Some Problems on Supply Chain Management

Thesis submitted to the

Vidyasagar University, Midnapore

for the award of the degree of

DOCTOR OF PHILOSOPHY (SCIENCE)

by

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2016

Dedicated to my parents

Mr. Santanu Majumder
Mrs. Manju Majumder

DECLARATION

I, Arunava Majumder, do hereby declare that, the results embodied in this thesis – "**Some Problems on Supply Chain Management**" have not been submitted to any other University or Institute for the award of any degree or diploma.

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CERTIFICATE

This is to certify that the thesis entitled **"Some Problems on Supply Chain Management"** being submitted to the Vidyasagar University by **Mr. Arunava Majumder** for the award of the degree of **Doctor of Philosophy in Science** is a record of bonafide research work carried out by him under my supervision and guidance. **Mr. Majumder** has done this research work in the **Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University** as per regulations of this University.

In my opinion, this thesis is of the standard required for the award of the degree of **Doctor of Philosophy in Science.**

The results, embodied in this thesis, have not been submitted to any other University or Institute for the award of any degree or diploma.

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ACKNOWLEDGMENTS

It is pleasure for me to express my sincere and deepest gratitude to Dr. Biswajit Sarkar, Assistant Professor of Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, India, under whose kind supervision and guidance the entire work contained in this thesis have been carried out. I convey my respect to my supervisor for helping me in research work and providing me continuous support. His suggestion on both my research work and my career is truly valuable. He provided me excellent suggestions for presenting the results of the thesis. I would like to appreciate his contributions of time, guidance, and suggestions to develop my Ph.D. experience throughout the research work. I feel myself very fortunate to carry on my thesis study under the direction of my supervisor, who helps me to learn the real technique for doing research work. My research would not have been completed without his supports, helps, and guidance.

I am thankful to Mrs. Mitali Sarkar, Mr. Bikash Koli Dey, and Miss Gargi Roy for their collaboration to contribute their ideas with me, which was very helpful for constructing my research work.

I convey my appreciation to Prof. Madhumangal Pal, respected Head of the Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Dr. Shyamal Kumar Mondal, Dr. Sankar Kumar Roy, Dr. Dilip Kumar Maiti, Mr. Ganesh Ghorai, Mr. Raghu Nandan Giri for their encouragement in different stages and also indebted to the University for help and support and giving me the opportunity to utilize the infrastructure to proceed my research work. I am also thankful a substantial debt to each members of the Doctoral Scrutiny Committee for their valuable suggestions, advice, and encouragement.

I appreciatively thankful to my co-researchers Mr. Sumon Sarkar, Miss Sarmila Saren, Mr. Buddhadev Mandal, Mr. Bimal Kumar Sett, Mr. Hiranmoy Gupta, Mr. Amalendu Singha Mahapatra, Miss Gargi Roy, Mr. Bikash Koli Dey, Miss Rekha Guchhait, Miss Baisakhi Ganguly, Mr. Shaktipada Bhunia, Mr. Bijoy Kumar Shaw, Miss Debjani Sinha for providing friendly environment and encouragement.

I convey my appreciation to Techno India College of Technology, Rajarhat, Kolkata, India, and all faculty members and staffs of the Department of Basic Science and Humanities for their continuous support and encouragement to carry on my research.

I gratefully acknowledge my parents, Mrs. Manju Majumder and Mr. Santanu Majumder; all family members, Mr. Pradip Majumder, Mrs. Krishna Majumder, Mr. Amitava Majumder, Mr. Rana Majumder, Miss Anuradha Nath, Mr. Madhab Nath, Mr. Akshar Nath and my aunt Miss Swapna Das for their constant encouragement which has been a source of inspiration to me.

I acknowledge all my teachers who supported me from the beginning of my education and helped me to gather knowledge and wisdom to build a successful human being.

I acknowledge all my friends Mr. Debodeep Raychaudhuri, Mr. Chirantan Dey and my colleagues Mr. Tanmay Chakraborty, Miss Barnima Ghosal, Mr. Sujay Das, Mr. Subhajit Pal, and Mr. Sayantan Majumder for their continuous support and encouragement.

I am very much thankful to everyone who is important to me for successful realization of the thesis. I also convey my apology as I could not acknowledge each person individually.

I gratefully acknowledge to the Editor-in-Chief of the journals Economic Modelling, Applied Mathematics and Computation, and Journal of Industrial and Management Optimization for publishing my papers.

Finally, I am grateful to the authors of all research articles from which I studied many valuable contributions to gather knowledge and wisdom to help me for writing this thesis.

Arunava Majumder

List of Publications and communicated papers

Published and accepted papers

1. B. Sarkar, A. Majumder, A study on three different dimensional facility location problems, *Economic Modelling*, SSCI, I.F. 0.997. 2013, 30, 879-887.

2. B. Sarkar, A. Majumder, Integrated vendor-buyer supply chain model with vendor's setup cost reduction, *Applied Mathematics and Computation*, SCIE, I.F. 1.345. 2013, 224, 362-371.

3. B. Sarkar, A. Majumder, M. Sarkar, B. K. Dey, G. Roy, Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction, *Journal of Industrial and Management Optimization*, SCIE, I.F. 0.84. Accepted.

Communicated papers

1. Manufacturing quality improvement and setup cost reduction in an integrated vendor-buyer supply chain system. Under review in *European Journal of Industrial Engineering.*

2. Joint effect of price and demand on decision making in a supply chain management. Submitted to *Journal of Industrial and Management Optimization.*

3. Distribution free newsvendor model with consignment policy and retailer's royalty reduction. Submitted to *International Journal of Production Research.*

4. A multi-retailer supply chain model with backorder and variable production cost. Submitted to *RAIRO-Operations Research.*

5. Relation between quality of products and production rate in a single-vendor multi-retailer joint economic lot size model with variable production cost. Under review in *International Journal of Production Research.*

Contents

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Notation

Decision variables

S	setup cost of the vendor per setup (\$/setup)
Q	quantity ordered by the buyer/production lot size per batch−cycle (unit)
q	delivery lot size for lot splitting policy (unit)
R	reorder point of the buyer (unit)
L	length of the lead time for the buyer (time unit)
m	an integer number of lots delivered from vendor/supplier to buyer/retailer in one production cycle (unit)
P	production rate per unit item (unit) (for Chapter 8 and 9)
V	variable cost per unit for order handling and receiving (\$/unit)
T	duration of inventory cycle (time unit)
T_1	production time duration for the supplier (time unit)
T_2	the duration of non-production time for supplier (time unit)
T_3	duration between two successive deliveries (time unit)
$D(p)$	price dependent demand of retailer (unit)
k	safety factor
r	reorder point of retailer (unit)
α	commission for each item sold (\$/unit)

A	fixed cost given by the manufacturer to the buyer (\$)
q_i	order quantity delivered by the vendor to buyer i in a single lot (unit)
r_i	reorder point for buyer i (unit)
s_i	safety stock for buyer i (unit)
k_i	safety factor for buyer i
L_i	length of lead time for buyer i (time unit)
$C(P)$	unit variable production cost (\$)
N	number of defective goods in a production cycle (unit)
t	actual production run time (time unit)
$\eta(P)$	elapsed time that the process goes *out-of-control* (time unit) (exponential random variable)
X_i	normally distributed lead time demand for buyer i with mean $d_i L_i$ and standard deviation $\sigma_i \sqrt{L_i}$ (unit)
x_{ij}	fraction (with respect to D_i) of product delivered to retailer i from warehouse j
x_{ijp}	fraction (with respect to D_{ip}) of product p delivered to retailer i from warehouse j
x^t_{ijp}	fraction (with respect to D_{ip}) of product p delivered to retailer i from warehouse j via transportation mode t
y_{jk}	fraction (with respect to WC_j) of the product delivered to warehouse j from plant k
y_{jkp}	fraction (with respect to WC_j) of product p delivered to warehouse j from plant k
y^t_{jkp}	fraction (with respect to WC_j) of product p delivered to warehouse j from plant k via transportation mode t

s_i fraction (with respect to D_i) of the product delivered to retailer i from an outside supplier

s_{ip} fraction (with respect to D_{ip}) of product p delivered to retailer i from an outside supplier

s_{ip}^t fraction (with respect to D_{ip}) of product p delivered to retailer i from an outside supplier via transportation mode t

I_j inventory holding of a product at warehouse j (unit)

I_{jp} inventory holding of product p at warehouse j (unit)

J_k inventory holding of a product at plant k (unit)

J_{kp} inventory holding of product p at plant k (unit)

Parameters

D average demand per unit time of the buyer (unit)

A ordering cost of the buyer per order ($/order)

P production rate per unit time (unit)

S_0 initial setup cost of the vendor per setup ($/setup)

C_v production cost per unit paid by vendor ($/unit)

C_b purchase cost per unit paid by buyer, $C_b > C_v$ (money unit)

r_v fractional holding cost of the vendor per unit item per unit time

r_b fractional holding cost of the buyer per unit item per unit time

H_s non-fractional holding cost for the supplier per unit time per unit item ($/unit/unit time)

H_b non-fractional holding cost for the buyer per unit time per unit item ($/unit/unit time)

h_v non-fractional holding cost for vendor per unit item per unit time ($/unit/unit time)

π unit backlogging cost for the buyer ($/unit)

θ probability that the production process may go '*out-of-control*'

s replacement cost per unit defective item ($/unit)

σ standard deviation of demand per unit time (unit)

A_b area under the buyer's inventory

A_s area under the supplier's inventory

F transportation cost per delivery ($/delivery)

d deterioration rate

C_d deterioration cost per unit ($/unit)

p unit selling price of retailer/retail price ($/unit)

w unit wholesale price ($/unit)

h_r^{TS} holding cost of the retailer under the traditional system ($/unit/unit time)

h_r^{CP} holding cost of the retailer under the consignment policy ($/unit/unit time)

h_m^{CP} holding cost of the manufacturer under the consignment policy ($/unit/unit time)

s_r goodwill loss per unit item for the retailer ($/unit)

s_m goodwill loss per unit item for the manufacturer ($/unit)

c manufacturing cost for unit item ($/unit)

WC_j capacity of warehouse j (unit)

PC_k capacity of plant k (unit)

D_i demand of retailer i (unit)

D_{ip} demand of product p of retailer i (unit)

TCW_j total cost of warehouse j open (\$)

TCP_k total cost of plant k open (\$)

PTC_{jk} production and transportation cost per unit of product from plant k to warehouse j (\$/unit)

PTC_{jkp} production and transportation cost per unit of product p from plant k to warehouse j (\$/unit)

PTC^t_{jkp} production and transportation cost per unit of product p from plant k to warehouse j via transportation mode t (\$/unit)

TC_{ij} transportation cost per unit of product from warehouse j to retailer i (\$/unit)

TC_{ijp} transportation cost per unit of product p from warehouse j to retailer i (\$/unit)

TC^t_{ijp} transportation cost per unit of product p from warehouse j to retailer i via transportation mode t (\$/unit)

IC_j unit inventory holding cost of a product at warehouse j (\$/unit)

IC_{jp} unit inventory holding cost of product p at warehouse j (\$/unit)

JC_k unit inventory holding cost of a product at plant k (\$/unit)

JC_{kp} unit inventory holding cost of product p at plant k (\$/unit)

OSC_i transportation cost per unit of product to retailer i from an outside supplier (\$/unit)

OSC_{ip} transportation cost per unit of product p to retailer i from an outside supplier (\$/unit)

OSC_{ip}^t transportation cost per unit of product p to retailer i from an outside supplier via transportation mode t (\$/unit)

M monetary value per unit of lead time (\$/unit)

M_p monetary value per unit of lead time for product p (\$/unit)

TWR_{ij} delivery lead time per unit of product from warehouse j to retailer i (time unit)

TWR_{ijp} delivery lead time per unit of product p from warehouse j to retailer i (time unit)

TWR_{ijp}^t delivery lead time per unit of product p from warehouse j to retailer i via transportation mode t (time unit)

TPR_{jk} delivery lead time per unit of product from plant k to warehouse j (time unit)

TPR_{jkp} delivery lead time per unit of product p from plant k to warehouse j (time unit)

TPR_{jkp}^t delivery lead time per unit of product p from plant k to warehouse j via transportation mode t (time unit)

n number of buyers (unit)

d_i average demand/unit time for buyer i (unit)

A_{bi} ordering cost of i^{th} buyer per order (\$/order)

h_{bi} non-fractional holding cost of i^{th} buyer per unit item per unit time (\$/unit/unit time)

σ_i standard deviation of the demand/unit time (unit)

π_i unit backorder cost for buyer i (\$/unit)

$E(\cdot)$ mathematical expectation

x^+ maximum value of x and 0

X lead time demand having a cumulative distribution function (c.d.f) with a finite mean and standard deviation (unit)

π_r^{TS} retailer's profit for the traditional policy (\$)

π_m^{TS} manufacturer's profit for the traditional policy (\$)

π_r^{CP} retailer's profit for the consignment policy (\$)

π_m^{CP} manufacturer's profit for the consignment policy (\$)

π_t^{CP} joint profit for the consignment policy (\$)

I set of retailer $i \in I$

J set of potential warehouse sites $j \in J$

K set of plant sites $k \in K$

P set of different product types $p \in P$

T set of available transportation modes $t \in T$

Chapter 1
Abstract

1 Abstract

The research study consists of recent development on supply chain model which is entitled as '**Some problems on supply chain management**'. A supply chain is generally formed with various stages, most commonly named as players or parties. Players of the supply chain always maintain some rules or policies (cooperative or non-cooperative) to gain more for any purpose as cost, benefits, and others. The supply chain is a flow of goods, information, and funds among its different stages. This research successfully resolves a number of critical problems which are hurdles to form a successful supply chain. This dissertation consists of nine (excluding Chapter 1) chapters. Every chapter, except the introductory part, contains a descriptive mathematical model related to supply chain management under various realistic circumstances. The topic of each chapter is stated below.

