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PG/IS/MTM - 102/13

M.Sc. 1st Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Complex Analysis*)

PAPER— MTM - 102

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *four* questions from the following : 2×4

(*i*) Show that the function e^z has an isolated singularity at $z = \infty$.

(*ii*) Show that

$$f(z) = \frac{(z+3i)^5}{(z^2 - 2z + 5)^2}$$

has double pole at $z = 1 \pm 2i$ and a simple pole at ∞.

(*iii*) What do you mean by mesomorphic function ? Give an example.

(*iv*) What kind of singularity exists in the function $f(z) = e^{\frac{1}{z-a}}$?

(*v*) How do you determine a simple pole of a complex function $f(z) = \dfrac{\phi(z)}{\psi(z)}$?

(*vi*) If $f(z)$ has a pole at $z = a$, then show that $|f(z)| \to \infty$ as $z \to \infty$.

2. (*a*) Find the necessary and sufficient conditions for $f(z)$ to be analytic. 4

(*b*) If $f(z) = u + iv$ is an analytic function of z and $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$; find $f(z)$ subject to $f\left(\dfrac{\pi}{2}\right) = 0$. 4

(*c*) State and prove Cauchy integral formula for complex function $f(z)$. 4

(*d*) If $f(z)$ is analytic function of z and if $f'(z)$ is continuous within and on a closed contour C then show that $\int_C f(z)dz = 0$. 4

3. (*a*) Evaluate

$$\int_C \frac{e^{3z}}{z-\pi i}\, dz$$

if c is a circle $|z-1|=4$. 4

(*b*) Find the Laurent's series of the function

$$f(z)=\frac{z^2-1}{(z+2)(z+3)} \text{ when}$$

$2<|z|<3$ 4

(*c*) Find the Möbius transformation which transforms the circle $|z|=1$ into $|w|=1$ and makes the point $z=1,-1$ corresponds to $w=1,-1$ respectively. 4

(*d*) By contour integration show that

$$\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}=\frac{2\pi}{\sqrt{a^2-b^2}}.$$ 4

4. (*a*) State and prove Cauchy's residue theorem for a function $f(z)$. 4

(*b*) Locate and name all singularities of

$$f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3(3z+2)^2}.$$ 4

(*c*) Using Rouche's theorem show that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$. 4

(*d*) If $f(z)$ is analytic within and on a closed contour except at a finite number of poles and is not zero on C then

$$\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}dz = N - P$$

where N is the number of zeros and P is the number of poles inside C. 4

[*Internal Assessment* : 10 Marks]