2017

PHYSICS

[Honours]

PAPER – VI

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP – A

Answer any **two** questions : 15×2

1. (*a*) For a one dimensional harmonic oscillator with mass m and angular frequency w, define

$$\hat{a}_{\mp} = \sqrt{\frac{mw}{2\hbar}}\left(\hat{x} \pm i\frac{\hat{p}_x}{mw}\right),$$

where the other terms and symbols are of usual meaning. Show that $\hat{a}_+\hat{a}_-$ represents number operator, such that $\hat{N}\psi_n = n\psi_n$ when $\hat{N} \equiv \hat{a}_+\hat{a}_-$ and n is the eigenvalue in the number state specified by ψ_n. Based on the defined operators, find the expectation values of kinetic and potential energy in the ground state of the harmonic oscillator. 6

(*b*) Let $|0\rangle$ and $|1\rangle$ denote the normalized eigen states corresponding to the ground state and the first excited states of the one-dimensional harmonic oscillator. Prove that the uncertainty Δx in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is $\sqrt{\frac{\hbar}{2mw}}$. 5

(*c*) For the mentioned oscillator, show that the probability of finding the particle within classical turning points is greater than 0·8. 4

2. (*a*) What do you mean by Q value of a nuclear reaction $X(x, y)Y$? 2

(*b*) From the conservation principles, establish the Q value eqn. for the above reaction. 4

(*c*) Define reaction cross-section and give its geometrical significance. 2

(*d*) The masses of the different nuclei taking part in ${}_{3}^{7}\text{Li}(p,n)\,{}_{z}^{A}\text{Y}$ reaction in $a{\cdot}u$ are as follows :

$$M\left({}_{3}^{7}\text{Li}\right)=7\cdot01822,\ M\left({}_{1}^{1}H\right)=1\cdot00814,$$

$$M\left({}_{0}^{1}n\right)=1\cdot00898$$

and the mass of the product nucleus = 7·01915. Calculate the Q value of this reaction in MeV. Is it exoergic or endoergic ? What is the threshold energy of the reaction ? (1 U = 931 MeV) 4

(*e*) Using the law of conservation of lepton no., find which one of the following reactions is possible 3

(*i*) $p+\overline{\nu}_e \rightarrow n+\mu^-$

(*ii*) $p+\overline{\nu}_e \rightarrow n+e^+$

3. (*a*) Two particles are distributed in an energy level having 3-fold degeneracy. What would be the possible microstates if the particles are (*i*) distinguishable, (*ii*) indistinguishable bosons, and (*iii*) indistinguishable fermions ? Complete the answer with possible diagram in each case. 2 + 2 + 2

(*b*) What will be the phase trajectory of a linear harmonic oscillator moving with a constant energy ? How is the phase trajectory modified if the oscillation are damped ? 2 + 1

(*c*) Derive the partition function of a classical ideal gas of N indistinguishable molecules. 3

(*d*) From this partition function determine pressure, mean energy, and specific heat of the system. 1 + 1 + 1

4. (*a*) Considering only nearest neighbour interaction, derive dispersion relation for the lattice waves in a monoatomic linear lattice. Hence find the maximum frequency that can be propagated through the lattice and the allowed values of the phonon wave vector. 3 + 2

(b) Derive an expression for the lattice heat capacity of a two dimensional solid at a low temperature region following Debye model and compare it with experimental observations. 5 + 2

(c) Nickel (fcc) has the lattice parameter of 3·5 Å. Calculate the atomic planar density on (100), (110) and (111) planes. 3

GROUP – B

Answer any five questions : 8 × 5

5. (a) Briefly explain about the conservation of isospin and parity involved in fundamental interactions. How can we classify the elementary particles ? $(1\frac{1}{2}+1\frac{1}{2})+2$

(b) Distinguish between quark, antiquark and c-quark. 3

6. (a) Find the ratio of number of atoms in an unit cell of sc, fcc and bcc lattices. If [h k l] be the Miller indices of a lattice plane with axial

units a, b and c, then show that the separation between two successive planes is given by

$$d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}.$$

Show that the density of a crystal material is proportional to the number of atoms per unit cell. 2 + 2 + 2

(*b*) Distinguish between lattice and crystal with necessary explanation and diagram. 2

7. (*a*) Write and plot the ground state probability distribution of the hydrogen atom as a function of the distance *r*. 2