Chapter 2 contains the introduction of the dissertation. This part includes overview and mathematical background of supply chain management, definitions, brief description of various supply chain models and their industrial benefits. **Chapter 3** deals with a single-vendor single-buyer supply chain model with single-setup multi-delivery (SSMD) policy. An effort for vendor's setup cost reduction is considered to gain more at the optimum level. **Chapter 4** is an extended version of the first one with imperfect quality of products. **Chapter 5** extends all previous models with setup cost reduction, quality improvement, and deterioration of products under just-in-time manufacturing system. **Chapter 6** considers the price dependent variable demand, which is a decreasing function of selling-price. The model is developed under both fixed and variable purchasing cost. **Chapter 7** consists of a different delivery policy named as consignment policy. The model is solved using a distribution free approach with known mean and standard deviation. An effective approach is also proposed to enhance the agreement policy of consignment contract. **Chapter 8** is constructed with a single-vendor multi-buyer supply chain model under variable production cost, which is dependent on the production rate. **Chapter 9** extends the previous model with the reliability of the production process. **Chapter 10** consists of a three-echelon facility location model. This research emphasizes on a comparative study among three different dimensional facility location problems.

Chapter 2
Introduction

2 Introduction

Now a days, in almost every social and business sectors supply chain management plays an important role. A proper development of supply chain leads to a revolution of world economy which results the prosperity of the social-economic background of the world. In this section, the definition, examples, and various types of supply chain management problems are described.

2.1 Overview and definition of supply chain management

A supply chain is composed of all contributors involved in the production of goods from raw materials to finished goods. It includes manufacturing, framing, packaging, design, transportation. Retailers manage supply chain to control the inventory level, timing, product quality, and costs. Thus, an uninterrupted flow of materials, information, and fund is one of the most important attributes to achieve a successful supply chain.

Definition

Supply chain management (SCM) is handling of the flow of goods, data, and information among the multiple nodes of an extended industry sector. These nodes include manufacturers, vendors, retailers, customers or other players with several facilities. The ultimate goal of the supply chain management is to satisfy the customer's demand. Whatever uncertainties are, the demand has to be met at the exact location, with exact amount, and with insignificant delay in time.

All facilities include every function of receiving and satisfying the demand of any other facility or organization. Functions performed to satisfy the demand are marketing, operations, distribution, development of product, finance, and customer service.

An example of a Wall-Mart store is considered. A customer enters a store to purchase a product. The supply chain begins with the demand of the customer. The next step is the Wall-Mart retailer store, where the customer goes to purchase product. The inventory of the

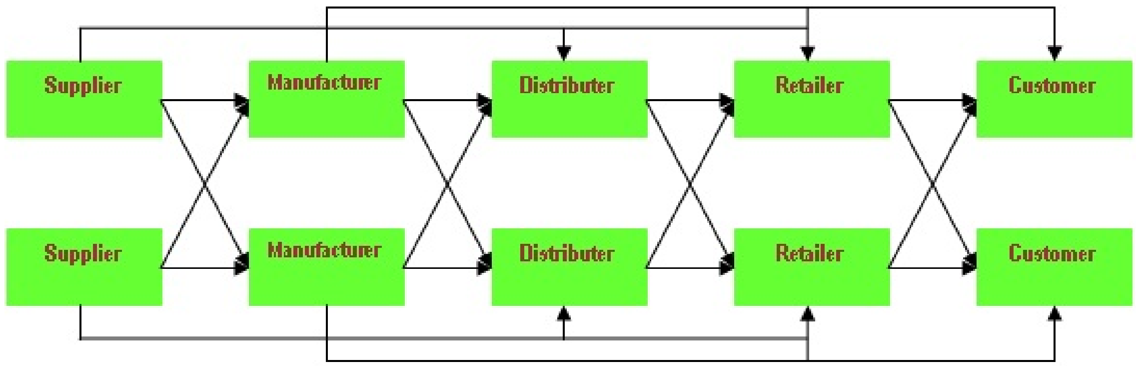

Figure 1 Coordination in various supply chain stages

retailer store comes from a warehouse or distributor through any transportation mode and the stock of goods is maintained by the manufacturing plant. The manufacturing plant receives raw materials from various suppliers.

2.2 Stages of a supply chain

The primary aim of a supply chain is to satisfy the customer's need and at the same time make a profit for itself. The term supply chain indicates that it is a flow of information, products or materials among suppliers, manufacturers or distributors, retailers, and customers. Therefore, stages of supply chain are categorized as follows:

1. Customers
2. Retailers
3. Warehouses/distributers
4. Manufactures
5. Raw material suppliers

2.3 Objective of a supply chain

The objective of every supply chain is to optimize i.e. to maximize or to minimize the total profit or cost of the chain, respectively. The aim of a supply chain is to maximize of the overall profit generated by every stage of the chain. The profit of the supply chain is the difference between the revenue earned from the customer and total costs of all stages of the chain to satisfy the customer's demand.

• *Supply chain profitability*

The supply chain profitability or surplus refers to the difference between the revenue generated from customers and total cost throughout the supply chain. The cost of each stage is highly correlated with the supply chain profitability in most of the commercial environment.

If a customer purchases a product for $100, the revenue of the supply chain is $100. Costs incurred by the chain are for production, transportation, conveying information. The difference between the price paid by the customer i.e. $100 and the total cost of the overall supply chain to meet the customer's demand of that particular product represents is supposed $80, then the supply chain profitability is $20 for a particular product.

• *Source of revenue and cost*

The customer is generally the one and only one source of revenue in a supply chain. Whenever a customer pays an amount, provides positive cash flow in a supply chain, all the other cash flows are the exchange of fund, provided by customers, between stages of the supply chain, retailers take a part of the fund given by customers and pass the rest to their suppliers. The same procedure happens for suppliers and distributors.

2.4 Decisions in a supply chain

Decision phases in a supply chain to convey information, flow of products and raw materials for high supply chain profitability are as follows:

1. Supply chain design or strategy

2. Supply chain planning

3. Supply chain operation

Above three decisions plays a vital role for a successful supply chain. Each decision is to be made to increase the supply chain profitability.

• *Supply chain design*

All strategic decisions to construct the supply chain over next several time periods are made during this phase. This decision phase includes the configuration of the overall chain, processes that each stage of the chain will perform and allocate several of resources. Locations of facilities (manufacturing plants, warehouses, retailers), capacities of plants and warehouses, type of product to be manufactured, type of transportation mode and the information system to be utilized are decided in this phase to maintain fruitful supply chain profitability.

• *Supply chain planning*

This phase of supply chain includes making the decision of what inventory policy is to be followed, the subcontracting of manufacturing, locating the market sites where products will be supplied from which locations, timing and size of marketing and price promotions. Uncertainties in demand, exchange rates are also regarded in this phase. The frame of time horizon is assumed as quarter of a year.

• *Supply chain operation*

In this phase, satisfying the customer's order is to be performed. For each individual order, the allocation of inventory and production are carried out. Firms assign an order to particular transportation mode, generate pick lists at some warehouses and place replenishment orders. The aim of this supply chain decision phase is to reduce the uncertainty and optimize performance. The time horizon considered is daily or weekly or sometime monthly.

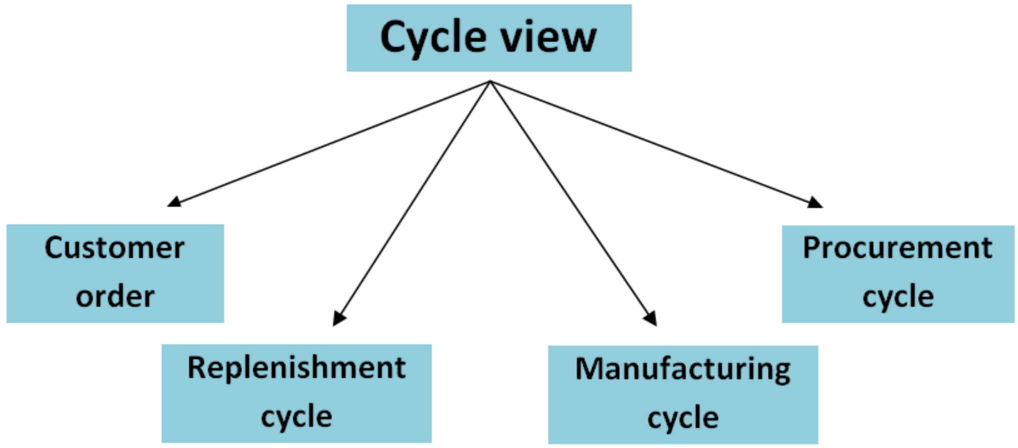

Figure 2 Cycle view

2.5 Supply chain process views

A supply chain consists of many processes and flows. These processes take place between various stages of the supply chain to satisfy customers demand. The supply chain processes can be viewed by two ways.

1. Cycle view

2. Push-pull view

Processes in a supply chain can be divided into cycles such that each cycle can be defined in between any two stages of the chain. If the supply chain has five stages, then there are four cycles. They are generally customer order cycle, replenishment cycle, manufacturing cycle, procurement cycle.

The execution of push and pull processes depend on awareness of the demand of the customer. The execution of push process is initiated in anticipation of the customer's demand i.e. when the actual demand is not known or forecasted. When the customer's demand is known, the pull process execution is initiated. Thus, the pull process takes place in response to the customer's demand.

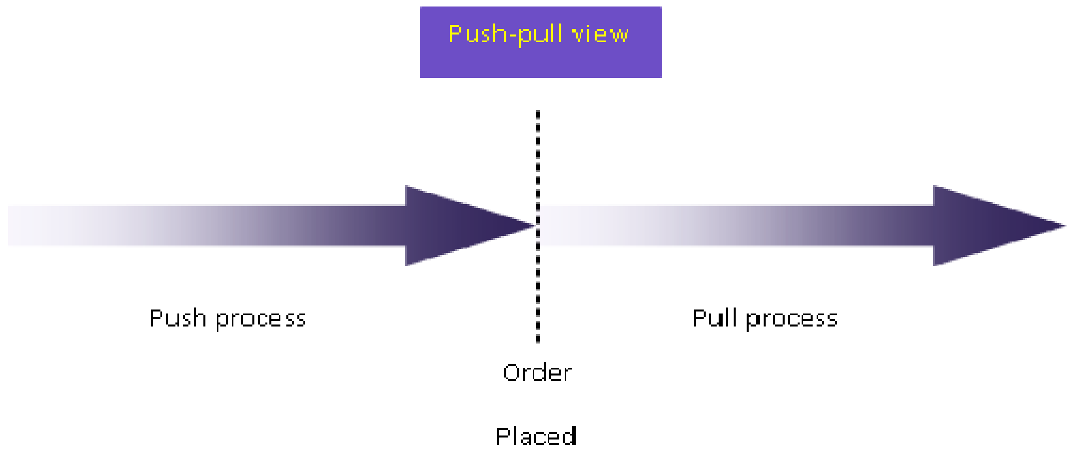

Figure 3 Push-pull view

2.6 Introduction of modelling within supply chain

There are many different models in the existing literature, which describe the decision making in a supply chain under different circumstances. Various situations include demand uncertainty, imperfect production, product deterioration, variable production rate, and variability of demand. Supply chain models under foretold situations are studied in next consecutive chapters. This section gives a thorough idea about some of the most widely used models in the modern marketing environment.

