Journal of Physical Sciences, Vol. 23, 2018, 97-109
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/publication/journal
Published on 24 December 2018

Feasible Degenerate Pivoting and Optimal Non-Degenerate Pivoting for Solving Fuzzy Linear Sum Assignment Problems

A.Nagoor Gani[1] and T.Shiek Pareeth[2]

PG and Research Department of Mathematics,
Jamal Mohammed College (Autonomous),
Tiruchirappalli – 620020,Tamilnadu, India.
E-mail:[1]ganijmc@yahoo.co.in and [2]shiekpareeth.t@gmail.com

Received 12 September 2018; accepted 10 December 2018

ABSTRACT
In this paper, we compute the fuzzy optimal assignment cost for solving fuzzy linear sum assignment problem through rooted in a strongly feasible tree. In this problem, c_{ij} is denotes as cost for perfect matching the jth job to the ith person. Here, $(\widetilde{c_{ij}})$ is ω-trapezoidal fuzzy numbers and we discussed ranking technique for solving fuzzy linear sum assignment problem.

Keywords: Strongly Feasible Tree, ω-Trapezoidal Fuzzy Numbers, Fuzzy Dual Variables, Fuzzy Linear sum Assignment Problems, Fuzzy Ranking Technique.

1. Introduction

In 1965, Zadeh initiated the concept of fuzzy set theory [14]. Further many authors the extended the concept of fuzzy set**.** In 1987, Wang, developed the concept of Fuzzy optimal assignment problem [13]. In 1976, Cunningham, initiated the concept of the network simplex method [8]. In1981, Bertsekas proposed a new algorithm for the assignment problem [3]. In 1995, Goldberg and Kennedy proposed An efficient cost scaling algorithm for the assignment problem [9].

In 2007, Chen, Chen proposed fuzzy risk analysis on the ranking of generalised trapezoidal fuzzy numbers [3]. In 2010, Kumar, Singh and Kaur proposed a generalised simplex algorithm for solving special type of fuzzy linear programming problems by using proposed ranking method [2]. In 2011, Kaur and Kumar proposed a new approach developed for solving fuzzy transportation problem by using generalised trapezoidal fuzzy numbers [1].

In 2013, Thorani and Shankar proposed fuzzy assignment problem with generalised fuzzy numbers, it is developed the classical algorithm from fundamental theorems and to compute minimum fuzzy cost and also for the variations in fuzzy assignment problems [12]. In 2014, Kar et al. proposed solution of generalised fuzzy assignment problem with restriction on costs under fuzzy environment [11]. Many applications of fuzzy assignment problems applied in real life situations, scientific, uncertainty and engineering.

Let us consider the linear problem associated with Linear Sum Assignment Problem [LSAP]. There is one-to-one correspondence between primal basic solutions and spanning trees on the associated bipartite graph G = (U,V; E).Given any basic feasible solution, and the associated spanning tree T consists of the 2n-1 edges [i,j] corresponding to the basic columns. If the reduced costs corresponding to the edges of E/T are non-negative, the basis is optimal, otherwise is not optimal. Barr, Glover and Klingman reported computational testing's showing that approximately 90 percent of the simplex pivots may be degenerate for LSAP instances. The linear problem associated with LSAP has 2n-1 rows and n^2 columns and that any primal feasible basic solution has exactly n variables with non-zero value, and hence, is highly degenerate.

In this paper, a new method is proposed for solving Fuzzy Linear Sum Assignment Problem [FLSAP]. Here, the assignment cost is represented as ω- trapezoidal fuzzy numbers andx_{ij} is the decision variables for the fuzzy assignment of the i^{th} person to the j^{th} job. Here,$\widetilde{c_{ij}}$ is theω- trapezoidal fuzzy cost element for solving the perfect matching in j^{th} job to the i^{th} person. Main objective is to minimize the total ω- trapezoidal cost is performed i^{th} person perfect matched to the available j^{th} job. Since the objectives are minimize the total cost or maximize the total profit, subject to some crisp constraints. In some ranking method is used for ranking ω- trapezoidal fuzzy numbers. A ranking method is used to transform the fuzzy linear sum assignment problem into crisp one and after proposed the procedure is taken by the decision maker.

