# 2017

## STATISTICS

**[ Honours ]**

PAPER – VI

*Full Marks* : 90

*Time* : 4 hours

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

*Illustrate the answers wherever necessary*

[ NEW SYLLABUS ]

GROUP – A

( *Statistical Inference*-II)

[ *Marks* : 35 ]

Answer **Q. No. 1** and any **one** from **Q. Nos. 2 & 3**

( Turn Over )

1. Answer any *five* from the following questions : 5×5

(*a*) Show that if $\{T_n\}$ be a sequence of statistics such that

$$\sqrt{n}(T_n - \theta) \xrightarrow{L} N(0, \sigma^2(\theta)),$$

then $\sqrt{n}(g(T_n) - g(\theta)) \xrightarrow{L} N(0, (g'(\theta))^2 \sigma^2(\theta))$, provided $g(\cdot)$ admitting first order derivative with $g'(\theta) \neq 0$.

(*b*) Derive shortest confidence interval for the mean of a normal population with unknown variance.

(*c*) Define power, level of significance and size of a non-randomized test.

(*d*) On the basis of a random sample of size 5 from Poisson distribution with unknown parameter $\lambda (0 < \lambda < \infty)$, construct the MP test of exact size $\alpha (0 < \alpha < 1)$ for testing $H_0 : \lambda = 3$ against $H_1 : \lambda = \lambda_1 (> 3)$.

(*e*) Let $(X_1, X_2, ..., X_n)$ is a random sample of size $n$ from $U(\theta, \theta+1)$. To test $H_0 : \theta = 0$

against $H_1 : \theta > 0$ the following test was used :

Reject $H_0$ iff $X_{(1)} > 1$ or $X_{(n)} > C$ where $C$ is a constant and $X_{(1)} = \min_{1 \leq i \leq n}\{X_i\}$, $X_{(n)} = \max_{1 \leq i \leq n}\{X_i\}$. Determine $C$ so that the size of the test is $\alpha$.

(f) Describe how you use the variance stabilization transformation for testing the equality of variances of two normal populations.

(g) Define most powerful, uniformly most powerful and uniformly most powerful unbiased tests.

(h) Describe Mann-Whitney test procedure.

2. (a) State Neyman-Pearson fundamental lemma.

(b) Suppose $(X_1, X_2, ..., X_n)$ is random sample of size $n$ from a population having pdf

$$f(x) = \begin{cases} \theta e^{-\theta x} & ; \ 0 < x < \infty \\ 0 & ; \ \text{otherwise}, \end{cases}$$

where $0 < \theta < \infty$.

Derive UMP critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. Show that the test can be done using a chi-square statistic. 10

3. Describe the likelihood ratio (LR) test procedure based on a random sample of size $n$ drawn from $N(\mu, \sigma^2)$, where $\mu$ and $\sigma^2$ are both unknown, for testing $H_0 : \mu = \mu_0$ against the alternatives $H_1 : \mu < \mu_0, H_2 : \mu > \mu_0$. 10

GROUP – B

( *Theory of Sample Survey*)

[ *Marks* : 35 ]

Answer Q. No. 4 and any one from Q. Nos. 5 & 6

4. Answer any *five* from the following questions : 5×5

(*a*) State some advantages of sample survey over complete census.

(*b*) For an SRSWOR of size $n$ drawn from a population of size $N$, consider a class of estimators of population mean $\left(\overline{Y}\right)$,

$$\overline{y}' = \sum_{r=1}^{n} a_r y_r,$$

where $a_r$ is the constant depending at the $r$th draw, $y_r$ is the value of study variable $y$ on the unit selected at $r$th draw. Show that $\overline{y}'$ is unbiased for $\overline{Y}$ iff $\sum_{r=1}^{n} a_r = 1$.

Show also that, under this condition,

$$V(\overline{y}') = S^2\left[\sum_{r=1}^{n} a_r^2 - \frac{1}{N}\right],$$

where $S^2$ is population variance with divisor $(N-1)$.

(c) Let $Y$ be the population total of the study variable $y$. Define ratio estimator $\hat{y}_R$ of $Y$. Show that $\hat{y}_R$ is a biased estimator of $Y$. Obtain an approximate expression for bias of ratio estimator $\hat{\overline{y}}_R$ of population mean $\overline{Y}$.

(*d*) Distinguish between two-stage sampling and stratified Random Sampling. Indicate the cases when a systematic sampling is equivalent to stratified random sampling.

(*e*) Obtain an unbiased estimator of the variance of sample proportion in case of simple random sampling drawn with replacement (SRSWR) from a finite population.

(*f*) Give an unbiased estimator of population total under two-stage sampling with SRSWOR at both the stages. Show that variance of the estimator is greater than the variance of the corresponding single-stage estimator.

(*g*) Obtain a ratio estimator of population mean under double sampling and also obtain its mean square error.

(*h*) Mention the basic principles of sample survey. Discuss the non-sampling errors that may arise in a large-scale sample survey.

5. (*a*) In stratified random sampling using SRSWOR in each stratum, obtain an unbiased estimator of population mean.

(*b*) Show that, with usual notations, for estimating population mean of a stratified population under SRSWOR in each stratum

$$V_{\text{rand}} \geq V_{\text{prop}} \geq V_{\text{opt}}.$$ 10

6. (*a*) Describe the method of Linear systematic sampling. Also indicate the modification for circular systematic sampling.

(*b*) Show that the variance of the unbiased estimator of population mean may be expressed as

$$\frac{(N-1)}{nN} S^2 [1+(n-1)\rho],$$

where $S^2$ is population variance with divisor $(N-1)$ and $\rho$ is the intra-class correlation coefficient between the values of the pairs of units. Hence show that systematic sampling is better than SRSWOR if

$$\rho \leq -\frac{1}{N-1}.$$ 10

## GROUP – C

( *Statistical Quality Control* (SQC))

**[ Marks : 20 ]**

**Answer Q. No. 7 *Or* Q. No. 8 and Q. No. 9**

**7.** **Answer any *two* from the following questions : 5×2**

**(*a*) How do you calculate control limits for a C-chart ?**

**(*b*) Distinguish clearly between**

**(*i*) Producer's risk and Consumer's risk.**

**(*ii*) 3σ-limits and specification limits.**

**(*c*) Derive the expression of OC curve in double sampling inspection plan.**

**(*d*) Describe the technique of single sampling plan for inspection by variables assuming normal distribution with known standard deviation.**

**8.** (*a*) Explain the construction of $\bar{X}$ chart for detection of lack of control in a continuous flow of manufactured product.

(*b*) When S-chart used in place of R-chart ?

(*c*) Give practical examples where attribute control chart can be used. 10

9. Describe single sampling inspection plan in detail. Give a general outline of methods for determining the constants involved in single sampling plan. 10