2.6.1 Inventory and integrated inventory models

Inventory is defined as the stock of items such as goods in warehouses or retailer shops. The inventory is accumulated by production lot from the manufacturing plants. This inventory can be used to satisfy customer's demand through supply chain.

As an example, in a retailer store, an average of 5 items of a product is purchased by customers per day. The storekeeper orders 150 items to the manufacturer each time of placing an order. Thus, 150 is the lot or batch size in this case. Now, according to the daily sale of the retailer's store, it takes an average of 30 days to sell all items and a new lot of 150 items to be purchased from the manufacturer. There is a time horizon during which the level of inventory rises up and reaches down to zero and again the lot size comes and goes and so on. Without shortages, each time period, in which the lot reaches at maximum inventory level just after receiving shipment and gradually decreasing and finally reaches to zero, is called the cycle period. Besides this costs incurred by the retailer are ordering cost, inventory holding cost, lead time cost, shortage cost, deterioration cost etc. The mathematical model for an inventory management is to obtain the optimal inventory level such that the total cost incurred during a cycle period is minimized. There are generally two types of inventory model. They are given by

1. Economic order quantity model

2. Economic manufacturing quantity model

An EOQ (Economic order quantity) model is developed to optimize the level of inventory such that the total inventory cost in a cycle period is minimized. In this model, whenever the inventory level comes to zero, each new order is delivered and the replenishment rate is infinite.

The EMQ (Economic manufacturing quantity) has the same objective. But unlike EOQ model, each order is produced and the replenishment rate is finite.

Safety inventory

In modern global economic environment, the demand of the customer is not always fixed. Every company manages their level of inventories by means of forecasting of demand. Due to the uncertainty of demand, the actual demand of the customer does not match to the forecasting demand. Here is the necessity of safety stock. Safety stock is the additional stock after the nullifying of actual inventory level. It satisfies the customer's demand if the inventory level reaches to zero during delivery lead time.

The integrated vendor buyer model is the combined model of vendors and buyers where the objective is to optimize the inventory level and safety stock (if considered) in order to minimize the expected joint total cost of buyer and vendor.

Several inventory costs

- ***Ordering cost***

The ordering cost is incurred every time the order is placed. In the other word this cost is needed for the replenishment of the inventory. The ordering cost can be divided into two parts.

Fixed cost for ordering

This is a fixed cost incurred when the order is placed and independent of the amount of product delivered. This cost consists of the cost of accounting, communication or the delivery cost when the order is placed.

Logistics cost

This cost refers to the variable cost for transportation while delivering items from one party to another.

- ***Holding cost/carrying cost***

This cost is incurred for holding or maintaining the inventory in one's shelter. As the inventory level is being changed by time, this cost depends on the average inventory level. This includes financing or capital cost which includes every kind of cost related to the investment, cost for storage spaces i.e., the cost incurred for maintaining goods (air conditioning, heating etc.) and lease or property taxes, different service costs which include insurance, costs for IT services or hardware and application etc., deterioration or inventory risk cost.

- ***Shortage cost***

Shortages occur when the supply of product is less than the customer's demand. Shortages can be explained by two ways one is fully backordered and another one is partial backordered.

Fully backordering

In this case customers wait until the delivery of the item. Therefore, there is no chance of occurrence of any lost sale. The exact magnitude of backorder cost is unknown, generally it is incurred by the extra paper or managerial cost in processing the order.

Partial backordering

In this condition, the customer does not wait for the delivery of the item which leads to lost sales in other words the retailer faces goodwill loss. This goodwill loss is expected to increase proportional to the time of delay.

Newsvendor problem

The newsvendor model is widely used in inventory and supply chain management. The intension of this model is to decide the order quantity under the a single purchasing opportunity before the selling period starts and the demand of the customer is random (Gallego and Moon, 1993). The newsvendor may face the overage or underage costs if the order is very high or very low, respectively compared to the customer demand (Gallego, 1995). A classical newsvendor problem with perishable item has become an interesting topic among researchers worldwide as a high managerial importance lies underneath the overstocking problem in case of product of short lifetime. The perishable product may damage if any extra items are left in stock after satisfying the customer demand. This model is useful for the industry related to any perishable agricultural product, baked or dairy food (Pal *et al.*, 2013). This model is also often used in order to make decisions in fashion and sports goods industries (Gallego and Moon, 1993).

Integrated vendor-buyer problem

A single buyer or single vendor problem consists of only single inventory model of either vendor or buyer. But, in modern global markets, every industry has to choose new technologies and strategies to survive. Integrated vendor-buyer model is the combined optimization of vendor's and buyer's inventory decisions together. Its main objective is to optimize the expected joint total cost/profit of both vendor and buyer. Thus, managing manufacturing, inventory and as well as supply chain are matters of concern too.

JIT (Just in time) production has the capability to strengthen the competiveness of the firm by reducing the waste, improving product quality and efficiency of production. It was Japanese manufacturing technique and adopted by various Japanese manufacturing organizations since

1970. Benefits of achieving JIT are,

1. Reduction of the level of inventory, purchased goods and finished goods
2. Reduction of the requirement of space
3. Reduction of manufacturing lead time
4. Increase of product quality
5. Increase of productivity level and machine utilization

JIT has an important role in integrated inventory systems. It reduces the manufacturing time and delivery lead time between different stages in supply chain. Fig 4 represents an integrated supply chain system with single-vendor and single-buyer. The notation P, Q, D, and S indicates production rate of the vendor, order quantity of the buyer, demand of the buyer, and safety stock, respectively.

2.6.2 Facility location problem

Facility location problem is one of the most important parts in SCM. Supply chain stages like supplier, manufacturer, warehouse, retailer etc are denoted by facilities. The objective of FLP (Facility location problem) is to locate facilities in different locations in such a way that the total network cost of the supply chain is to be minimized. To satisfy the customer's demand each and every facility must communicates with each other.

The echelon of a FLP is referred to the steps. As an example, if the supply chain consists of any two of the facilities such as supplier to manufacturer or warehouses to retailers then the FLP is said to be one echelon FLP and if it has three facilities, then is called two echelon FLP and so on. Any FLP containing more than two facilities is said to be multi-echelon FLP.

Location problems generally are of two types such as,

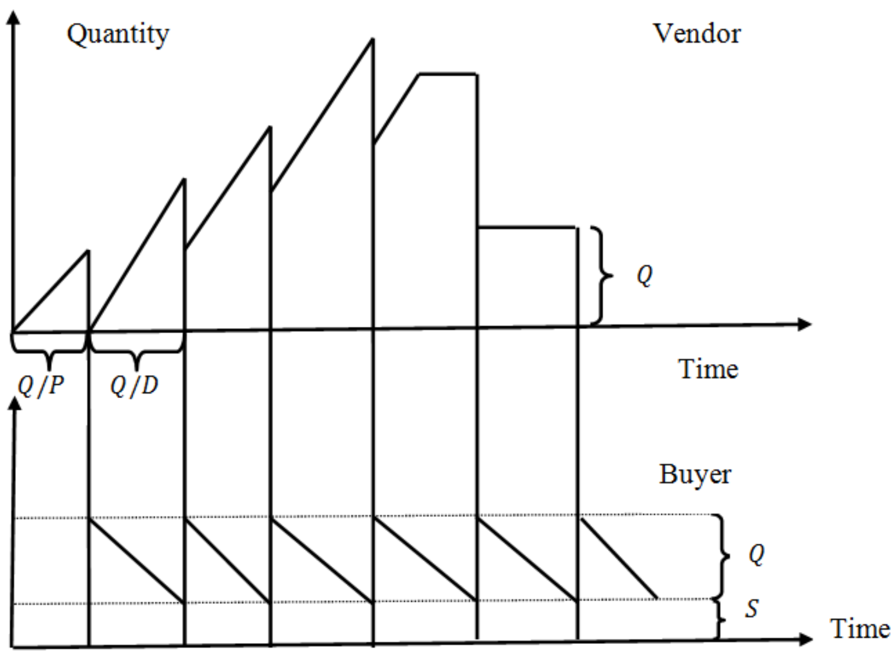

Figure 4 Integrated vendor-buyer model

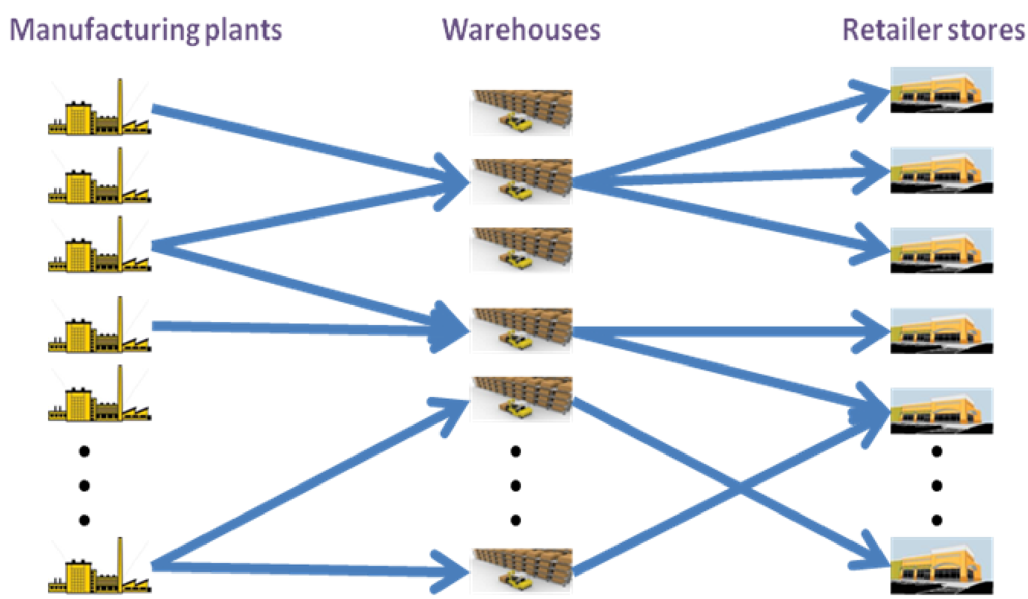

Figure 5 Facility location problem

1. Unapacitated facility location problem

2. Capacitated facility location problem

Uncapacitated FLP is that type of location problem where the capacity of facilities is not taken under consideration. In this type of problem, limitations of production of the plant or the inventory holding capacity of warehouse or retailer are negligible. But in case of capacitated FLP all the above capacities for facilities are assumed.

When a FLP model is developed, is transformed into a MILP (mixed integer linear programming problem). The objective of MILP is to optimize a linear objective function with respect to some linear constraint and some of the variables are treated as integers.

2.7 Delivery policies among parties in a supply chain

To achieve a successful supply chain, a suitable delivery policy must be selected by parties such that lead time as well as the cost is minimized. There are several policies to deliver products between parties. Every policy has its own importance depending on different marketing environment. A brief discussion on some of these policies is stated below.

2.7.1 Single-setup single-delivery (SSSD) policy

This policy is also known as 'lot-for-lot' policy or single time delivery policy and was proposed by Banerjee (1986). The manufacturer produces a lot whenever an order comes from the retailer. In this case, the manufacturer makes just one lot in one setup, which is demanded by the retailer and no extra item is produced. The final product or lot is delivered to the retailer. Costs incurred by both the parties are

Manufacturer	Retailer
1. Setup cost 2. Holding cost	1. Ordering cost 2. Holding cost

2.7.2 Single-setup multi-delivery (SSMD) policy

This delivery policy was introduced by Goyel (1986). Other than SSSD, this policy supports delivery over multiple times. When the retailer's order comes, the manufacturer produces an integer multiple of the retailer's order quantity and delivers over multiple segment of times (Ouyang *et al.*, 2004). In 'lot-splitting' case, the manufacturer splits the order quantity of the retailer and delivers over multiple times. In this case, the manufacturer makes the same amount as ordered by the buyer, but divides it into an equal parts. The cost components are same as SSSD policy for both parties.

