2008

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA 1201

Full Marks : 50

Time : 2 hours

Answer **Q. No. 5** and any **three** from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

(*Fluid Mechanics*)

1. (*a*) Find the complex potential and stream function when an elliptic cylinder rotating in an infinite mass of liquid at rest at infinity.

   (*b*) The circle $|z + a| = a$ is placed in an on coming wind of velocity U and there is a circulation k. Find the complex potential and show that the moment about the origin is ρkaU. 6 + 6

(*Turn Over*)

2. (*a*) Determine the velocity potential and stream function at any point of a liquid contained between two coaxial cylinders of radius a and b $(a < b)$, when the cylinders are moving suddenly parallel to themselves in direction at right angle with velocities U and V respectively.

(*b*) Consider the case of single row of vortices each of strength k at the points

$$z = 0, \pm a, \pm za, \ldots$$

in the complex z-plane, z being $x + iy$. Show that as $y \to \infty$, there is a uniform stream of speed $\frac{k}{2a}$ in the negative x- direction. 6 + 6

3. (*a*) An incompressible viscous fluid flows steadily under an uniform axial pressure gradient in a straight long pipe of elliptic cross-section. Find the flux of fluid through the pipe.

(*b*) An elliptic cylinder, the semi-axes of whose cross-sections are a and b, is moving with velocity U parallel to the major axis of the cross-section through an infinite liquid of density ϱ which is at rest at infinity, the pressure there being Π. Prove that in order that the pressure may everywhere be positive

$$\varrho U^2 < \frac{2a^2\Pi}{2ab + b^2}. \qquad 6+6$$

4. (*a*) Show that the motion of a liquid due to a set of line vortices, each of strength k, at the points $z = \pm na$, $(n = 0, 1, 2, 3, \ldots)$, is given by

$$w = \frac{ik}{2\pi} \log \sin\left(\frac{\pi z}{a}\right).$$

Hence find the velocity components.

(*b*) Show that a rectilinear vortex whose cross-section is an ellipse and whose spin is constant can maintain its form rotating as if it were a solid cylinder in an infinite liquid. 6 + 6

5. Answer any *two* questions: 2×2

(*a*) State Blasins theorem.

(*b*) Find the complex potential due to a constant circulation about a circular cylinder.

(*c*) Show that vortex lines and tubes cannot originate or terminate at internal points in a liquid.

6. (*a*) Assuming necessary stress-strain rate relations, deduce Navier - Stokes' equations of motion for a viscous incompressible fluid in Cartesian coordinates.

(*b*) Discuss the pulsatile motion of an incompressible viscous fluid between two horizontal parallel surfaces. 6 + 6

[*Internal Assessment*: 10]