 MSC/IIS/MATH/MA 1202/08

2008

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA 1202

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

(*Numerical Analysis*)

1. Answer any *four* questions : 2x4

(*a*) Prove that

$$hD \equiv \sinh^{-1}(\mu\delta)$$

where the symbols have their usual meaning.

(*Turn Over*)

(*b*) Is the function

$$f(x)=\begin{cases} -\frac{11}{2}x^3+26x^2-\frac{75}{2}x+18, & 1\leqslant x\leqslant 2 \\ \frac{11}{2}x^3-40x^2+\frac{189}{2}x-70, & 2\leqslant x\leqslant 3 \end{cases}$$

cubic spline ?

(*c*) Find the value of the following tri–diagonal determinant by using an efficient method

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & -2 \\ 0 & -3 & 4 \end{vmatrix}.$$

(*d*) If $f(x)$ is a polynomial of degree 2, prove that

$$\int_0^1 f(x)\,dx=\frac{1}{12}[5f(0)+8f(1)-f(2)].$$

(*e*) Given $y'=y^2-x^2$, where $y(0)=2$. Find $y(0{\cdot}1)$ by fourth order Runge – Kutta method.

(*f*) Express the polynomial $x^3 + 2x^2 - 7$ in terms of Chebyshev polynomials.

2. (*a*) Deduce Aitken's iterative method for polynomial interpolation.

(*b*) Use Gram - Schmidt orthogonalization process to determine the first four orthogonal polynomials on [-1, 1] with respect to the weight function $w(x) = 1$.

(*c*) Describe LU decomposition method to solve a system of linear equations. 5 + 5 + 6

3. (*a*) Describe power method to find the largest eigenvalue (in magnitude) and corresponding eigenvector of an arbitrary method.

(*b*) Describe Crank – Nicolson implicit method to solve the partial differential equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

with boundary conditions $u(0, t) = f_1(t)$, $u(1, t) = f_2(t)$ and initial condition $u(x, 0) = g(x)$.

(*c*) Find the value of $y(0{\cdot}20)$ for the initial value problem $\frac{dy}{dx} = y^2 \sin x$ with $y(0) = 1$ using Milne's predictor – corrector method, taking $h = 0{\cdot}05$. 5 + 6 + 5

4. (*a*) Describe Birge – Vieta method to find the roots of a polynomial equation.

(*b*) Discuss the stability of second order Runge – Kutta method and draw the region of stability.

(*c*) Economize the power series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots$$

correct to four significant digits. 6 + 5 + 5

[*Internal Assessment* : 10]