2.7.3 Consignment policy

Consignment policy (CP) is a new approach in supply chain management which became popular especially in heath care field. Many departmental stores and online shopping industries such as Wall-Mart, amazon.com etc., have been adopting this policy. The major advantage of CP is to reduce the vendor's/supplier's inventory level as vendor uses buyer's/retailer's warehouse to stock items.

CP is is formed by an agreement of two parties namely the 'consignor' and the 'consignee'. The 'consignor' is mainly the owner of goods (manufacturer) who delivers products to the other party 'consignee' (retailer). The following assumptions are considered to develop a successful CP.

1. Manufacturer uses retailer's inventory to stock its produced items but retains the ownership of goods.
2. The inventory holding cost is shared between two parties. The operational part of the holding cost is incurred by the retailer and manufacturer bears the financial part.
3. No fund transfer is occurred until a product is sold.
4. Retailer gets a per unit commission from manufacturer for each sold item sold.

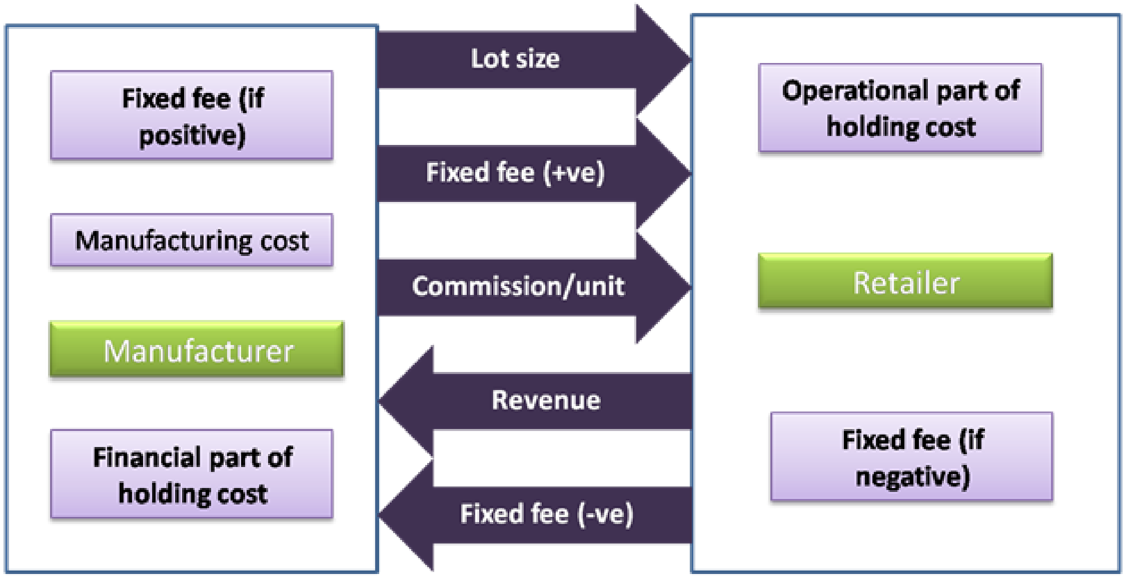

Figure 6 Outline of consignment policy

As there is a matter of gaining per unit commission for the sell of each item, retailer accepts the contract only if the consignment agreement favours the 'consignee' party.

2.8 Supply chain model under uncertain and variable demand

Demand is the most vital factor for any industrial model not only for supply chain. No business would ever exist if there is no demand. Thus, understanding the nature of demand is one of the key factors to achieve a successful supply chain. Even, the profit or loss of an industry directly depends on this factor.

Initially, some models considered the annual demand as a fixed quantity. But, in reality, it is quite impossible to predict the exact figure. In most of the cases, demand is uncertain and can be predicted under some probability distribution, sometimes demand is variable and varies over some factors like stock, selling-price etc. Even, both uncertainty and variability may occurs simultaneously. Remembering the above points this section describes some realistic nature of demand.

2.8.1 Uncertain demand

A vary common phenomenon is that the customer demand per unit time (year, week, day etc.,) is not fixed. To manage the supply chain and to maximize the supply chain profitability the manager of every industry must estimate the customer demand. An useful methodology to estimate demand is to consider it as a random variable. Analyzing the previous demand data a suitable probability distribution of demand can be formed. By the help of that specific distribution a mathematical model can be formed and managerial decisions can be made easily.

2.8.2 Variable demand

There is a difference between randomness and variability. An uncertain random demand can vary over many factors. Some examples should be given in this context.

1. Customer demand may increase over increasing stock.

2. Demand may decrease over increasing selling-price.

In the first example, demand is an increasing function of stock i.e., if the retail outlet/shop has a significant number of product in its stock, customer demand may increase. On the other hand, demand may decrease with increasing selling-price which implies it is a decreasing function of price. Both examples show the customer demand as a continuous dependent variable in stock and selling-price, respectively.

2.8.3 Distribution free approach

This is a very efficient procedure to make decisions under uncertainty. This method is applicable when the information about the random variable is insufficient and difficulty occurs in order to obtain the actual probability distribution of the random variable. As an example, in a classical continuous or periodic review inventory model, an uncertain the lead time demand is treated to be an uncertain entity which leads to the requirement of demand information. However, the exact probability distribution of the lead time is often difficult to obtain. Sometimes the only information available are just a conjectured mean and variance. In this case the tendency of considering a normal distribution is quite habituated which does not provide the best fit against the occurrence of other distribution. Therefore, only two ways are left to solve this problem.

1. Obtaining the proper information about lead time demand in order to get knowledged about exact probability distribution. This process requires a lot of fund to gather the lead time demand data.

2. Discovering a procedure to obtain managerial decisions without having the appropriate demand distribution such that one can solve the model with just on hand mean and variance of lead time demand.

Distribution free approach gives us the opportunity to solve the model even if no proper information about lead time demand distribution is available. Thus, the manager can make decisions with just an educated guess of mean and variance of lead time demand.

Chapter 3
Integrated vendor-buyer supply chain model with vendor's setup cost reduction

3 Integrated vendor-buyer supply chain model with vendor's setup cost reduction

This chapter deals with an integrated vendor-buyer supply chain model. Two models are constructed based on the probability distribution of the lead time demand. The lead time demand is stochastic following a normal distribution in the first model and in the second model, the lead time demand is considered as random, but without any specific probability distribution except for educated mean and standard deviation. The aim of this research is to reduce the total system cost by considering the setup cost reduction of the vendor.

3.1 Literature review

The supply chain model is used to minimize the cost or to maximize the profit throughout the network under the condition that demands of each facilities have to be met. Thus, the integrated inventory control policy is a matter of concern (for instances Villa (2001), Yang and Wee (2001), Viswanathan (1998), and Bylka (2003)). Goyal (1976) developed the first research work on the integrated vendor-buyer problem. Banerjee (1986) extended Goyal's (1976) model with lot-for-lot (LFL) policy. Goyal (1988) extended Banerjee's (1986) model by assuming the manufacturing quantity of the vendor is an integer multiple of the order quantity of the buyer as single-setup-multiple-delivery (SSMD) policy. Huang (2002) developed an integrated vendor-buyer model in an imperfect production process. Cárdenas-Barrón *et al.* (2011) used the arithmetic-geometric inequality to solve a vendor-buyer integrated inventory model with a closed−form solution. Teng *et al.* (2011) considered a vendor-buyer inventory model with a closed-form optimal solution.

Controlling the lead time plays an important role for any inventory model. Tersine (1994) introduced the lead time as a partition of five components as the supplier's lead time, order

B. Sarkar, A. Majumder, Integrated vendor-buyer supply chain model with vendor's setup cost reduction, Applied Mathematics and Computation. 2013, 224, 362-371.

preparation, order transit, delivery time, and the setup time. Liao and Shyu (1991) considered the lead time as a unique decision variable in their inventory model. Ben-Daya and Raouf (1994) explained both the ordering quantity and the lead time as decision variables without shortages. Ouyang *et al.* (1996) modified Ben-Daya and Raouf's (1994) model in view of shortages but they made a mistake which is corrected by Moon and Choi (1998). Hariga and Ben-Daya (1999) developed some stochastic inventory models with variable lead time. Pan and Yang (2002) considered the lead time as a controllable factor to obtain the expected joint total cost.

Scarf (1958) first established the min-max distribution free approach for the newsvendor problem without any information about the distribution of the lead time demand except mean and standard deviation. Gallego and Moon (1993) made Scarf's (1958) ordering rule very easy. After Gallego and Moon's (1993) proof, the distribution free approach becomes a very famous approach for solving inventory models without any specific distribution of the lead time demand except mean and standard deviation.

Moon and Gallego (1994) found out some valuable applications of the distribution free approach for different types of inventory models. After this model, the distribution free approach has been applied by many researchers from the different sectors. Moon and Yun (1997) considered the distribution free job control problem. Moon and Choi (1997) explained the distribution free procedure for make-to-order (MTO), make-in-order (MIA), and composite policies. Ouyang *et al.* (2002) developed an inventory model with the product's quality improvement and the vendor's setup cost reduction. Ouyang *et al.* (2004) explained an integrated production inventory model with the controllable lead time and shortages including a long-term strategic supply chain between the buyer and the vendor. They simultaneously optimized the lead time, reorder point, number of lots delivered, and the ordering quantity. The setup cost for the vendor was treated to be fixed in that model.

Lin (2011) proposed a min-max distribution free approach for the integrated inventory model with the defective goods and the probabilistic lead time demand. Using min-max distribution free approach Liao *et al.* (2011) discussed a newsvendor model with lost sales penalty and the balking policy. Lee *et al.* (2006) developed an inventory model with the negative exponential backorder cost and the mixture of distribution for the lead time demand. Hsu and Lee (2009) investigated a single-manufacturer multiple-retailer supply chain model with the distribution free approach. Jha and Shankar (2009) considered a single-vendor single-buyer supply chain model with a service level constraint. Annadurai and Uthayakumar (2010) developed a *(T,R,L)* inventory model with the controllable lead time and lost sales reduction. Lin *et al.* (2012) developed an integrated supplier-retailer inventory model with defective items and trade credit policy.

3.2 Model formulation

This section contains assumptions to formulate a mathematical model, entire model description, propositions, and solution algorithms.

3.2.1 Assumptions

Following assumptions are considered to develop the model

1. An integrated vendor-buyer model is considered.
2. When the buyer orders a lot size Q, the vendor manufactures the lot mQ with finite production rate $P(P > D)$ at one setup but delivers the quantity Q over m times.
3. The buyer places an order when the level of inventory reaches to the reorder point R.
4. The reorder point is $R = DL + k\sigma\sqrt{L}$, where $DL =$ the expected demand during the lead time, $k\sigma\sqrt{L} =$ safety stock, and $k =$ safety factor.
5. Shortages are allowed and fully backordered.
6. The lead time L consists of n mutually independent components. For the ith component, $a_i =$ minimum duration, $b_i =$ normal duration, and $c_i =$ crashing cost per unit time. For the sake of convenience, we assume $c_1 \leq c_2 \leq ... \leq c_n$.

7. We assume $L_0 \equiv \sum_{j=1}^{n} b_j$ and L_i be the lead time length having components $1, 2, ..., i$ crashed to their minimum duration, then, $L_i = L_0 - \sum_{j=1}^{i}(b_j - a_j)$, $i = 1, 2, ...n$. The lead time crashing cost/cycle $C(L)$ is expressed as $C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.

8. The transportation cost per unit time from the vendor to the buyer is constant and independent of the quantity ordered. Thus, the total transportation cost per unit time is neglected.

