2008

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA 2101

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

(*Partial Differential Equations*)

1. Answer any *two* questions: 4×2

(*a*) If a harmonic function $\psi(x, y)$ is continuous in some closed bounded region $\bar{S} = S + C$, then the value of $\psi(x, y)$ in S can not be less than its minimum value on the boundary C.

(*Turn Over*)

(*b*) Find the general solution of

$$(z^2 - 2yz - y^2)p + x(y+z)q = x(y-z)$$

where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$.

(*c*) Find the solution of

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y.$$

2. Answer any *four* questions: 8×4

(*a*) Solve the interior Dirichlet boundary value problem for the Laplace's equation for the rectangle $0 \leq x \leq a$, $0 \leq y \leq b$ with the conditions

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x \leq a,\ 0 \leq y \leq b$$

$$u(0, y) = 0, \quad 0 < y < b$$
$$u(a, y) = 0, \quad 0 < y < b$$
$$u(x, 0) = 0, \quad 0 < x < a$$
$$u(x, b) = f(x), \quad 0 < x < a$$

(*b*) If $u(x, y, z)$ is a harmonic function in the region Ω, show that the value of u at an interior point of Ω can be determined with the help of the corresponding Green's function.

(*c*) Solve the one-dimensional diffusion equation

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}.$$

in the region $0 \leq x \leq \pi, \quad t > 0$ when

(*i*) T remains finite as $t \to \infty$;

(*ii*) $T = 0$ if $x = 0$ and π, for all values of t;

(*iii*) At $t = 0, \begin{cases} T = x & 0 \leq x \leq \dfrac{\pi}{2} \\ T = \pi - x & \dfrac{\pi}{2} \leq x \leq \pi \end{cases}$.

(*d*) Discuss the d'Alembert's solution of the following initial value problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

subject to the initial conditions

$$u(x, 0) = f(x),$$
$$\text{and} \quad u_t(x, 0) = g(x).$$

show that this solution is unique.

(*e*) Solve the Gousat problem

$$u_{tt} = c^2 u_{xx}$$

subject to the following conditions

$$u(x, t) = f(x) \quad \text{on} \quad x - t = 0$$
$$u(x, t) = g(x) \quad \text{on} \quad t = t(x).$$

(*f*) Find the Riemann-Volterra solution of one-dimensional wave equation

$$u_{tt} = c^2 u_{xx}$$

subject to the condition

$$u|_{\Gamma} = f$$

$$\text{and} \qquad \left.\frac{\partial u}{\partial n}\right|_{\Gamma} = g.$$

(*g*) Reduce the following equation to a canonical form and hence solve it

$$u_{xx} - 2(\sin x)\,u_{xy} - (\cos^2 x)\,u_{yy} - (\cos x)\,u_y = 0.$$

[*Internal Assessment* —10]

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