2. Preliminaries

Definition 2.1. A fuzzy set Ã defined on the universal set of real numbers R , is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}$: R →[0,1] is continuous.
(ii) $\mu_{\tilde{A}}$ (x) = 0 ∀ x ∈(-∞ , a] ∪ [d,∞).
(iii) $\mu_{\tilde{A}}$ (x) is strictly increasing on [a,b] and strictly decreasing on [c,d].
(iv) $\mu_{\tilde{A}}$ (x) = 1∀ x∈ [b,c], where a≤b≤ c ≤d.

Definition 2.2. A fuzzy number Ã = (a,b,c,d) is said to be a trapezoidal fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a}{b-a}\right) & if\ a \le x \le b \\ 1 & if b \le x \le c \\ \left(\frac{d-x}{d-c}\right) & if\ c \le x \le d \end{cases}$$

Definition 2.3. A fuzzy set Ã defined on the universal set of real numbers R , is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}$: R →[0,1] is continuous.
(ii) $\mu_{\tilde{A}}$ (x) = 0 ∀ x ∈ (-∞ , a] ∪ [d,∞).
(iii) $\mu_{\tilde{A}}$ (x) is strictly increasing on [a,b] and strictly decreasing on [c,d].
(iv) $\mu_{\tilde{A}}$ (x) =ω∀ x∈ [b,c], where $\omega \epsilon$ [0,1].

Definition 2.4. A fuzzy number Ã = (a,b,c,d: ω) is said to be a ω-trapezoidal fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} \omega\left(\frac{x-a}{b-a}\right) \; if\; a \le x \le b \\ \omega if b \le x \le c \\ \omega\left(\frac{d-x}{d-c}\right) \; if\; c \le x \le d \end{cases}$$
$$where \omega \epsilon\ [0,1)$$

3. Arithmetic operations on ω-trapezoidal fuzzy numbers

Arithmetic operations on ω-trapezoidal fuzzy numbers between twoω-trapezoidal fuzzy numbers,defined on universal set of real numbers R is following characteristics:

Let $\tilde{A}$ = $(p_1, q_1, r_1, s_1; \omega_1)$ and $\tilde{B}$ = $(p_2, q_2, r_2, s_2; \omega_2)$ be any two ω-trapezoidal fuzzynumbers, then

(i) $\tilde{A} + \tilde{B} = (p_1+p_2, q_1+q_2, r_1+r_1, s_1+s_2;\ \min(\omega_1,\omega_2))$

(ii) $\tilde{A} - \tilde{B} = (p_{1-}s_2, q_1-r_2, ,r_1-q_2, s_1-p_2\ ;\ \min(\omega_1,\omega_2))$

(iii) $\delta\tilde{A} = \begin{cases} \delta p_1, \delta q_1, \delta r_1\ , \delta s_1;\ \omega_1), \delta > 0 \\ \delta s_1, \delta r_1, \delta q_1, \delta p_1;\ \ \omega_2), \delta < O \end{cases}$

(iv) $\tilde{A}X\tilde{B} = (p', q', r', s';\ \min(\omega_1,\omega_2))$, where $p' = \min(p_1p_2, p_1s_2, p_2s_1, s_1s_2)$
$q' = \min(q_1q_2, q_1r_2, ,r_1q_2, r_1r_2)$, $r' = \max(q_1q_2, q_1r_2, ,r_1q_2, r_1r_2)$,
$s' = \max\ (p_1p_2, p_1s_2, p_2s_1, s_1s_2)$

Definition 3.1. Fuzzy Linear Sum Assignment Problem [FLSAP]

Let a bipartite graph G=(U,V;E) having a vertex of U for each row, a vertex of V for each column and fuzzy cost $(\widetilde{c_{ij}})$ associated with edge[i, j] (i,j = 1,2...n)then the problem is todetermine a minimum fuzzy cost perfect matching in G.

Definition 3.2. Fuzzy Degenerate pivoting [FDP]

From **t**he FLSAP with cost $\widetilde{c_{ij}}$ is ω-trapezoidal fuzzy number and if the reduced from cost matrix elements at least any one cost is negative then, the basis corresponding to tree is producing degenerate pivoting.

Definition 3.3. Fuzzy Non-Degenerate pivoting [FNDP]

From the FLSAP with cost $\widetilde{c_{ij}}$ is ω-trapezoidal fuzzy number and if the reduced from cost matrixelements for all the cost is non-negative, then the basis corresponding to tree is producing non-degenerate pivoting.

Definition 3.4. Strongly feasible tree (T)

Given a feasible solution x, a tree T in G = (U,V;E) rooted at r ϵ U is a strongly feasible tree ifx_{ij} = 1 ∀ odd edges [i, j] ϵ T and x_{ij} = 0 ∀ even edges [i, j] ϵ T.