We consider the single-setup-multiple-delivery (SSMD) policy in an integrated vendor-buyer model i.e., if the buyer orders quantity Q, then the vendor produces the quantity mQ where m is any positive integer, and the quantity Q is delivered to the buyer over m times. The vendor produces the quantity mQ in one production cycle. Thus, the expected cycle length for the vendor and the buyer are $\frac{mQ}{D}$ and $\frac{Q}{D}$, respectively. The ordering cost per unit time is $\frac{AD}{Q}$. When the inventory level reaches to the reorder point R, an order of quantity Q is placed by the buyer. The expected inventory level before receipt an order is $R - DL$ and the expected inventory level immediately after the delivery of quantity Q is $Q + (R - DL)$. Hence, the average inventory over a cycle can be written as $\frac{Q}{2} + R - DL$ which implies that the buyer's expected holding cost per unit time becomes $r_b C_b[(\frac{Q}{2}) + R - DL]$.

X is normally distributed with finite mean DL and standard deviation $\sigma\sqrt{L}$. We assume that X has a cumulative distribution function F and the reorder point $R = DL + k\sigma\sqrt{L}$. If $X > R$, then shortage occurs. Hence, the expected shortage at cycle end is $E(X - R)^+ = \int_R^\infty (x-R)dF(x)$. The expected shortage cost per unit time is $\frac{\pi D E(X-R)^+}{Q}$. The lead time crashing cost per unit time is $\frac{DC(L)}{Q}$.

Therefore, the total expected cost per unit time to the buyer is

$$\begin{aligned} ATC_b(Q, R, L) &= \text{ordering cost} + \text{holding cost} + \text{shortage cost} + \text{lead time crashing cost} \\ &= \frac{AD}{Q} + r_b C_b \left(\frac{Q}{2} + R - DL\right) + \frac{\pi D}{Q} E(X - R)^+ + \frac{DC(L)}{Q} \qquad (1) \end{aligned}$$

The total expected cost/unit time for vendor is

ATC_v = setup cost + holding cost.

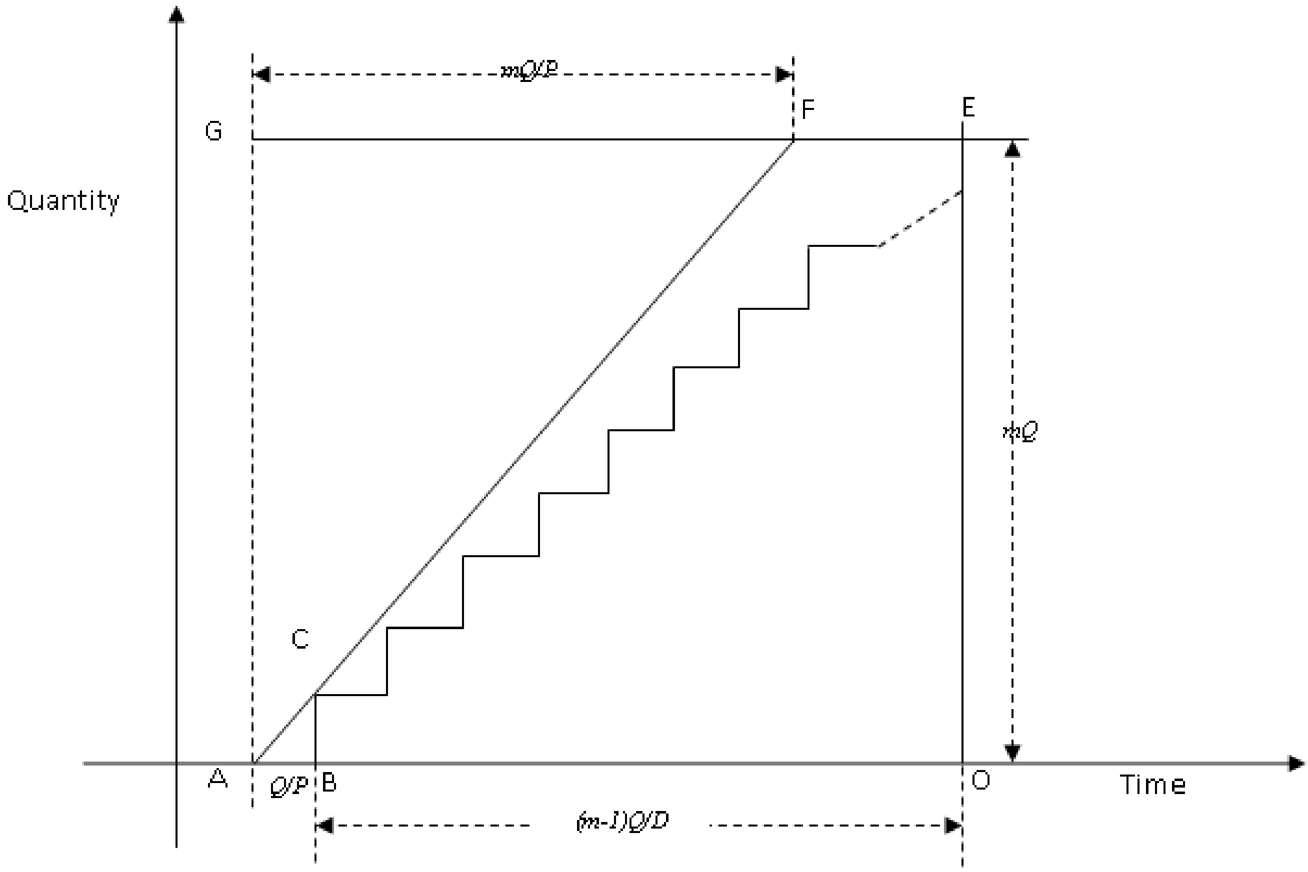

Figure 7 Vendor's inventory position

The expected cost for the vendor/unit time is $\frac{SD}{mQ}$

From figure 7 the average inventory of the vendor is

$$= \left[\left\{mQ\left(\frac{Q}{P}+(m-1)\frac{Q}{D}\right)-\frac{m^2Q^2}{2P}\right\}-\left\{\frac{Q^2}{D}(1+2+...+(m-1))\right\}\right]\frac{D}{mQ}$$

$$= \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] \text{ See Appendix A}$$

Hence, the expected holding cost per unit time for the vendor is

$$= r_vC_v\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]$$

Therefore, the total expected cost per unit time for the vendor is

$$ATC_v(Q,m) = \frac{SD}{mQ} + r_v C_v \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \tag{2}$$

Investment in setup cost reduction

We now investigate the effect of an investment on the vendor's setup cost reduction. In relation (2), the equation represents the vendor's total expected cost per unit time with fixed setup cost. But for more realistic issues, this cost can be considered as variable. We consider a capital investment for vendor's setup cost reduction.

If I_S is an investment for the setup cost reduction, then it can be expressed as

$$\begin{aligned} I_S &= B\ln\left(\frac{S_0}{S}\right) \\ &= B(\ln S_0 - \ln S) \text{ for } 0 < S \le S_0 \end{aligned}$$

where S_0 is the original setup cost, $B = \frac{1}{\delta}$, and δ = a decrease of percentage in S/dollar increase in I_S.

The total expected cost for the vendor per unit time is

$$ATC_v = \alpha B(\ln S_0 - \ln S) + \frac{SD}{mQ} + r_v C_v \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \tag{3}$$

where α is the annual fractional cost of the capital investment.

The resulting total expected cost per unit time for the buyer is

$$ATC_b = \frac{AD}{Q} + r_b C_b\left(\frac{Q}{2} + R - DL\right) + \frac{\pi D}{Q}E(X-R)^+ + \frac{D}{Q}C(L) \tag{4}$$

The expected shortage at the cycle end is

$$\begin{aligned} E(X-R)^+ &= \int_R^\infty (x-R)dF(x) \\ &= \sigma\sqrt{L}\psi(k) \end{aligned}$$

where $\psi(k) = \phi(k) - k[1-\Phi(k)]$, ϕ = standard normal probability density function, and Φ = cumulative distribution function of the normal distribution. The safety factor k is assumed as a decision variable.

The expected joint total cost/unit time for the vendor and buyer can be expressed as

$$\begin{aligned} JATC(Q,k,S,L,m) &= ATC_b(Q,k,L)+ATC_v(Q,S,m) \\ &= \alpha B(\ln S_0-\ln S)+\frac{D}{Q}\left[A+\frac{S}{m}+\pi\sigma\sqrt{L}\psi(k)+C(L)\right] \\ &+ \frac{Q}{2}\left[r_bC_b+r_vC_v\left\{m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\}\right]+r_bC_bk\sigma\sqrt{L} \end{aligned} \tag{5}$$

The problem can be written as

$$\begin{aligned} \text{Min}\;\; JATC(Q,k,S,L,m) &= \alpha B(\ln S_0-\ln S)+\frac{D}{Q}\left[A+\frac{S}{m}+\pi\sigma\sqrt{L}\psi(k)+C(L)\right] \\ &+ \frac{Q}{2}H(m)+r_bC_bk\sigma\sqrt{L} \\ \text{subject to} \quad & 0<S\le S_0 \\ \text{where}\;\; H(m) &= r_bC_b+r_vC_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] \end{aligned} \tag{6}$$

It is a non-linear program and in order to solve it we just relax the constraint $0<S\le S_0$. For a fixed positive integer m, we take the partial derivatives of $JATC(Q,k,S,L,m)$ with respect to Q,k,S, and L to obtain the optimal solution.

$$\frac{\partial JATC(Q,k,S,L,m)}{\partial Q} = -\frac{D}{Q^2}\left[A+\frac{S}{m}+\pi\sigma\sqrt{L}\psi(k)+C(L)\right]+\frac{1}{2}H(m) \tag{7}$$

$$\frac{\partial JATC(Q,k,L,S,m)}{\partial k} = \frac{D}{Q}\pi\sigma\sqrt{L}[\Phi(k)-1]+r_bC_b\sigma\sqrt{L} \tag{8}$$

$$\frac{\partial JATC(Q,k,S,L,m)}{\partial S} = -\frac{\alpha B}{S}+\frac{D}{mQ} \tag{9}$$

$$\begin{aligned} \frac{\partial JATC(Q,k,S,L,m)}{\partial L} &= \frac{D}{2Q\sqrt{L}}\pi\sigma\psi(k)+\frac{D}{Q}\frac{\partial C(L)}{\partial L}+\frac{r_bC_bk\sigma}{2\sqrt{L}} \\ &= \frac{D}{Q}\left[\frac{\pi\sigma\psi(k)}{2\sqrt{L}}-c_i\right]+\frac{r_bC_bk\sigma}{2\sqrt{L}} \end{aligned} \tag{10}$$

For fixed Q,k,S, and m, the function $JATC(Q,k,L,S,m)$ is concave in L as

$$\frac{\partial^2 JATC(Q,k,S,L,m)}{\partial L^2} = -\frac{D}{4Q}\pi\sigma\psi(k)L^{-3/2}-\frac{1}{4}r_bC_bk\sigma L^{-3/2}<0.$$

Hence, for fixed Q,k,S, and m, the minimum value of $JATC(Q,k,L,S,m)$ is attained at the end of the interval $[L_i,L_{i-1}]$. Now for fixed positive integer m, values of Q, $\Phi(k)$, and S

are obtained by equating equations (7), (8), and (9) to zero as

$$
\begin{aligned}
Q &= \left\{ \frac{2D\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right]}{H(m)} \right\}^{\frac{1}{2}} && (11) \\
\Phi(k) &= 1 - \frac{r_b C_b Q}{D\pi} && (12) \\
S &= \frac{\alpha B Q m}{D} && (13)
\end{aligned}
$$

3.2.2 Proposition 1

If we denote Q^*, k^*, and S^* as optimal values of Q, k, and S, then for fixed m and $L \in [L_i, L_{i-1}]$, the expected joint total cost function $JATC(Q, k, S, L, m)$ has a global minimum at (Q^*, k^*, S^*) obtained from equations (11)-(13) if the constraint $0 < S \leq S_0$ is relaxed.

Proof See Appendix B.

The optimal value of m can be obtained when

$$JATC(m^* - 1) \geq JATC(m^*) \leq JATC(m^* + 1)$$

where m^* is the optimal value of m.

Now we consider the constraint $0 < S \leq S_0$. S^* is already assumed as the optimal value of S. From equation (13) it is clear that S^* is positive as values of α, B, Q, m, and D are all positive. If $S^* > S_0$, then no investment should be made for the reduction of the setup cost and we set $S = S_0$. An algorithm is developed to obtain the optimal values for Q, k, L, S, and m. The optimal reorder point can be found using the optimal value of k.

3.2.3 Solution algorithm 1

Step 1 Set $m = 1$.

