2017

MATHEMATICS

[Honours]

PAPER – VII

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Elements of Computer Science*)

[*Marks* : 30]

1. Answer any *two* questions : 8 × 2

(*a*) (*i*) Write a flowchart to print a given number in reverse order. 4

(*ii*) Perform the arithmetic operations : (+42) + (−13), (+ 42) − (−13) with binary numbers in signed-2's complement representation. 4

(*b*) (*i*) Explain machine language, assembly language and high level language. 4

(*ii*) Explain half-adder with truth table. How a full-adder can be costructed with the help of half-adder(s) ? 4

(*c*) Write an algorithm to find mode of a simple sample of size n. Also draw its flowchart. 5 + 3

2. Answer any *two* questions : 4 × 2

(*a*) Write a program to find the largest number among the three numbers using nested if statement.

(*b*) Write an algorithm and corresponding flowchart to exchange contents of two variables without using third variable.

(*c*) Give a brief description on input and output statements in FORTRAN or in C.

3. Answer any *two* questions : 3 × 2

(*a*) Write about (*i*) Assignment operator (*ii*) Logical operator, (*iii*) Relational operator in C or in FORTRAN.

(*b*) Write a program to check whether a number is prime or not.

(*c*) Give a brief descriptions on IF statements in FORTRAN using flowchart.

Or

Give a brief description on switch statement in C using flowchart.

GROUP – B

(*Mathematical Theory of Probability*)

[*Marks* : 35]

4. Answer any *one* question : 15 × 1

(*a*) (*i*) If $\{A_n\}$ be a monotonic sequence of events, then show that

$$P(\lim A_n) = \lim P(A_n).$$ 5

(*ii*) The chance that a doctor will diagnose a certain disease correctly is 60%. The charice that a potient will die under his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor who had the disease died. What is the probability that his disease was diagnosed correctly ? 5

(*iii*) A and B alternately throw a pair of dice. A starting the game. A wins if he throws six before B throws seven and B win's if he throws seven before A throws six. What is the probability of A's winning ? 5

(*b*) (*i*) If $F(x)$ denotes the distribution function of a random variable X, then show that 5

$$P(a < X < b) = F(b-0) - F(a)$$

$$P(a \leq X \leq b) = F(b) - F(a-0)$$

(*ii*) A point chosen at random in a given interval divides it into two sub-intervals. Find the probability that the ratio of the length of the left sub-interval to that of the right sub-interval is less than a constant K. 5

(*iii*) Let X and Y are independent variates, each uniformly distributed over the interval (0, 1). Find the probability that the greater of X and Y is less than a fixed number K $(0 < K < 1)$. 5

5. Answer any *two* questions : 8 × 2

(*a*) (*i*) The joint probability density function of the random variables X, Y is given by

$$f(x,y) = \begin{cases} x+y, & 0<x<1,\ 0<y<1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the distribution of $X + Y$. 5

(*ii*) Find the expected number of failures preceding the first success in an infinite sequence of Bernoulli trials with probability of success p. 3

(*b*) (*i*) If X_1 is a $B(n_1, p)$ and X_2 is a $B(n_2, p)$ variates then show that $X_1 + X_2$ is also a binomial variate. 4

(*ii*) If $\rho = \rho(X, Y)$ be the correlation coefficient between two random variables X and Y then find the correlation coefficient between

$$U = aX + bY \text{ and}$$
$$V = cY$$

where a, b, c are positive constants. 4

(*c*) (*i*) If $X_1, X_2 \ldots. X_n$ are mutually independent standard normal variates, then find the mean value of $\min[|X_1|, |X_2|, \ldots |X_n|]$. 5

(*ii*) If a random variable X possesses a finite second order moment and c is any fixed number, then show that for any $\varepsilon > 0$.
$P(|X - c| \geq \varepsilon) \leq E[(X-c)^2]/\varepsilon^2$. 3

6. Answer any *one* question : 4 × 1

(*a*) For the poisson distribution with parameter λ, prove that

$$\mu_{k+1} = \lambda\left(k\mu_{k-1} + \frac{d\mu_k}{d\lambda}\right)$$

where μ_k is the k-th order central moment. 4

(*b*) State Bernoulli's theorem. Hence give the frequency interpretation of probability. 2 + 2

GROUP – C

(*Mathematical Statistics*)

[*Marks* : 25]

7. Answer any *one* question : 15 × 1

(*a*) (*i*) Show that the sample variance is a consistent estimator of the population variance but it is not an unbiased estimator. 5

(*ii*) Show that the sample mean and sample variance are uncorrelated if $\mu_3 = 0$. 6

(*iii*) Find the confidence interval for the parameter '*m*' in $N(m, \sigma)$ population when σ is known. 4

(*b*) (*i*) Briefly describe the interval estimation of a statistic. What do you mean by confidence coefficient ? 4

(*ii*) Find the maximum likelihood estimate of σ^2 for a $N(m, \sigma)$ population if *m* is known. Show that the estimate is unbiased. 6

(*iii*) The weekly wages of 144 workers of a large factory were recorded, and the sample mean and standard deviation were found to be Rs. 23·52 and Rs. 6·71 respectively. Find the 95% confidence limits for the mean wage (population is not normal). (given $P(z > 1{\cdot}96) = {\cdot}025$). 5

8. Answer any *one* question : 8×1

(*a*) (*i*) What do you mean by Statistical hypothesis? 2

(*ii*) Fit a curve of the form $y = x^2 + ax + b$ to the following data by the method of least squares : 6

x :	2	3	4	5	6
y :	7·2	3·9	3·0	4·4	6·3

(*b*) (*i*) A drug is given to 10 patients and the increments in their blood pressure were recorded to be 3, 6, –2, 4, –3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on change of blood pressure? Test at 5% significance level, assuming the population to be normal (given : $P(t > 2 \cdot 262) = \cdot 025$). 4

(*ii*) Let *A* be an event connected with a random experiment *E*. If in 192 repetitions of *E* under identical conditions A occur 61 times, can we reasonably conclude that the probability of *A* is $\frac{1}{4}$? Use 5 % level of significance. 4

9. Answer any *one* question: 2×1

(*a*) Describe the sampling distribution of a statistic.

(*b*) Discuss the two types of errors appear in testing of hypotheses.