Definition 3.5. An edge[i, j]ϵ E\T with iϵ U and jϵ V is a forward edge if i lies on the Path that connects r to j in T, is a backward edge if j lies on the path that connects r to i in T, and is a cross edge if it is neither forward nor backward.

4. ω- trapezoidal fuzzy linear sum assignment problem [ω-TFLSAP]

Suppose there are 'm' jobs to be performed and 'n' persons (m = n) are available for doing the jobs. Assume that each person can do each job at a time, depending on their efficiency to do the job. Let ($\widetilde{c_{ij}}$)be the ω −trapezoidal fuzzy linear sum assignment cost, then the objective is to minimize the total ω- trapezoidal cost is performed ith person perfect matched to the available jth job or assigning all the jobs to the available persons(one job to one person).

The ω-trapezoidal fuzzy linear sum assignment problem can be stated in the form of an mxn (m = n) Cost matrix [$\widetilde{c_{ij}}$] of ω-trapezoidal fuzzy numbers as given in the following table:

Persons/jobs	1	2	3	. . . . .	N
1	$[\widetilde{c_{11}};\ \omega_{11}]$	$[\widetilde{c_{12}};\ \omega_{12}]$	$[\widetilde{c_{13}};\ \omega_{13}]$	. . . . .	$[\widetilde{c_{1n}};\ \omega_{1n}]$
2	$[\widetilde{c_{21}};\ \omega_{21}]$	$[\widetilde{c_{22}};\ \omega_{22}]$	$[\widetilde{c_{23}};\ \omega_{23}]$	. . . . .	$[\widetilde{c_{2n}};\ \omega_{2n}]$
3	$[\widetilde{c_{31}};\ \omega_{31}]$	$[\widetilde{c_{32}};\ \omega_{32}]$	$[\widetilde{c_{33}};\ \omega_{33}]$	. . . . .	$[\widetilde{c_{3n}};\ \omega_{3n}]$
. ..	. ..	. ..	. ..	. ..	. .
M	$[\widetilde{c_{m1}};\ \omega_{m1}]$	$[\widetilde{c_{m2}};\ \omega_{m2}]$	$[\widetilde{c_{m3}};\ \omega_{m3}]$		$[\widetilde{c_{mn}};\ \omega_{mn}]$

Table 1: ω- Trapezoidal fuzzy linear sum assignment problem

4.1. Mathematical formulation of fuzzy linear sum assignment problem

Minimize $\tilde{z} = \sum_{i=1}^{n}\sum_{j=1}^{n}\widetilde{c_{ij}}x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = 1$, i = 1,2,...n

$\sum_{i=1}^{n} x_{ij} = 1$, j = 1,2,...n

where,

$x_{ij} = 1,(if the i^{th} person is assigned to j^{th} job)$ and

$x_{ij} = 0,(if the i^{th} person is not assigned to j^{th} job)$.

x_{ij} is the decision variable for the fuzzy assignment of the person i to job j.$\widetilde{c_{ij}}$ is the cost of the perfect matching in jth job to the ith person. The new algorithm to solve the fuzzy linear sum assignment problem is based on the following theorems.

Theorem 4.2. If reduced costs $\overline{c_{ij}} = c_{ij} - u_i - v_j \geq 0$ where 1≤ i,j≤n is corresponding LSAP it is optimal solution and T is producing a non-degenerate pivot on a forward edge.

Proof: Let us take a strongly feasible tree T and from the balanced linear sum assignment problem and arbitrarily fixing the value of u_r to zerothen computed dual variables from T. Next execution by using duality theory, $\overline{c_{ij}} = c_{ij} - u_i - v_j$ is non- negative. Since, the LSAP is optimal solution and T is producing non-degenerate pivot on a forward edge.■

Theorem 4.3. If $\overline{c_{ij}} = c_{ij} - u_i - v_j$< o where 1≤ i,j≤n is corresponding LSAP it is not optimal solution and T is producing a degenerate on a backward edge.