Step 2 For each $L_i, i = 1, 2, ...n$; perform Step 2a−2f.

Step 2a Set $S_{i1} = 0$, $k_{i1} = 0$ (implies $\psi(k_{i1}) = 0.39894$).

Step 2b Substitute $\psi(k_{i1})$ into equation (11) and evaluate Q_{i1}.

Step 2c Utilize Q_{i1} to determine the value of $\Phi(k_{i2})$ from equation (12).

Step 2d For the value of $\Phi(k_{i2})$, find k_{i2} from the normal table and hence evaluate $\psi(k_{i2})$.

Step 2e Utilize Q_{i1} and find S_{i2} from equation (13).

Step 2f Repeat 2b to 2e until no changes occur in the values of Q_i, k_i, and S_i as well as denote these values by (Q_i^*, k_i^*, S_i^*).

Step 3 If $S_i^* < S_0$, then go to Step 4. Else set $S_i^* = S_0$ and utilize (11) and (12) to determine new optimal values of (Q, k) denoted by $(Q_i^{'*}, k_i^{'*})$ by substituting S by S_0 and using the same procedure stated as Step 2. Then go to Step 4.

Step 4 Find $JATC(Q_i^*, k_i^*, S_i^*, L_i, m)$ and $\text{Min}_{i=1,2,\ldots n} JATC(Q_i^*, k_i^*, S_i^*, L_i, m)$.

Step 4a If $JATC(Q_i^*, k_i^*, S_i^*, L_i, m) = \text{Min}_{i=1,2,\ldots n} JATC(Q_i^*, k_i^*, S_i^*, L_i, m)$,
then $JATC(Q_i^*, k_i^*, S_i^*, L_i, m)$ is the optimal solution for fixed m.

Step 5 Set $m = m + 1$.
If $JATC(Q_m^*, k_m^*, S_m^*, L_m, m) \leq JATC(Q_{m-1}^*, k_{m-1}^*, S_{m-1}^*, L_{m-1}, m-1)$, repeat Step 2, Step 3, and Step 4. Otherwise go to Step 6.

Step 6 Set $JATC(Q_m^*, k_m^*, S_m^*, L_m, m) = JATC(Q_{m-1}^*, k_{m-1}^*, S_{m-1}^*, L_{m-1}, m-1)$.
Then $(Q^*, k^*, L^*, S^*, m^*)$ is the optimal solution. The optimal reorder point can be obtained from $R^* = DL^* + k^*\sigma\sqrt{L^*}$, where R^* denotes the optimal reorder point.

3.2.4 Distribution free approach

We consider the distribution free approach for the same model stated above. We do not make any assumption for the distribution of the lead time demand X except that the cumulative distribution function (c.d.f.) F of the lead time demand belongs to the class $\Im$ of c.d.f. with mean DL and standard deviation $\sigma\sqrt{L}$. Only mean and standard deviation are known. The value of $E(X-R)^+$ cannot be determined exactly as the exact probability distribution of the lead time demand, X is unknown. Thus, a min-max distribution free approach is applied to solve this problem. The min-max distribution free approach is to determine the least favorable c.d.f. F in class $\Im$ for each (Q, R, S, L, m) and then to minimize the total expected joint annual cost over (Q, R, S, L, m), i.e.,

$$\begin{aligned} \text{Min Max}_{F\in\Im} \quad & JATC(Q, R, S, L, m) \\ \text{subject to} \quad & 0 < S \leq S_0 \end{aligned} \tag{14}$$

The following proposition is used to approximate the value of $E(X-R)^+$ which was proposed by Gallego and Moon (1993).

3.2.5 Proposition 2

For any $F \in \Im$, the following inequality always holds.

$$E(X-R)^+ \leq \frac{1}{2}\left[\sqrt{\sigma^2 L + (R-DL)^2} - (R-DL)\right] \tag{15}$$

Moreover the upper bound of the above equation is tight.

We have $R = DL + k\sigma\sqrt{L}$. By using relation (6) and inequality (15), model (14) can be written as

$$\begin{aligned} \text{Min} \quad JATC_f(Q, k, S, L, m) \quad &= \quad \alpha B(\ln S_0 - \ln S) + \frac{D}{Q}\Big[A + \frac{S}{m} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2} - k) \\ &+ \quad C(L)\Big] + \frac{Q}{2}H(m) + r_b C_b k\sigma\sqrt{L} \end{aligned} \tag{16}$$

where f denotes the distribution free case.

According to the previous normal distribution case, it can be shown that for fixed (Q, k, S)

$$\frac{\partial^2 JATC_f(Q, k, S, L, m)}{\partial L^2} < 0$$

i.e., $JATC_f(Q, k, S, L, m)$ is concave in L. Therefore, the minimum value of $JATC_f(Q, k, S, L, m)$ is attained at end points of the interval $[L_{i-1}, L_i]$. Again, for fixed m and $L \in [L_i, L_{i-1}]$, the minimum value of Q, k, and S is obtained by equating

$$\begin{aligned}\frac{\partial JATC_f(Q,k,S,L,m)}{\partial Q} &= 0\\ \frac{\partial JATC_f(Q,k,S,L,m)}{\partial k} &= 0\\ \frac{\partial JATC_f(Q,k,S,L,m)}{\partial S} &= 0\end{aligned}$$

We obtain the values as

$$Q = \left\{\frac{2D\left[A+\frac{S}{m}+\frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2})+C(L)\right]}{H(m)}\right\}^{\frac{1}{2}} \tag{17}$$

$$\frac{k}{\sqrt{1+k^2}} = 1-\frac{2Qr_bC_b}{D\pi} \tag{18}$$

$$S = \frac{\alpha BQm}{D} \tag{19}$$

3.2.6 Proposition 3

If we denote Q^{**}, k^{**}, and S^{**} as optimal values of Q, k, and S, then for fixed $L \in [L_i, L_{i-1}]$ and m, the expected joint total cost function for the distribution free approach $JATC_f(Q,k,L,S,m)$ has a global minimum at (Q^{**}, k^{**}, S^{**}) obtained from equations (17)-(19) if the constraint $0 < S \leq S_0$ is relaxed.

Proof Similar proof as **Proposition 1.**

The optimal value of m (say m^{**}) can be obtained from

$$JATC_f(m^{**}-1) \geq JATC_f(m^{**}) \leq JATC_f(m^{**}+1)$$

Taking into consideration the constraint $0 < S \leq S_0$, we set $S = S_0$ if the optimal value of S (say S^{**}) is higher than S_0. An algorithm for the distribution free approach is considered to determine $(Q^{**}, k^{**}, S^{**}, L^{**}, m^{**})$.

3.2.7 Solution algorithm 2

Step 1 Set $m = 1$.

Step 2 For each $L_i, i = 1, 2, ...n$; perform Step 2a$-$2e.

Step 2a Set $S_{i1} = 0$, $k_{i1} = 0$.

Step 2b Evaluate Q_{i1} from equation (17).

Step 2c Utilize Q_{i1} to determine the value of k_{i2} from equation (18).

Step 2d Utilize Q_{i1} and find S_{i2} from equation (19).

Step 2e Repeat 2b to 2d until no changes occur in the values of Q_i, k_i, and S_i as well as denote these values by $(Q_i^{**}, k_i^{**}, S_i^{**})$.

Step 3 If $S_i^{**} < S_0$, then go to Step 4. Else set $S_i^{**} = S_0$ and utilize (17) and (18) to determine new optimal values of (Q, k) denoted by $(Q_i^{'**}, k_i^{'**})$ by substituting S by S_0 and using the same procedure as stated in Step 2. Then go to Step 4.

Step 4 Find $JATC(Q_i^{**}, k_i^{**}, S_i^{**}, L_i, m)$ and $\text{Min}_{i=1,2,...n} JATC(Q_i^{**}, k_i^{**}, S_i^{*}, L_i, m)$.

Step 4a If $JATC(Q_i^{**}, k_i^{**}, S_i^{**}, L_i, m) = \text{Min}_{i=1,2,...n} JATC(Q_i^{**}, k_i^{**}, S_i^{**}, L_i, m)$, then $JATC(Q_i^{**}, k_i^{**}, S_i^{**}, L_i, m)$ is the optimal solution when m is fixed.

Step 5 Set $m = m + 1$.

If $JATC(Q_m^{**}, k_m^{**}, S_m^{**}, L_m, m) \leq JATC(Q_{m-1}^{**}, k_{m-1}^{**}, S_{m-1}^{**}, L_{m-1}, m-1)$, repeat Step 2, Step 3, and Step 4. Else go to Step 6.

Step 6 Set $JATC(Q_m^{**}, k_m^{**}, S_m^{**}, L_m, m) = JATC(Q_{m-1}^{**}, k_{m-1}^{**}, S_{m-1}^{**}, L_{m-1}, m-1)$.

$(Q^{**}, k^{**}, L^{**}, S^{**}, m^{**})$ is the optimal solution. The optimal reorder point can be obtained from $R^{**} = DL^{**} + k^{**}\sigma\sqrt{L^{**}}$, where R^{**} denotes the optimal reorder point for the distribution free model.

We note that for both algorithms in Step 3, if the reduced setup cost S for the vendor greater than the initial setup cost S_0, we will take S_0 as the vendor's setup cost and there will be no need to use vendor's setup cost reduction.

Table 3.1

Lead time data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

3.3 Numerical experiments

The parameter values used in the following examples are obtained from Ouyang *et al.* (2004).

Example 1

We consider an example to illustrate the algorithm described above. This example deals with the model having the lead time demand which follows the normal distribution. Let us consider $D = 600$ units/year, $A = \$200$/order, $C_b = \$100$/unit, $\pi = \$50$/unit, $\sigma = 7$ units/week, $P = 2000$ units/year, $S_0 = \$1500$/setup, $C_v = \$70$/unit, $r_b = \$0.2$/unit/year, $r_v = \$0.2$/unit/year, $\alpha = 0.1$/dollar/year, and $B = 18000$. The lead time has three components and the data for the lead time are shown in Table 3.1. We obtain some results which are described in Table 3.2.

Applying algorithm 1, we obtain the following results for Example 1. The optimal ordering quantity $Q^* = 134$ units, the lead time $L^* = 28$ days, the optimal setup cost for the vendor $S^* = \$1202.6$, the number of lots delivered from the vendor to the buyer $m^* = 3$, the reorder point $R^* = 65$ units, the minimum joint cost is \$6627.4/year.

Now we compare our numerical results to Banerjee's (1986) and Ouyang *et al.*'s (2004) model in order to examine the effect of considering the vendor's setup cost as variable. The summarization of these comparisons is shown in Table 3.3 and Table 3.4.

From Table 3.3 and Table 3.4, our model indicates the lowest cost compared to that of Banerjee's (1986), Goyal's (1988), and Ouyang *et al.*'s (2004) model for both $m = 1$ and $m > 1$.

Table 3.2

Solution of Example 1 (for normal distribution case)

m	L^*	Q_m^*	S_m^*	R_m^*	$JATC(Q_m^*, R_m^*, S_m^*, L_m^*, m)$
1	28	212	637.2	61	6981.7
2	28	162	972.7	63	6638.2
3	28	134	1202.6	65	6627.4 $^{a\leftarrow}$
4	28	115	1380.7	66	6716.0

$^{a\leftarrow}$ indicates the minimum expected joint total cost.

Table 3.3

Comparison table (for $m = 1$)

	Banerjee (1986)	Ouyang *et al.* (2004)	This model
Reorder point (R) (units)	–	58	61
Buyer's ordering quantity (Q) (units)	290	299	212
$JATC$ ($)	7948.9	7466.7	6981.7

– indicates the reorder point was not considered as a decision variable in Banerjee (1986).

Table 3.4

Comparison table (for $m > 1$)

	Goyal (1988)	Ouyang *et al.* (2004)	This model
Number of lots delivered (m)	2	3	3
Reorder point (R) (units)	–	64	65
Buyer's ordering quantity (Q) (units)	164	144	134
Vendor's lot size (mQ) (units)	328	432	402
$JATC$ (\$)	7875.1	6660.4	6627.4

– indicates the reorder point was not considered as a decision variable in Goyal (1988).

