NEW

2017

Part-I 3-Tier

STATISTICS

PAPER—I

(General)

Full Marks : 100

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group A

[Marks : 25]

1. Answer any *one* question : 1×10

(a) State and prove Bayes' theorem.

There are two urns containing respectively 4 white, 6 black balls and 3 white, 6 black balls. A ball is

drawn at random from 1st urn and put into the 2nd urn. Then a ball is drawn at random from the 2nd urn and found to be black. What is the probability that the ball drawn from the 1st urn is white?

(b) Write down the pmf of a Binomial (n, p) distribution. Derive its variance. Find out the mean deviation about mean for this distribution. 2+3+5

2. Answer any *three* questions : 3×5

(a) Derive Poisson distribution as a limiting form of Binomial distribution, clearly stating the assumptions. 5

(b) Three biased coins with probability of heads 0.3, 0.4 and 0.6 respectively are tossed together. What is the probability of getting at least one tail? 5

(c) If the joint distribution of two random variables X and Y be

$$f(x, y) = \frac{1}{18}(x+y), \quad x = 0, 1, 2; \quad y = 0, 1, 2$$
$$= 0 \quad , \quad \text{otherwise}$$

Find the marginal distributions of X and Y. Also find $E(X)$. 2+2+1

(d) The cdf of a random variable X is given by

$$F(x) = \begin{cases} 0 , & \text{if } x < 0 \\ x^4 , & \text{if } 0 \le x \le 1 \\ 1 , & \text{if } x > 1 \end{cases}.$$

Derive pdf of X. Also find $E(X)$ and median of X. 1+2+2

(e) If for a random variable X,

$$\frac{f(x)}{f(x-1)} = \frac{\lambda}{x}, \quad x = 1, 2, 3, \ldots ; \ \lambda > 0,$$

find the pmf of X.

(f) Write down the limitations of the classical definition of probability. State the axiomatic definition of probability. 3+2

Group B

(Descriptive Statistics)

[Marks : 45]

3. Answer any *two* questions : 2×10

(a) (i) Distinguish between divided bar diagram and multiple bar diagram. Also state their uses.

(ii) What is a histogram ? How can a histogram be drawn in case of unequal class widths ? (3+2)+(2+3)

(b) What is skewness ? Write down different measures of skewness. Prove that Pearson's measure lies between –3 and 3. 1+4+5

(c) What is a scatter diagram ? What idea about the correlation do we get from a scatter diagram ? Explain the method of least squares for finding out the regression equation of y on x. 2+2+6

(d) (i) Given two groups of values of a variable with sizes n_1 and n_2, means $\bar{x}_1$ and $\bar{x}_2$, and variances s_1^2 and s_2^2 respectively, derive the variance of the composite group.

(ii) What will be the maximum possible value of the standard deviation of marks obtained in a paper of 100 marks with no negative marks ?

4. Answer any *five* questions : 5×5

(a) Find the mean and the standard deviation of a set of n observations in which two observations are 'a' and 'b' and the remaining $(n-2)$ observations are all equal to $\frac{'a+b'}{2}$. 5

(b) Let x be a variable assuming values 1, 2, ..., k and let F_1', F_2', ... F_k' be the corresponding cumulative frequencies of the 'greater-than' type.

Show that, $\bar{x} = \sum_{i=1}^{k} \frac{F_i'}{F_1'}$. 5

(c) Prove that the reciprocal of the arithmetic mean can not exceed the arithmetic mean of reciprocals of a set of positive values. 5

(d) What is an ogive ? How can it be drawn ? How can the first, the second and the third quartiles be determined from an ogive ? 1+1+3

(e) If the standard deviations of x and y are s_1 and s_2 respectively and x and y are uncorrelated, prove that correlation coefficient between x and $x + y$ is

$$\frac{s_1}{\sqrt{s_1^2 + s_2^2}}.$$ 5

(f) Prove that $b_2 \geq b_1 + 1$. 5

(g) Prove that the standard deviation is independent of change of origin but dependent on change of scale. 5

(h) If X_1 and X_2 are uncorrelated, correlation coefficient between X_1 and X_3 is 0.6 and that between X_2 and X_3 is 0.7 then find the partial correlation coefficient between X_1 and X_2 after eliminating the effect of X_3. 5

(i) Write down the formula of Spearman's rank correlation coefficient (r_R). Explain the situation of perfect disagreement and find the value of r_R. 5

(j) When will the two regression lines be the same ? State the value(s) of the correlation coefficient in that situation. 5

Group C

(Economic Statistics and Official Statistics)

[Marks : 20]

5. Answer any *one* question : 1×10

(a) (i) Define an index number with an example.

(ii) Describe the time-reversal test and factor-reversal test of an index number. Check whether Fisher's formula satisfies both the tests. 3+7

(b) (i) Describe the merits and demerits of moving average method of determining trend in a time series.

(ii) Describe the method of fitting an exponential trend line of the form $T_t = a.b^t$ by the method of least squares. 5+5

6. Answer any *two* questions : 2×5

(a) Explain the method of ratio-to-moving average in determining seasonal indices of a time series. 5

(b) State the different sources of official statistics relating to industry and prices in India. Mention the organisations who collect data on these areas. 5

(c) What are price relatives ? Show that Paasche's formula can be expressed as weighted average of price-relaltives. 5

(d) Write down different uses of an index number. 5