Proof: LetT be a strongly feasible tree and from the balanced linear sum assignment problem and arbitrarily fixing the value of u_r to zero then obtain dual variables from T. Then next executing a dual solution satisfying the complementary slackness condition,

$\overline{c_{ij}}$ = c_{ij} - u_i - v_j is negative. i.e, the LSAP is not optimal solution and T is producing degenerate pivot on a backward edge.■

5. Ranking of ω-trapezoidal fuzzy numbers

An effective algorithm developed before for comparing ω- trapezoidal fuzzy numbers is by use of ranking function [1]. The decision maker first we take ω- trapezoidal fuzzy number must be ranked and after the decision. R: F(R) →R, where F(R) is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line. The following comparisons are exists i.e.,

(i) $\tilde{A}>_R\tilde{B}$ if and only if R($\tilde{A}$) >R ($\tilde{B}$)
(ii) $\tilde{A}<_R\tilde{B}$ if and only if R ($\tilde{A}$) <R ($\tilde{B}$)
(iii) $\tilde{A}=_R\tilde{B}$ if and only if R ($\tilde{A}$) = R ($\tilde{B}$)

Let $\tilde{A}$ = $(p_1, q_1,r_1,s_1;\omega_1)$ and $\tilde{B}$ = $(p_2, q_2,r_2,s_2;\omega_2)$ be any two ω-trapezoidal fuzzy numbers and ω = min(ω_1,ω_2) then

$$R(\tilde{A}) = \frac{\omega_1(p_1 + q_1 + r_1 + s_1)}{4} \text{ and } R(\tilde{B}) = \frac{\omega_2(p_2 + q_2 + r_2 + s_2)}{4}$$

6. A new algorithm for solving fuzzy linear sum assignment problems with costs asω-trapezoidal fuzzy numbers with strongly feasible tree

Step 1: First test whether the givenω-trapezoidal fuzzy cost matrix of an linear sum assignment Problem is an balanced one (or) not. If it is balanced one (i.e. number of persons are equal to the number of jobs), then go to step3. If it is unbalanced one. (Since, number of persons are not equal to the number of jobs), then go to step2.
Step 2: Introduce dummy rows (or) dummy columns with zero ω–trapezoidal fuzzy Cost, so as form a balanced one.
Step 6: Examine the rank of ω-trapezoidal fuzzy cost matrix ($\widetilde{c_{ij}}$) and is defined as [R ($\widetilde{c_{ij}}$)] and compute the rank of each cell of the ω -trapezoidal fuzzy cost matrix [R($\widetilde{c_{ij}}$)]
Step 3: Form a Strongly Feasible Tree(T) ; Let T be a strongly feasible tree corresponding to X, and r be any vertex of $\widetilde{u_i}$*,and let us start with solution $\widetilde{x_{ij}}$ = 1and associated strongly feasible tree with r ∈ $\widetilde{u_i}$ *.
Step 4: Compute Rank of Fuzzy Dual Variables ($\widetilde{u_i}$ * and$\widetilde{v_j}$*). Let us take balanced rank of fuzzy linear Sum assignment problem and then arbitrarily fixing the value of rank of fuzzy dual variable from strongly feasible tree T and set $\widetilde{u_i}$***** = 0 then calculate $\widetilde{v_j}$*****= $\widetilde{c_{ij}}$- $\widetilde{u_i}$ *; and$\widetilde{u_{i-1}}$*****= $\widetilde{c_{ij}}$ - $\widetilde{v_j}$*****and next $\widetilde{v_{j-1}}$***** = $\widetilde{c_{ij}}$- $\widetilde{u_{i-1}}$*****and so on, similarly compute $\widetilde{u_{i+1}}$*****= $\widetilde{c_{ij}}$ - $\widetilde{v_j}$*****; $\widetilde{v_{j-1}}$***** = $\widetilde{c_{ij}}$- $\widetilde{u_{i-1}}$*****and so on.
Step5: [Compute reduced rank of fuzzy cost matrix [R($\widetilde{\widetilde{c_{ij}}}$)]
The reduced rank of fuzzy cost matrix($\widetilde{\widetilde{c_{ij}}}$)] is R[R**(**$\widetilde{\widetilde{c_{ij}}}$)])]= R[$\widetilde{c_{ij}}$- $\widetilde{u_i}$- $\widetilde{v_j}$]
Step 7: (Assigning the zeros)

a. Compute the row **[**R**(**$\widetilde{\widetilde{c_{ij}}}$)] successively until a row with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross all other zeros in the column**[**R**(**$\widetilde{\widetilde{c_{ij}}}$)]of this encircled zero.

b. Compute the column **[**R($\widetilde{\widetilde{c_{ij}}}$)] successively until a column with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross any other zeros in its row **[**R($\widetilde{\widetilde{c_{ij}}}$)]of this encircled zeros.

c. Continue the process until in each row **[**R($\widetilde{\widetilde{c_{ij}}}$)]and each column **[**R($\widetilde{\widetilde{c_{ij}}}$)]exactly one encircled zero.