Example 2

The data for Example 2 are the same as Example 1. This is for the distribution free case. Applying algorithm 2, we obtain results as follows. The optimal ordering quantity $Q^{**} = 204$ units, the lead time $L^{**} = 28$ days, the optimal setup cost for the vendor $S^{**} = \$1227.4$, the number of lots delivered from the vendor to the buyer $m^{**} = 2$, the reorder point $R^{**} = 61$ units, and the minimum joint cost is \$6994/year. Solutions for Example 2 are summarized in Table 3.5.

3.3.1 Sensitivity analysis

For sensitivity analysis, we change each key parameter by -50%, -25%, $+25\%$, and $+50\%$. Each parameter is changed one at a time keeping other parameters fixed. The effect of changes of the key parameters are illustrated in Table 3.7 and Table 3.8.

Table 3.5

Solution of Example 2 (for the distribution free case)

m	L^{**}	Q_m^{**}	S_m^{**}	R_m^{**}	$JATC_f(Q_m^{**}, R_m^{**}, S_m^{**}, L_m^{**}, m)$
1	28	258	775.4	58	7244.2
2	28	204	1227.4	61	6994.4 $^{b\leftarrow}$
3	28	173	1555.6	63	7066.3

$^{b\leftarrow}$ indicates the joint total cost for the distribution free case.

Table 3.6

Sensitivity analysis for normal distribution case

Parameters	Changes(in %)	EAC^N	Parameters	Changes(in %)	EAC^N
A	-50%	-6.32	r_b	-50%	-14.55
	-25%	-2.99		-25%	-6.59
	$+25\%$	$+3.00$		$+25\%$	$+6.47$
	$+50\%$	$+6.33$		$+50\%$	$+11.89$
C_b	-50%	-16.65	r_v	-50%	-7.46
	-25%	-7.24		-25%	-4.00
	$+25\%$	$+7.11$		$+25\%$	$+4.39$
	$+50\%$	$+14.01$		$+50\%$	$+8.51$
C_v	-50%	-7.56	S_0	-50%	-19.21
	-25%	-3.89		-25%	-6.56
	$+25\%$	$+3.23$		$+25\%$	$+5.64$
	$+50\%$	$+6.32$		$+50\%$	$+11.91$

Table 3.7

Sensitivity analysis for distribution free case

Parameters	Changes(in %)	EAC^N	Parameters	Changes(in %)	EAC^N
A	-50%	-5.06	r_b	-50%	-16.15
	-25%	-2.33		-25%	-8.69
	$+25\%$	$+2.36$		$+25\%$	$+7.57$
	$+50\%$	$+4.87$		$+50\%$	$+15.02$
C_b	-50%	-15.99	r_v	-50%	-9.37
	-25%	-7.79		-25%	-3.86
	$+25\%$	$+6.97$		$+25\%$	$+4.01$
	$+50\%$	$+14.23$		$+50\%$	$+8.96$
C_v	-50%	-8.06	S_0	-50%	-17.53
	-25%	-4.56		-25%	-8.33
	$+25\%$	$+4.27$		$+25\%$	$+6.70$
	$+50\%$	$+8.14$		$+50\%$	$+11.26$

3.3.2 Evaluation of expected value of additional information (EVAI)

Now we compare results of the distribution free case to the normal distribution case. The lead time demand distribution is not known. Therefore, some additional information is needed to get familiar with the nature of the lead time demand distribution. From Table 3.2 and Table 3.5, we obtain $(Q^*, R^*, S^*, L^*, m^*) = (134, 65, 1202.6, 28, 3)$ and $(Q^{**}, R^{**}, S^{**}, L^{**}, m^{**}) = (204, 61, 1227.4, 28, 2)$. Therefore, the added cost will be $JATC(Q^{**}, R^{**}, S^{**}, L^{**}, m^{**})$ $-JATC(Q^*, R^*, S^*, L^*, m^*) = \$6651.86 - \$6627.4 = \$24.46$ which is less than 1% of the joint total cost for the distribution free case. This amount is said to be the expected value of additional information (EVAI) for the buyer. This is the largest amount that a buyer would be willing to incur to obtain the information of the lead time demand distribution. This concept was introduced by Moon and Gallego (1993).

3.4 Managerial insights

This chapter deals with a fruitful way for cost reduction in an integrated supply chain model. The managerial insights of this chapter are given as follows.

- The manager can reduce the setup cost of vendor by investing an amount of fund which results reduction of total supply chain cost also.
- The reduced cost shown in this study is lesser than the total cost shown in existing literature.
- Manager can decide whether to invest fund or not in collecting market demand information.

3.5 Concluding remarks

This study considered an integrated vendor-buyer supply chain model with the lead time, ordering quantity of the buyer, reorder point, quantity shifted from the vendor to the buyer, and the setup cost for the vendor as decision variables. An investment function was used to minimize the vendor's setup cost. The lead time demand is distributed normally. In the second model, the distribution free approach is applied for the lead time demand. We minimized the expected joint total cost for the buyer and the vendor for both the normal distribution and the distribution free cases. Finally, we saved more amount of money compared to the previous studies related to this problem. The model can be extended by improving the quality of products produced by the vendor. One immediate extension for this model is to consider an investment for buyer's ordering cost reduction. A fruitful research can be done by assuming a discrete investment to reduce vendor's setup cost instead of continuous investment. One another possible extension can be examined by considering the deterioration of the items.

3.6 Appendices of Chapter 3

Appendix A

Area ADEG is divided into two strips. These are $(mQ)(Q/P)$ and $(m-1)(Q/D)(mQ)$, respectively. Therefore,

$$\begin{aligned}
\text{Area ADEG} &= mQ(Q/P) + (m-1)(Q/D)(mQ) \\
&= (mQ)\{Q/P + (m-1)(Q/D)\} \\
\text{Area AGF} &= (1/2)(mQ/P)(mQ) \; = \; m^2Q^2/2P \\
\text{Area of the ladder} &= [Q + 2Q + 3Q + ... + (m-1)Q](Q/D) \\
\text{Total inventory of the vendor} &= \text{Area ADEG} - \text{Area AGF} - \text{Area of the ladder} \\
&= mQ\left[\frac{Q}{P} + (m-1)\frac{Q}{D}\right] - \frac{m^2Q^2}{2P} - \frac{Q^2}{D}[1 + 2 + ... + (m-1)]
\end{aligned}$$

Appendix B

Proof of Proposition 1.

For given value of L and m, the Hessian matrix H is as follows

$$H = \begin{bmatrix} \frac{\partial^2 JATC(\cdot)}{\partial Q^2} & \frac{\partial^2 JATC(\cdot)}{\partial Q \partial k} & \frac{\partial^2 JATC(\cdot)}{\partial Q \partial S} \\ \frac{\partial^2 JATC(\cdot)}{\partial k \partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial k^2} & \frac{\partial^2 JATC(\cdot)}{\partial k \partial S} \\ \frac{\partial^2 JATC(\cdot)}{\partial S \partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial S \partial k} & \frac{\partial^2 JATC(\cdot)}{\partial S^2} \end{bmatrix}$$

The symbol '.' represents independent variables of the function $JATC$. Now, the global minimum is attained only if principal minors of all order are positive. Expressions of all partial derivatives of the above Hessian matrix are derived below.

$$
\begin{aligned}
\frac{\partial^2 JATC(\cdot)}{\partial Q^2} &= \frac{2D}{Q^3}\left[A+\frac{S}{m}+\pi\sigma\sqrt{L}\psi(k)+C(L)\right]\\
\frac{\partial^2 JATC(\cdot)}{\partial Q\partial k} &= -\frac{D}{Q^2}\pi\sigma\sqrt{L}[\Phi(k)-1]\\
\frac{\partial^2 JATC(\cdot)}{\partial Q\partial S} &= -\frac{D}{mQ^2}\\
\frac{\partial^2 JATC(\cdot)}{\partial k\partial Q} &= -\frac{D}{Q^2}\pi\sigma\sqrt{L}[\Phi(k)-1]\\
\frac{\partial^2 JATC(\cdot)}{\partial k^2} &= \frac{D}{Q}\pi\sigma\sqrt{L}\phi(k)\\
\frac{\partial^2 JATC(\cdot)}{\partial k\partial S} &= 0\\
\frac{\partial^2 JATC(\cdot)}{\partial S\partial Q} &= -\frac{D}{Q^2 m}\\
\frac{\partial^2 JATC(\cdot)}{\partial S\partial k} &= 0\\
\frac{\partial^2 JATC(\cdot)}{\partial S^2} &= \frac{\alpha B}{S^2}
\end{aligned}
$$

The first order principal minor of $|H|$ is

$$
|H_{11}|_{(Q^*,k^*,S^*)} = \left|\frac{\partial^2 JATC(\cdot)}{\partial Q^2}\right|_{(Q^*,k^*,S^*)} = \frac{2D}{Q^{*3}}\left[A+\frac{S^*}{m}+\pi\sigma\sqrt{L}\psi(k^*)+C(L)\right] > 0
$$

The second order principal minor of $|H|$ is

$$
\begin{aligned}
|H_{22}|_{(Q^*,k^*,S^*)} &= \frac{\partial^2 JATC(\cdot)}{\partial Q^2}\frac{\partial^2 JATC(\cdot)}{\partial k^2} - \left(\frac{\partial^2 JATC(\cdot)}{\partial Q\partial k}\right)^2\\
&= \frac{2D}{Q^{*4}}\pi\sigma\sqrt{L}\psi(k^*)\left[A+\frac{S^*}{m}+\pi\sigma\sqrt{L}\psi(k^*)+C(L)\right] - \frac{D^2}{Q^{*4}}\pi^2\sigma^2 L\left[\Psi(k^*)-1\right]^2\\
&= \frac{2D^2}{Q^{*4}}\pi\sigma\sqrt{L}\psi(k^*)\left[A+\frac{S^*}{m}+C(L)\right]\\
&+ \frac{D^2}{Q^{*4}}\pi^2\sigma^2 L\left[2\psi(k^*)\phi(k^*)-(\Phi(k^*)-1)^2\right] > 0
\end{aligned}
$$

as $\phi(k)$, $\psi(k) > 0$ and $2\phi(k)\psi(k) - (\Phi(k)-1)^2 > 0$ for all $k > 0$ (Ouyang *et al.*, 2004).

The third order principal minor of $|H|$ is

$$
\begin{aligned}
|H_{33}|_{(Q^*,k^*,S^*)} &= \begin{vmatrix} \frac{\partial^2 JATC(\cdot)}{\partial Q^2} & \frac{\partial^2 JATC(\cdot)}{\partial Q \partial k} & \frac{\partial^2 JATC(\cdot)}{\partial Q \partial S} \\ \frac{\partial^2 JATC(\cdot)}{\partial k \partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial k^2} & \frac{\partial^2 JATC(\cdot)}{\partial k \partial S} \\ \frac{\partial^2 JATC(\cdot)}{\partial S \partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial S \partial k} & \frac{\partial^2 JATC(\cdot)}{\partial S^2} \end{vmatrix} \\
&= \frac{\alpha B}{S^{*2}} |H_{22}| - \frac{D^3}{Q^{*5} m^2} \pi \sigma \sqrt{L} \phi(k^*) \\
&> \frac{2D^2}{Q^{*4}} \pi^2 \sigma^2 L \phi(k^*) + \frac{2D^2}{Q^{*4}} \pi \sigma \sqrt{L} \psi(k^*) \left[A + \frac{S^*}{m} + C(L) \right] - \frac{D^3}{Q^{*5} m^2} \pi \sigma \sqrt{L} \phi(k^*) \\
&= \frac{D^2}{Q^{*4}} \pi \sigma \sqrt{L} \left[2\pi\sigma\sqrt{L}\phi(k^*) + 2\psi(k^*) \left\{ A + \frac{S^*}{m} + C(L) \right\} - \frac{D}{Qm^2} \phi(k^*) \right] > 0 \\
\text{as} \qquad & 2\pi\sigma\sqrt{L} + 2\psi(k^*) A + \frac{S^*}{m} + C(L) > \frac{D}{Q^* m^2}
\end{aligned}
$$

We see that all principal minors of the Hessian matrix are positive. Hence, the Hessian matrix, H is positive definite at (Q^*, k^*, S^*). Therefore, the total expected annual cost function has a global minimum at (Q^*, k^*, S^*).