(*b*) Show that the most probable position of the electron of the hydrogen atom in the ground state is given by the first Bohr radius. Also calculate the probability that the electron lies in a sphere of radius *R*. 2 + 2 (*c*) The wave function of a hydrogen atom is given by

$$\psi_{nlm}(r,\theta,\phi) = A\left(1-\frac{r}{6a}\right)\left(\frac{r}{a}\right)\exp\left(-\frac{r}{a}\right)\sin\theta e^{-i\phi}.$$

Identify n, l and m. Also find the parity of the wave function. 2

8. (*a*) Sketch and discuss the FD distribution function for $T = 0K$ and $T > 0K$. 2

(*b*) Calculate the Fermi energy of a Fermi gas in one dimension and the average energy of a particle in Fermi gas at absolute zero of temperature. 2 + 2

(*c*) The specific heat of silver at liquid helium temp. is represented by $C_v = \gamma T + \alpha T^3$ where γ and α are constants. Explain the origin of the two term. 2

9. (*a*) Distinguish between nuclear fission and nuclear fusion. How can the energy release in these processes be explained qualitatively with the help of packing fraction curve ? 4

(*b*) Explain the nuclear fission with the help of liquid drop model. Also explain why ^{235}U is fissile with slow neutrons but ^{238}U is not. 2 + 2

10. (*a*) What do you mean by Bose-Einstein condensation ? How does it differ from a vapour condensing into the liquid state ? 3

(*b*) Establish a condition on the no. of particles of a system at temp. *T* at which Bose condensation occurs. Also identify the Bose temperature. 5

11. (*a*) The result of Kronig-Penney model is given below

$$P\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$$

where $P=\frac{mV_0ab}{\hbar}$ and $\alpha^2=\frac{2mE}{\hbar^2}$, other symbols have their usual meanings. Plot the curve of the left hand side as a function of αa and draw the conclusions. 3

(*b*) Prove that the no. of possible states in an energy band of a finite crystal is equal to the no. of primitive cells in it. 2

(*c*) The energy near the valence band edge of a crystal is given by $E = -AK^2$, where $A = 10^{-39}\text{Jm}^2$. An electron with wave vector $\vec{K} = 10^{10}\hat{x}\, m^{-1}$ is removed from an orbital in the completely filled valence band. Determine the effective mass and momentum of the hole. 3

12. (*a*) Give an account of the essential characteristics of nuclear fission. How does the phenomenon find application in a nuclear reactor ? 2 + 2

(*b*) What are 'prompt' and 'delayed' neutrons ? Explain the role of 'delayed' neutrons in nuclear reactors. 1 + 1 + 2

GROUP – C

Answer any **five** questions : 4 × 5

13. A spin $\frac{1}{2}$ particle of mass m with charge $-e$ in an external magnetic field B.

(*Turn Over*)

(*i*) What is the Hamiltonian of the system ?

(*ii*) If S is the spin angular momentum vector, show that

$$\frac{d\vec{S}}{dt} = -\frac{e}{m}\left(\vec{S} \times \vec{B}\right) \text{ quantum mechanically.} \quad 1+3$$

14. For a system of two spin $-\frac{1}{2}$ particles, obtain the possible symmetric and antisymmetric particle wave functions. 4

15. (*a*) What are the composition of primary and secondary cosmic rays ? $1\frac{1}{2}+1\frac{1}{2}$

(*b*) What is the property of decay of π^0 ? 1

16. Find an expression for electronic polarisation of a gas atom of radius R. Calculate the electronic polarisation for argon gas of radius $R = 10^8$ cm with an applied field of 10 kV/cm.
(Given : $N = 10^{19}$ cm^{-3}] 2+2

17. What is Hall effect ? Find Hall coefficient in a metal. Why is the Hall coefficient positive in some metals ? 1+2+1

18. For the one dimensional random walk problem of total N steps calculate the mean values of n_1 steps to the right and dispersion or r.m.s. deviation of n_1. 4

19. (*a*) Calculate the ground state energy and wave function of a particle of mass m moving in the one dimensional potential well defined by

$$\begin{aligned} V(x) &= \infty \quad \text{for } x \le 0 \\ &= \frac{1}{2} m w^2 x^2 \quad \text{for } x > 0 \end{aligned}$$ 2

(*b*) An oscillator consists of a weight of 1 kg at the end of a light rod of length 1 m. If the amplitude of oscillation is 0·1 m. Calculate the approximate value of the quantum no. 2

20. Four particles of mass m each are inside a two-dimensional square box of side L. If each state obtained from the solution of the Schrödinger equation is occupied by only one particle, then prove that the minimum energy of the system is equal to $\frac{10\pi^2\hbar^2}{mL^2}$. 4