Step 8: (Apply optimal test)

(i) A strongly feasible tree is producing degenerate pivoting on a backward edge and The rank of reduced fuzzy cost matrix **[**R($\widetilde{\widetilde{c_{ij}}}$)] is negative and $\bar{E}$= {[i, j] / $\bar{E}$: R [$\widetilde{c_{ij}}$ - $\widetilde{u_i}$-$\widetilde{v_j}$]$<$ 0}then the current fuzzy linear sum assignment problem is notoptimal.

(ii)A strongly feasible tree is producing non-degenerate pivoting on a forward edge And the rank the reduced cost matrix **[**R($\widetilde{\widetilde{c_{ij}}}$)]is non-negative and $\bar{\bar{E}}$ = $\emptyset$ then the Current fuzzy linear sum assignment problem is optimal.

Step 9: The rank of reduced fuzzy cost matrix**[**R($\widetilde{\widetilde{c_{ij}}}$)] = R[$\widetilde{c_{ij}}$ - $\widetilde{u_i}$ - $\widetilde{v_j}$] $<$ 0 and$\bar{E}$= {[i, j] / $\bar{E}$: R[$\widetilde{c_{ij}}$ - $\widetilde{u_i}$ - $\widetilde{v_j}$] $<$ 0} form a matching in a bipartite graph from **[**R($\widetilde{\widetilde{c_{ij}}}$)]then select most negative edge from**[**R($\widetilde{\widetilde{c_{ij}}}$)]the most negative edge[i, j]$\epsilon\bar{E}$with i ϵ u and j ϵ v and removes from the basis the unique other edge [i, l] ϵ C (T , [i, j]) incident to i with l≠j and form a new matching in a bipartite graph and making a new strongly feasible tree. Again a strongly feasible tree T is producing degenerate pivoting on a backward edge, continue the process reached until T is producing the non-degenerate pivoting on a forward edge and optimal.

Step 10: Stop**.**

7. Numerical example

A company has four persons P_1, P_2, P_3, P_4 and four jobs J_1, J_2, J_3, J_4 with cost matrix[$\widetilde{c_{ij}}$] is given whose elements are ω-trapezoidal fuzzy numbers and then illustrated the proposed algorithm. The problem is to compute the best perfect matching (the total cost is minimum of the i^{th} persons to the j^{th} job assignment).

	J₁	**J₂**	**J₃**	**J₄**
P_1	(14,20,26,32;0.4)	(8,14,20,26;0.5)	(4,8,14,20;0.2)	(14,20,26,32;0.4)
P_2	(8,14,20,26;0.5)	(8,14,20,26;0.5)	(14,20,26,32;0.4)	(20,26,32,38;0.5)
P_3	(32,38,44,50;0.8)	(14,20,26,32;0.4)	(4,8,14,20;0.2)	(14,20,26,32;0.4)
P_4	(19,25,31,37;0.55)	(26,32,38,44;0.46)	(14,20,26,32;0.4)	(8,14,20,26;0.5)

Table 2:

Case (i): The given ω-trapezoidal fuzzy assignment problem is balanced one (number of persons are equal to the number of jobs). Here we first obtain the matrix **[**R(**$\widetilde{c_{ij}}$**)] by using the given ranking method.

[R($\widetilde{c_{ij}}$)**]**=

	J_1	**J_2**	**J_3**	**J_4**
P_1	9.20	8.50	2.30	9.20
P_2	8.50	8.50	9.20	14.5
P_3	32.8	9.20	2.30	9.20
P_4	15.4	16.1	9.20	8.50

Table 3:

Form a strongly feasible tree

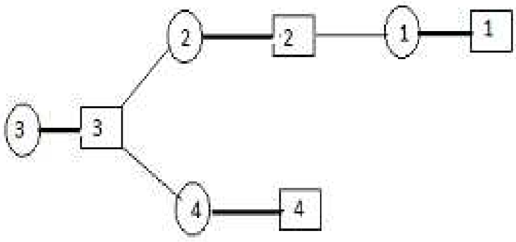

Figure 1:

