2017

STATISTICS

[Honours]

PAPER – I

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

(*Descriptive Statistics*)

1. Answer any *five* questions : 5 × 5

   (*a*) What is a histogram ? Describe the construction of a histogram.

(*b*) What is tabulation ? Mention the essential points to be remembered during tabulation.

(*c*) What do you mean by central tendency of a frequency distribution ? Which measures of central tendency is the best ? Justify your answer.

(*d*) What do you mean by a percentile of a frequency distribution ? How do you obtain a percentile graphically ?

(*e*) Show that the quartile deviation is independent of change of origin but dependent on scale.

(*f*) The arithmetic mean of a certain distribution is 5. The second and the third moments about 5 are 20 and 140 respectively. Find the third moment of the distribution about 10.

(*g*) For a set of data the correlation coefficient is 0·2 while for another set of data the correlation coefficient is 0·4. Would you agree that the correlation between the variables of the second set is double the correlation between the variables of the first set ? Explain.

(*h*) Find out correlation coefficient between (*i*) x and $-x$ and (*ii*) $-x$ and $-x$.

(*i*) Define the two regression coefficients in the context of linear regression. Show that the correlation coefficient cannot exceed the average of the two regression coefficients in magnitude.

(*j*) In the 2 × 2 case, explain independence and association between two attributes.

(*k*) State the formula of Spearman's rank correlation coefficient (r_R). Determine the value of r_R in case of perfect disagreement between the characteristics, under study,

2. Answer any *two* questions : 10 × 2

(*a*) (*i*) Prove, for any set of n real positive numbers, that AM ≥ GM ≥ HM.

(*ii*) Show that

$$\frac{R^2}{2n} \le S^2 \le \frac{R^2}{4},$$

where R and S are the range and standard deviation of n values of a variable.

(*b*) (*i*) Define skewness of a frequency distribution. Mention three measures of skewness and establish the range of variation for each of these measures.

(*ii*) What is an attribute ? Define Karl Pearson's coefficient of contingency to measure the association between two attributes. Also calculate the values of the measure when the two attributes are (*i*) independent and (*ii*) perfectly associated.

(*c*) (*i*) Define correlation ratio (e_{yx}) of y on x. If r is the product-moment correlation coefficient between x and y then prove that $0 \le r^2 \le e^2_{yx} < 1$.

(*ii*) What do you mean by the correlation index ? Show that it increases as the degree of the polynomial regression equation increases.

(*d*) What do you mean by intra-class correlation ? Derive the coefficient of intra-class correlation when a variable X is observed for 'p' families each having 'k' members. Show that its lies between

$$-\frac{1}{k-1} \text{ and } +1.$$

GROUP – B

(*Matrix Algebra*)

3. Answer any *two* questions : 5×2

(*a*) Explain the term 'Linearly independent set of vectors', illustrating your answer with a suitable example. Show that a subset of linearly independent set of vectors forms another linearly independent set.

(*b*) Define rank of a matrix. Show that if two rows of a matrix are interchanged then the rank remains unaltered.

(*c*) Prove that all the characteristic roots of a real symmetric matrix A are positive or non-negative according as A is positive definite or non-negative definite.

(*d*) Find the determinant of the matrix A, where

$$A = \begin{pmatrix} 1 & r & r & \cdots & \cdots & \cdots & \cdot\cdot & r \\ r & 1 & r & \cdots & \cdots & \cdots & \cdots & r \\ \vdots & & & \ddots & & & & \vdots \\ r & r & r & \cdots & \cdots & \cdots & \cdot\cdot & 1 \end{pmatrix}^{nxn}$$

4. Answer any *one* question : 10 × 1

(*a*) (*i*) Prove that any set of mutually orthogonal non-null vectors (with real elements) is necessarily linearly independent.

(*ii*) Prove that a necessary and sufficient condition for the existence of inverse of a matrix is that it is non-singular.

(*b*) (*i*) Obtain a set of necessary and sufficient conditions for a real quadratic form to be positive definite.

(*ii*) Examine the consistency or otherwise of the system $Ax = b$ by citing examples of both.

GROUP – C

(*Mathematical Analysis*)

5. Answer any *three* questions : 5 × 3

(*a*) When do you call a squence as convergent sequence ? Prove that every convergent sequence is bounded.

(*b*) Prove that if a function is differentiable at a point then it must be continuous at that point.

(*c*) If $f(h) = f(0) + hf'(0) + h^2/2!\, f''(\theta h), 0 < \theta < 1$, find θ, when $h = 1$ and $f(x) = (1 - x)^{5/2}$.

(*d*) Examine the convergence of the power series

$$\sum_{n=1}^{\infty} x^n / n^2.$$

(*e*) Show that there does not exist any value of p for which

$$\int_0^\infty x^p dx$$

is convergent.

(*f*) Evaluate

$$\int_0^\infty x^{n-1} e^{-cx} dx.$$

6. Answer any *one* question : 10 × 1

(*a*) (*i*) Find out the polynomial approximation to the function $f(x) = \log(1-x)$.

(*ii*) Show that

$$\lfloor 2n = \frac{2^{2n-1}}{\sqrt{\pi}} \lfloor n \lfloor n+\tfrac{1}{2}$$

when n is a positive integer.

(*b*) (*i*) Evaluate

$$\iint \ldots \int x_1^2 \, dx_1 dx_2 \ldots dx_n$$

$$0 \le x_1^2 + x_2^2 + \ldots . + x_n^2 \le 1.$$

(*ii*) If
$$\begin{aligned} Y_1 + Y_2 + \ldots \ldots + Y_n &= X_1 \\ Y_2 + \ldots \ldots + Y_n &= X_1 X_2 \\ Y_3 + \ldots + Y_n &= X_1 X_2 X_3 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Y_n &= X_1 X_2 X_3 \ldots\ldots X_n, \end{aligned}$$

Find the Jacobian of the transformation from Y to X.

[***Internal Assessment*** : **10 Marks**]