 PG/IIIS/MTM-301/13

M.Sc. 3rd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Partial Differential Equations*)

PAPER – MTM- 301

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **two** questions from the rest

The figures in the right hand margin indicate marks

1. Answer any *two* questions : 4 × 2

   (*a*) Find the general integral of the equation $(x-y)p+(y-z-x)q=z$ and the equation of the integral surface of the differential equation which passes through the circle $z=1, x^2+y^2=1$.

   (*b*) Solve the equation $(p+q)(px+qy)-1=0$ by Charpit's method.

(*Turn Over*)

(*c*) Solve :

$$(x^2D^2 + 2xy\,DD' + y^2D'^2)z = (x^2 + y^2)^{n/2}$$

where $D \equiv \dfrac{\partial}{\partial x}$, $D' \equiv \dfrac{\partial}{\partial y}$.

2. (*a*) Find the nature of the equation

$$e^x u_{xx} + e^y u_{yy} = u,$$

and reduce it to canonical form. 8

(*b*) (*i*) Let $u(x, t)$ be a solution of the wave equation $u_{tt} - u_{xx} = 0$ in a domain $D \subset \mathbb{R}^2$. Let a, b be real numbers such that the parallelogram with vertices $A_\pm = (x_0 \pm a, t_0 \pm b), B_\pm = (x_0 \pm b, t_0 \pm a)$ is contained in D. Prove the parallelogram identity :

$$u(x_0 - a, t_0 - b) + u(x_0 + a, t_0 + b) = u(x_0 - b, t_0 - a) + u(x_0 + b, t_0 + a).$$ 3

(*ii*) Using the parallelogram identity, solve the following initial boundary value problem :

$u_{tt} - u_{xx} = 0,\ 0 < x < \infty,\ t > 0,$

$u(0, t) = h(t), \ t > 0$
$u(x, 0) = f(x), \ 0 \le x < \infty,$
$u_t(x, 0) = g(x), \ 0 \le x < \infty,$
where $f, g, h \in C^2([0, \infty))$. 5

3. (*a*) Using the method of separation of variables solve the following problem :

$$u_{tt} - c^2 u_{xx} = 0, \ 0 < x < 2, \ t > 0$$
$$u(0, t) = u(2, t) = 0, \ t \ge 0$$
$$u(x, 0) = \sin^3\left(\frac{\pi x}{2}\right), \ 0 \le x \le 2$$
$$u_t(x, 0) = 0, \ 0 \le x \le 2.$$ 7

(*b*) Show that the Robin problem has at most one solution if $\alpha \ge 0$. 4

(*c*) State and prove mean value principle. 5

4. (*a*) Find the adjoint to the operator L where $L(u) = u_{xx} - u_t$. 2

(*b*) (*i*) State and prove weak maximum principle. Hence deduce weak minimum principle. 6

(*ii*) Solve the Dirichlet problem

$$\Delta u = 0, \ (x, y) \in Ba$$

$$u(x, y) = g(x, y), \ (x, y) \in \partial B_a,$$

where B_a is a disk of radius a around the origin. 8

[*Internal Assessment* – 10 Marks]