$\tilde{u}_i$***** / $\tilde{v}_j{}^*$	**2.30**	**1.60**	**2.30**	**1.60**
6.9	9.20	8.50	2.30	9.20
6.9	8.50	8.50	9.20	14.5
0	32.8	9.20	2.30	9.20
6.9	15.4	16.1	9.20	8.50

Table 4: Compute the fuzzy ranking dual variables $\tilde{u}_i$***** and $\tilde{v}_j$ *from R ($\widetilde{c_{ij}}$)

R ($\widetilde{\widetilde{c_{ij}}}$) =

(0)	0	***-6.9***	0.7
-0.7	**(0)**	0	6
30.5	7.6	**(0)**	7.6
6.2	7.6	0	**(0)**

Table 5: Compute the reduced rank of fuzzy cost matrix [R ($\widetilde{\widetilde{c_{ij}}}$)]

Here $\tilde{E}$ = {[1,3], [2, 1]} ; select [i, j] = [1,3] and [i,l] = [1,2]

A strongly feasible tree (T) is producing degenerate pivoting on a backward edge

Case (ii): The above reduced rank of fuzzy cost matrix [R ($\widetilde{\widetilde{c_{ij}}}$)] and strongly feasible tree(T) is producing degenerate pivoting on a backward edge. So, continue the process. The rank of fuzzy cost matrix is given by,

[R ($\widetilde{c_{ij}}$)**]**=

	J_1	**J_2**	**J_3**	**J_4**
P_1	9.20	8.50	2.30	9.20
P_2	8.50	8.50	9.20	14.5
P_3	32.8	9.20	2.30	9.20
P_4	15.4	16.1	9.20	8.50

Table 6:

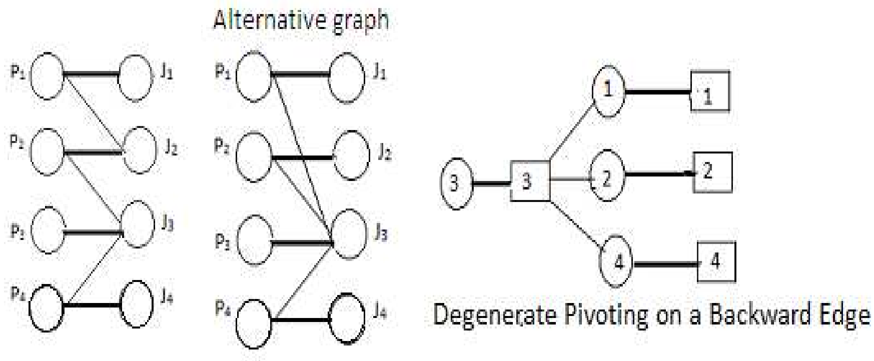

Figure 2:

$\tilde{u}_i$ * / $\tilde{v}_j$ *	**9.20**	**1.60**	**2.30**	**1.60**
0	9.20	8.50	2.30	9.20
6.9	8.50	8.50	9.20	14.5
0	32.8	9.20	2.30	9.20
6.9	15.4	16.1	9.20	8.50

Table 7: Compute the fuzzy Ranking dual variables $\tilde{u}_i$ * **and** $\tilde{v}_j$ *

R ($\widetilde{\widetilde{c_{ij}}}$) =

(0)	6.9	0	7.6
- 7.6	**(0)**	0	6
23.6	7.6	**(0)**	7.6
- 0.7	7.6	0	**(0)**

Table 8: Compute the reduced rank of fuzzy cost matrix [R ($\widetilde{\widetilde{c_{ij}}}$)]

Here, $\tilde{E}$ = {[4,1], [2, 1]} ; select [i, j] = [2,1] and [i,l] = [2,3]

A strongly feasible tree (T) is producing degenerate pivoting on a backward edge

Case (iii): The above reduced rank of fuzzy cost matrix [R ($\widetilde{\widetilde{c_{ij}}}$)] and strongly feasible tree (T) is producing degenerate pivoting on a backward edge. So, continue the process. The rank of fuzzy cost matrix is given by,

[R($\widetilde{c_{ij}}$)**] =**

Persons/Jobs	**J_1**	**J_2**	**J_3**	**J_4**
P_1	9.20	8.50	2.30	9.20
P_2	8.50	8.50	9.20	14.5
P_3	32.8	9.20	2.30	9.20
P_4	15.4	16.1	9.20	8.50

Table 9:

