M.Sc. 2nd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Continuum Mechanics*)

PAPER—MTM-204

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **four** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *two* questions : 4 ×2

(*a*) Define stress Quadric of Cauchy, Prove that the normal stress across any plane through the centre of stress Quadric is equal to the inverse of the square of the central radius vector of the quadric normal to the plane. 4

(*b*) Differentiate between the material and spatial methods of description of properties of a continuum. 4

(*Turn Over*)

(*c*) Define homogeneous linear elastic body and isotropic linear elastic body. Find the number of elastic module. 4

2. Prove that the principal stress values are all real and the corresponding stress directions are mutually orthogonal. 8

3. Define stream line. Test whether the motion specified by

$$\vec{V} = \frac{K^2(x_1\vec{j} - x_2\vec{i})}{x_1^2 + k_2^2}, \quad K = \text{Constant}$$

is a possible motion for an in compressible fluid. If so, determine the equation of stream lines. Also let the motion be of the potential kind and then determine the velocity potential. 2 + 6

4. Derive the law of transformation of stress tensor in matrix form. 8

5. Calculate the strain invariants from strain tensor

$$(E_{ij}) = \begin{pmatrix} 5 & -1 & -1 \\ -1 & 4 & 0 \\ -1 & 0 & 4 \end{pmatrix}$$

Determine the principal strains. Obtain strain invariants from them. Show the equivalence of strain invariants. Deduce Cauchy integrals in perfect fluid. 8

6. (*a*) Find the shearing stress and normal stress on the octahedral plane element through a point whose stress matrix is

$$(T_{ij}) = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}.$$

(*b*) State and prove Kelvin's circulation theorem. 4 + 4

7. Define source and sink with its strength. Two sources, each of strength m, are placed at the points $(-9, 0)$ and $(9, 0)$ and a sink of strength 2 m is placed at the origin. Show that the stream lines are curves

$$(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$$

where λ is a parameter, Also find the fluid speed at any point. 2 + 6

8. Find the differential equation of the boundary surface of a fluid. Show that

$$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$$

is a possible form of the boundary surface of a liquid and find an expression for the normal velocity. 3 + (4 + 1)

[*Internal Assessment* : 10 Marks]