2008

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA 2102

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **three** from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

(*Integral Transforms and Integral Equation*)

1. Answer any *five* of the following : 2 x 5

(*a*) If $F(p)$ denotes the Laplace transform of the function $f(t)$, $t \geq 0$, state the conditions which $f(t)$ must satisfy so that $F(p)$ exists.

(*b*) Define regular sequence and generalised function.

(*Turn Over*)

(*c*) Define singular integral equation. What do you mean by Fredholm alternative in integral equation ?

(*d*) Obtain the zero-order Hankel transform of $\frac{\delta(r)}{r}$, where $\delta(r)$ is the Dirac delta function.

(*e*) Find the Mellin transform of $\cos kx$.

(*f*) What do you mean inverse Fourier transform of a specified function ?

2. (*a*) Obtain the solution of the boundary value problem using Mellin transform

$$x^2 \frac{\partial^2 u}{\partial x^2} + x \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0, 0 \leq x < \infty, 0 < y < 1.$$

subject to the conditions

$$u(x, 0) = 0, \ u(x, 1) = \begin{cases} B, & 0 \leq x \leq 1 \\ 0, & x > 1. \end{cases}$$

where B is a constant. 6

(*b*) Using Parseval's relation for the Fourier cosine transform of

$$g(x) = e^{-ax}, \; f(x) = \begin{cases} 1, & 0 < x < \lambda \\ 0, & x > \lambda. \end{cases}$$

Show that

$$\int_0^\infty \frac{\sin \lambda\alpha}{\alpha(\alpha^2 + a^2)} \, d\alpha = \frac{\pi}{2}\left[\frac{1 - e^{-\lambda a}}{a^2}\right].$$ 4

3. (*a*) Solve the following ODE by Laplace transform technique

$$\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} = e^t + t + 1,$$

subject to the conditions

$$y(0) = y'(0) = y''(0) = 0.$$ 5

(*b*) Find the eigenvalues and the corresponding eigenfunctions of the integral equation

$$y(x) = \lambda \int_0^1 (2xt - 4x^2)\, y(t)\, dt.$$ 5

4. (*a*) Let $v(x)$ be a good function and $\phi(x)$ be a fairly good function then prove that their product will be a good function. 3

(*b*) Find the resolvent kernel of the following integral equation and then solve it : 5

$$y(x) = x + \int_0^x (t - x)\, y(t)\, dt.$$

(*c*) Find the Laplace transform of the following function : 2

$$f(t) = \begin{cases} \sin(t), & \text{if } t \geqslant 3 \\ 0 \quad , & \text{if } t < 3. \end{cases}$$

5. (*a*) State and prove FALTUNG Theorem on Fourier transform. 4

(*b*) Find the finite Hankel transforms of

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \frac{n^2}{r^2} f,$$

where $f(r)$ is a function of r defined in the interval $(0, a)$, restricting n to the case $n \geqslant 0$. 4

(c) Verify the truth of the initial value theorem for

$$F(p) = \frac{1}{p(p^2 + a^2)}$$ on Laplace transform. 2

[*Internal Assessment*: 10]