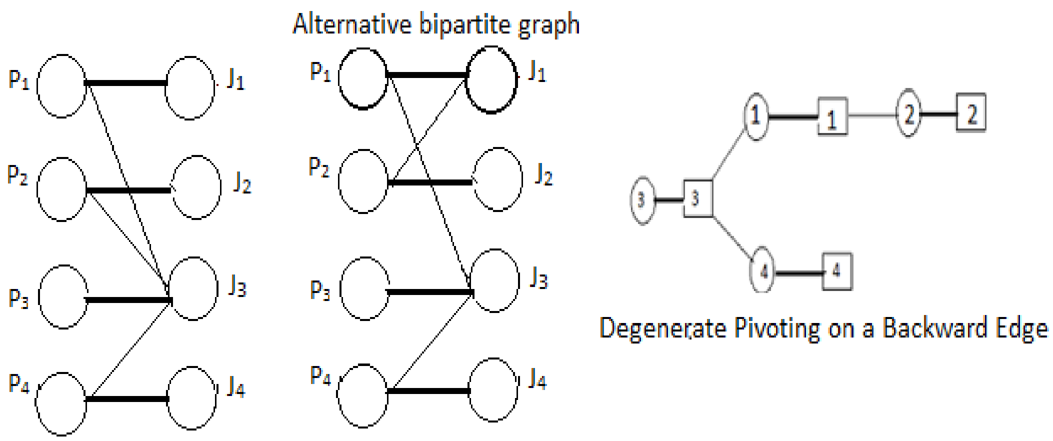

Figure 3:

$\widetilde{u_i}$ * / $\widetilde{v_j}$ *	**9.20**	**9.20**	**2.30**	**1.60**
0	9.20	8.50	2.30	9.20
- 0.7	8.50	8.50	9.20	14.5
0	32.8	9.20	2.30	9.20
6.9	15.4	16.1	9.20	8.50

Table 10: Compute the fuzzy Ranking dual variables $\widetilde{u_i}$ * **and** $\widetilde{v_j}$ *

[R ($\widetilde{\widetilde{c_{ij}}}$)]=

(0)	***- 0.7***	0	7.6
0	**(0)**	7.6	13.6
23.6	0	**(0)**	7.6
- 0.7	0	0	**(0)**

Table 11: Compute the reduced rank of fuzzy cost matrix [R ($\widetilde{\widetilde{c_{ij}}}$)]

Here,$\tilde{E}$= {[4,1], [1,2]} ; select [i, j] = [1,2] and [i,l] = [1,1]

A strongly feasible tree (T) is producing degenerate pivoting on a backward edge

Case (iv): The above reduced rank of fuzzy cost matrix [R ($\widetilde{\widetilde{c_{ij}}}$)] and strongly feasible tree (T) is producing degenerate pivoting on a backward edge. So, continue the process. the rank of fuzzy cost matrix is given by,

Persons /Jobs	**J_1**	**J_2**	**J_3**	**J_4**
P_1	9.20	8.50	2.30	9.20
P_2	8.50	8.50	9.20	14.5
P_3	32.8	9.20	2.30	9.20
P_4	15.4	16.1	9.20	8.50

Table 12:

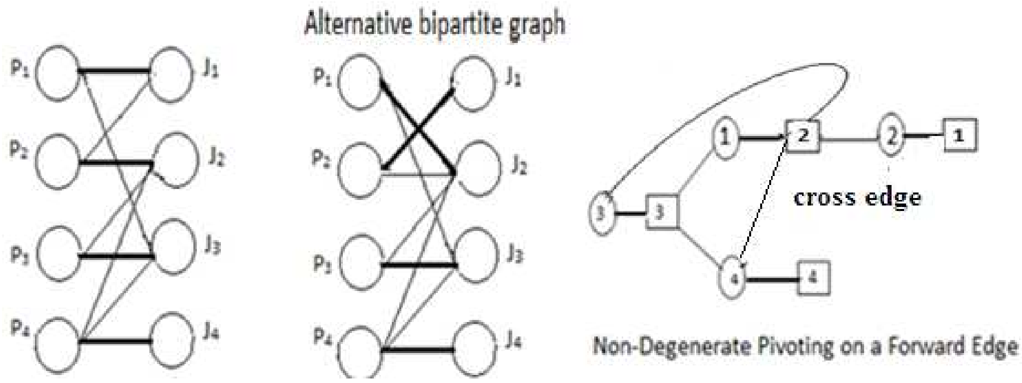

Figure 4:

