), t > 0.$$

- (b) Economize the power series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

correct to four significant figures, using Chebyshev polynomial. 4

9. (a) Define a cubic spline. Derive the governing equations for the spline function. 7
- (b) Find the Natural cubic splines for the data given below and estimate $f(1.5)$. 5

$x :$	0	2	3
$f(x) :$	5	8	20
			22

GROUP--C*[Computer Science)**[Marks: 30]*

10. Answer any *six* from the following :

- (i) Explain 'floating point representation' of a real number with the help of a register with a capacity of six digits and a sign bit. Pictorially, represent -0.004825×10^{-7} by the said register. Evaluate $0.01439 \times 10^4 - 0.1025 \times 10^2$. 5

- (ii) Using Karnaugh map, simplify the following function

$$f(A, B, C, D) = \sum (1,3,5,8,9,11,15)$$

$$+1 \quad (2,13)$$

4.

and draw the circuit. 5

- (if). A railway station with three platforms A, B, C is illustrated below. A train coming to the station in the direction of arrow is to be routed to platform A, B or C. Normally the train is to be routed to platform A if that platform is empty.

(iv) How do we initialize structures during declaration? Give an example. Explain the difference between the two member access operators '.' and '*'. What is a union? How is it different from a structure? 1+2+2

(v) Write a program to find out the value of sum of all prime numbers between two specified numbers in C. 5

(vi) Explain the role of the initialization, test and update expressions in a for loop. What is the output of the following statements: 4+1

```
for U= 0; i< 10; i++)
    printf("%d\  n ", l
```

(vii) Write a C program to test whether an integer is a prime or not. 5

(viii) Write down the general form of a function subprogram in C and discuss the different parts of it. 5

(ix) Write program segments that will read the value of x and evaluate the following function:

$$\begin{aligned}
 y &= 4x + 100 \text{ for } x < 40 \\
 &= 300 \quad \text{for } x = 40 \\
 &= 5x + 150 \text{ for } x > 40
 \end{aligned}$$

using (i) nested if statements, (ii) conditional operator. 5

(9)

- (x) Write a program to find the average of n numbers (i) not using array (ii) using array, with the help of two functions in C. 5
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