PG/IIS/MTM-201/13

M.Sc. 2nd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(Fluid Mechanics)

PAPER – MTM- 201

Full Marks : 50

Time : 2 hours

Answer **Q.No.6** and any **three** questions from the rest

The figures in the right hand margin indicate marks

1. (*a*) State and prove Blasius theorem. 2 + 5

(*b*) A circular cylinder is moving in a liquid at rest at infinity. Calculate the force on the cylinder owing to the presence of the fluid. Also show that the effect of the presence of the

liquid is to reduce the external force in the ratio.

$$(\sigma - \rho) : (\sigma + \rho)$$

where σ is the density of the cylinder and ρ is the density of the fluid. 5

2. (*a*) An infinite elliptic cylinder with semi-axes *a* and *b* is rotating round its axis with angular velocity ω in an infinite liquid of density ρ which is at rest at infinity. Show that if the fluid is under the action of no external forces, the moment of the fluid pressure on the cylinder about the centre is

$$\frac{1}{8}\pi\rho c^2 \frac{d\omega}{dt},$$

where $c^2 = a^2 - b^2$. 6

(*b*) Find the stream function and velocity potential when an elliptic cylinder moves in an infinite liquid with velocity U parallel to major axis of the cross-section. 6

3. (*a*) A cylinder of any shape is placed in a uniform stream of speed U. Show that the resultant thrust on the cylinder is a lift of magnitude $k\rho U$ per unit length and at right angles to the stream, where k is the circulation around the cylinder. 6

(*b*) Find the velocity potential when a liquid streaming part a fixed sphere with velocity U in a liquid. Hence obtain the equations to the line of flow. 6

4. (*a*) If

$$u = \frac{ax - by}{x^2 + y^2}, \quad v = \frac{ay + bx}{x^2 + y^2}, \quad w = 0,$$

investigate the nature of motion. Also find the velocity potential and the pressure at any point (x, y). 2 + 2 + 2

(*b*) Viscous incompressible fluid occupies the region $y > 0$ on one side of an infinite flat plate $y = 0$. The plate oscillates with a velocity $V_0 \cos nt$ in the x-direction. Show

that the velocity distribution of the fluid motion is given by

$$u = V_0 e^{-\eta} \cos(nt - \eta)$$

where $\eta = (n/\gamma)^{1/2} y$. Also find the shear stress at the plate $\eta = 0$. 6

5. (*a*) Determine the velocity distribution in an incompressible viscous fluid flowing steadily under a uniform pressure gradient in a pipe of elliptic cross-section. Find the volume flux of the fluid through pipe. 7

(*b*) Establish the Kārmáns momentum integral equation relating to motion in the boundary layer. 5

6. Answer any *two* questions : 2 × 2

(*i*) Define Momentum thickness.

(*ii*) Vortex lines

(*iii*) Define boundary layer thickness.

[*Internal Assessment* – 10 *Marks*]