$\widetilde{u_i}$ * / $\widetilde{v_j}$ *	**8.50**	**8.50**	**2.30**	**1.60**
0	9.20	8.50	2.30	9.20
0	8.50	8.50	9.20	14.5
0	32.8	9.20	2.30	9.20
6.9	15.4	16.1	9.20	8.50

Table 13: Compute the fuzzy ranking dual variables $\widetilde{u_i}$ *and $\widetilde{v_j}$ *

$[R(\widetilde{c_{ij}})]=$

0.7	**(0)**	0	7.6
(0)	0	6.9	12.5
24.3	0.7	**(0)**	7.6
0	0.7	0	**(0)**

Table 14: Compute the reduced rank of fuzzy cost matrix $[R(\widetilde{c_{ij}})]$

Here $\tilde{E}$ =∅;A strongly feasible tree (T) is Producing non- degenerate pivoting on a forward edge. Optimum reached and stop the procedure. The optimal assignment perfect Matching schedule is $\mathbf{P_1 \rightarrow J_2}$, $\mathbf{P_2 \rightarrow J_1}$, $\mathbf{P_3 \rightarrow J_3}$, $\mathbf{P_4 \rightarrow J_4}$, The fuzzy optimal assignment cost is computed as,

$(\widetilde{C_{12}};\ \omega_{12})+(\widetilde{C_{21}};\ \omega_{21})+(\widetilde{C_{33}};\ \omega_{33})+(\widetilde{C_{44}};\ \omega_{44})$ = (8,14,20,26;0.5) +(8,14,20,26;0.5)+ (4, 8, 14, 20; 0.2)+ 8, 14, 20, 26; 0.5) =***(*28,50,74,98;0.2)**

and also$[R(\widetilde{c_{ij}})]$= **(28,50,74,98;0.2) = *12.5.***

8. Conclusion

In this paper, the assignment cost as ω-trapezoidal fuzzy numbers. Here, ω-trapezoidal fuzzy assignment problem has been transformed into crisp assignment problem using some ranking method. A strongly feasible tree T is producing fuzzy degenerate pivoting on a backward edge and T is producing the non-degenerate pivoting on a forward edge and optimal are discussed.

Acknowledgements. This journal is unique in multiple ways and has innovative module. The decisions of the reviewers help me to further researches to a great extent.

REFERENCES

1. D.P.Bertsekas, A new algorithm for the assignment problem, Mathematical Programming, 21 (1981) 152–171.
2. M.S.Chen, On a fuzzy assignment problem, Tamkang J, 22 (1985) 407-411.
3. S.J.Chen and S.M.Chen, Fuzzy risk analysis on the ranking of generalised trapezoidal fuzzy numbers, Appl. Intell., 26(2007) 1-11.
4. S.H.Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, 17(1985) 113-129.
5. W.H.Cunningham, A network simplex method, Mathematical Programming, 11 (1976) 105–116.
6. V.Goldberg and R.Kennedy, An efficient cost scaling algorithm for the assignment problem, Andrew Programming, 71 (1995) 153-177.

7. S.Kar, K.Basu and S.Mukherjee, Solution of generalized fuzzy assignment problem with restriction on costs under fuzzy environment, International Journal of Fuzzy Mathematics and Systems, 2 (2014) 169-180
8. A.Kaur and A.Kumar, A new method for solving fuzzy transportation problems using ranking function, Applied mathematical modelling, 35(2011) 5652-5661.
9. A.Kumar, P.Singh and J.Kaur, Generalized simplex algorithm to solve fuzzy linear programming problems with ranking of generalized fuzzy numbers, Turkish Journal of Fuzzy Systems, 1(2) (2010) 80-103.
10. C.-J. Lin and Ue-Pyng Wen, A labelling algorithm for the fuzzy assignment problem, Fuzzy Sets and Systems, 142(2004) 373-391.
11. A.Nagoor Gani and V.N.Mohamed, Solution of a fuzzy assignment problem by using a new ranking method, Intern. J. Fuzzy Mathematical Archive, 2 (2013) 8-16.
12. Y.L.P.Thorani and R.Shanker, Fuzzy assignment problem with generalised fuzzy numbers, Applied Mathematical Sciences, 7(71) (2013) 3511-3537.
13. X.Wang, Fuzzy optimal assignment problem, Fuzzy Mathematics, 3 (1987) 101-108.
14. L.A.Zadeh, Fuzzy sets, Inform and Control, 8 (1965) 338-365.