Chapter 4
Manufacturing quality improvement and setup cost reduction in an integrated vendor-buyer supply chain system

4 Manufacturing quality improvement and setup cost reduction in an integrated vendor-buyer supply chain system

In this chapter, a method to improve the quality of a single type of product and reducing vendor's setup cost in a single-vendor and single-buyer model, is established. The buyer's demand is deterministic, but the lead time demand follows firstly a normal distribution and then follows no specific distribution except known mean and standard deviation. Based on the nature of lead time demand distribution, this research considers two different models. The procedure of reducing the vendor's setup cost and the manufacturing quality improvement of products are established analytically. In any production system with long run process, the occurrence of defective items arises. These items are reworked, rejected, or refused based on different policies of industries. Therefore, it is realistic that a cost to be incurred in order to improve quality of products. An investment can be done to improve the production process. The role of an investment is utilized to reduce the setup cost.

4.1 Literature review

Some literatures similar to Chapter 3 has been omitted as this chapter is an extension of the previous one. Some of the important literatures based on this topic is described. Goyal (1976) developed a single-supplier single-retailer integrated inventory model as the first research work in this field. Banerjee (1986) extended Goyal's (1976) model by assuming a joint economic lot size model. Goyal (1988) extended Banerjee's (1986) model with the vendor's production quantity as an integer multiple of buyer's ordering quantity. Huang (2002) introduced an imperfect quality products in an integrated vendor-buyer model.

Porteus (1985) introduced the concept of setup cost reduction in the inventory literature. Porteus (1985) and Rosenblatt and Lee (1986) first established the relation between the lot size and quality imperfection. Porteus (1986) investigated the effect of investment in reducing the setup cost and quality improvement cost jointly. Ouyang *et al.* (2002) utilized an investment for reducing setup cost in an inventory model with the imperfect production and controllable lead time. Sana (2011) considered a three−layer supply chain model with imperfect production. Soni and Patel (2012) investigated an integrated inventory model with defective items and variable production rate under retailer's partial trade credit policy.

4.2 Model formulation

This chapter is and extension of chapter 3. The assumptions to formulate the model are discarded as they are similar to the assumptions in previous chapter.

The buyer orders the quantity Q to the vendor and the vendor produces the quantity mQ. In order to reduce the setup cost whole mQ amount is produced within a single setup, but Q quantity is delivered to the buyer after its production. Thus mQ quantity will be delivered after m units of time. The expected cycle length of the vendor and buyer are $\frac{mQ}{D}$ and $\frac{Q}{D}$, respectively. The system is continuously investigated in such a manner that whenever the inventory level reaches the reorder point R, an order quantity Q is placed.

X is distributed as normally with finite mean DL and standard deviation $\sigma\sqrt{L}$. The shortage occurs if $X > R$. The expected shortage at cycle end can be written as $E(X-R)^+ = \int_R^\infty (x-R)dF(x)$. The expected net inventory level just before placing an order and after the delivery of item are $R - DL$ and $Q + R - DL$, respectively.

Thus, the expected holding cost per unit time is $r_bC_b[(Q/2) + R - DL]$. The expected shortage cost per unit time is $\frac{\pi D}{Q}E(X-R)^+$.

The ordering cost per unit time $\frac{AD}{Q}$ and lead time crashing cost per unit time is $\frac{DC(L)}{Q}$. The total expected cost per unit time for the buyer can be calculated as follows

$$TEC_b = \frac{AD}{Q} + \frac{DC(L)}{Q} + r_bC_b[(Q/2) + R - DL] + \frac{\pi D}{Q}E(X-R)^+ \tag{20}$$

The vendor's expected setup cost per unit time is $\frac{SD}{mQ}$.

The average inventory of the vendor is obtained from the similar derivation of Chapter 3.

$$
\begin{aligned}
&= \left[\left\{mQ\left(\frac{Q}{P}+(m-1)\frac{Q}{D}\right)-\frac{m^2Q^2}{2P}\right\}-\left\{\frac{Q^2}{D}(1+2+...+(m-1))\right\}\right]\frac{D}{mQ} \\
&= \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]
\end{aligned}
$$

Hence, the expected holding cost per unit time for vendor is

$$
= r_vC_v\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]
$$

Now, we introduce a possible relationship among defective items, quality, and lot size. During production process, there may be the possibility of producing imperfect items. During the time of producing a lot size mQ, expected defective units can be approximated by $mQ\theta/2$ (Porteus, 1986). Therefore, the expected annual cost for imperfect items is $sDmQ\theta/2$. Hence, the total expected cost per unit time for vendor is

$$
ATC_v(Q,m)=\frac{SD}{mQ}+r_vC_v\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+\frac{sDmQ\theta}{2} \tag{21}
$$

Investment in vendor's setup cost reduction

Now, we investigate the effect of investment on vendor's setup cost reduction. In relation (2), the equation represents the vendor's total expected cost per unit time. The setup cost is assumed as a fixed parameter. But for more realistic issues, this cost must be a variable and this can be done by varying the capital investment assigned to reduce the vendor's setup cost. If I_S is the investment for setup cost reduction, then it can be expressed as

$I_S = B\ln(\frac{S_0}{S})$ for $0 < S \leq S_0$, i.e., $I_S = B(\ln S_0 - \ln S)$, where S_0 is the initial setup cost, $B = \frac{1}{\delta}$, and $\delta =$ The decrease of percentage in S/dollar increase in I_S.

Investment in quality improvement of the product

Similar to the reduction of vendor's setup cost, the investment in quality improvement of the product is also taken into account. We assume the capital investment as I_θ for the reduction of the *out-of-control* probability θ. Thus, I_θ can be expressed as

$I_\theta = b\ln(\frac{\theta_0}{\theta})$ for $0 < \theta \leq \theta_0$, i.e., $I_\theta = b(\ln\theta_0 - \ln\theta)$, where θ_0 is the initial probability for which the production process may go *out-of-control* and $b = \frac{1}{\Delta}$, where Δ represents the percentage decrease in θ per dollar increase in I_θ.

Thus the total expected cost for the vendor per unit time is

$$\begin{aligned} ATC_v &= \alpha\left[B(\ln S_0 - \ln S) + b(\ln\theta_0 - \ln\theta)\right] + \frac{SD}{mQ} \\ &+ r_v C_v \frac{Q}{2}\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + \frac{sDmQ\theta}{2} \end{aligned} \tag{22}$$

where α is the annual fractional cost of capital investment.

The expected shortage at cycle end can be written as

$$\begin{aligned} E(X-R)^+ &= \int_R^\infty (x-R)dF(x) \\ &= \sigma\sqrt{L}\psi(k) \end{aligned}$$

where $\psi(k) = \phi(k) - k[1 - \Phi(k)]$, ϕ stands for the standard normal probability density function, and $\Phi =$ stands for the cumulative distribution function of normal distribution. In this context, we consider k as a decision variable. The optimal reorder pint can be calculated by the formula $R = DL + k\sigma\sqrt{L}$.

The expected joint total cost of the buyer and vendor is

$$\begin{aligned} JATC(Q,k,S,\theta,L,m) &= ATC_b(Q,k,L) + ATC_v(Q,S,m) \\ &= \alpha\left[B(\ln S_0 - \ln S) + b(\ln\theta_0 - \ln\theta)\right] \\ &+ \frac{D}{Q}\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right] \\ &+ \frac{Q}{2}\left[r_b C_b + r_v C_v\left\{m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right\}\right] \\ &+ r_b C_b k\sigma\sqrt{L} + \frac{sDmQ\theta}{2} \end{aligned} \tag{23}$$

Now the problem is to minimize the expected joint total cost function with respect to two constraints as $0 < S \le S_0$ and $0 < \theta \le \theta_0$ i.e.,

$$\begin{aligned}
\text{Min} \quad JATC(Q,k,S,\theta,L,m) &= \alpha\left[B(\ln S_0 - \ln S) + b(\ln\theta_0 - \ln\theta)\right] \\
&+ \frac{D}{Q}\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right] \\
&+ \frac{Q}{2}H(m) + r_b C_b k\sigma\sqrt{L} + \frac{sDmQ\theta}{2} \\
\text{subject to} \quad & 0 < S \le S_0 \\
& 0 < \theta \le \theta_0 \qquad (24) \\
\text{where} \quad H(m) &= r_b C_b + r_v C_v\left[m(1-\frac{D}{P}) - 1 + \frac{2D}{P}\right]
\end{aligned}$$

This is a constrained non-linear programme. In order to obtain the solution of the problem, first we neglect two constraints $0 < S \le S_0$ and $0 < \theta \le \theta_0$. The objective function of the above problem is a function of six variables $Q,\ k,\ S,\ \theta$, and L where m is a positive integer which indicates it is a discrete variable. Therefore, taking partial derivatives of the objective function with respect to $Q,\ k,\ S,\ \theta$, and L, we obtain

$$\begin{aligned}
\frac{\partial JATC(Q,k,S,\theta,L,m)}{\partial Q} &= -\frac{D}{Q^2}\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right] \\
&+ \frac{1}{2}H(m) + \frac{sDm\theta}{2} \qquad (25) \\
\frac{\partial JATC(Q,k,L,S,\theta,m)}{\partial k} &= \frac{D}{Q}\pi\sigma\sqrt{L}[\Phi(k) - 1] + r_b C_b\sigma\sqrt{L} \qquad (26) \\
& [\phi'(k) = -k\phi(k),\ \Phi'(k) = \phi(k)] \\
\frac{\partial JATC(Q,k,S,\theta,L,m)}{\partial S} &= -\frac{\alpha B}{S} + \frac{D}{mQ} \qquad (27) \\
\frac{\partial JATC(Q,k,S,\theta,L,m)}{\partial \theta} &= -\frac{\alpha b}{\theta} + \frac{sDmQ}{2} \qquad (28) \\
\frac{\partial JATC(Q,k,S,\theta,L,m)}{\partial L} &= \frac{D}{2Q\sqrt{L}}\pi\sigma\psi(k) + \frac{D}{Q}\frac{\partial C(L)}{\partial L} + \frac{r_b C_b k\sigma}{2\sqrt{L}} \\
&= \frac{D}{Q}\left[\frac{\pi\sigma\psi(k)}{2\sqrt{L}} - c_i\right] + \frac{r_b C_b k\sigma}{2\sqrt{L}} \qquad (29)
\end{aligned}$$

Again

$$\frac{\partial^2 JATC(Q,k,S,\theta,L,m)}{\partial L^2} = -\frac{D}{4Q}\pi\sigma\psi(k)L^{-3/2} - \frac{1}{4}r_b C_b k\sigma L^{-3/2} < 0$$

which shows that for fixed $Q,\ k,\ S,\ \theta$, and m, the function $JATC(Q,k,S,\theta,L,m)$ is concave in L. Thus for fixed $Q,\ k,\ S,\ \theta$, and m, the minimum value of $JATC(Q,k,S,\theta,L,m)$ attends at the end point of the interval $[L_i, L_{i-1}]$. Now for fixed positive integer m, the values of Q, $\Phi(k)$, S, and θ are obtained by equating the equations (25), (26), (27), and (28) to zero as

$$
\begin{aligned}
Q &= \left\{ \frac{2D\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right]}{H(m) + sDm\theta} \right\}^{\frac{1}{2}} && (30)\\
\Phi(k) &= 1 - \frac{r_b C_b Q}{D\pi} && (31)\\
S &= \frac{\alpha B Q m}{D} && (32)\\
\theta &= \frac{2\alpha b}{sDmQ} && (33)
\end{aligned}
$$

4.2.1 Proposition 1

If we denote $Q^*, k^*,\ S^*$, and θ^* as optimal values of $Q,\ k,\ S$, and θ then for fixed $L \in [L_i, L_{i-1}]$ and m, the Hessian matrix for $JATC(Q,k,S,\theta,L,m)$ is positive definite at the point $(Q^*, k^*, S^*, \theta^*)$ obtained from equations (30)$-$(33) if the constraints $0 < S \le S_0$ and $0 < \theta \le \theta_0$ are relaxed.

Proof: See Appendix A.

The optimal value of m can be obtained when

$$JATC(m^* - 1) \ge JATC(m^*) \le JATC(m^* + 1)$$

where m^* is the optimal value of m.

We now take two constraints into consideration. S and θ are both positive quantity as α, B, Q, m, D, b, and s are all positive. If $S^* < S_0$ and $\theta^* < \theta_0$, then S^* and θ^* are optimal solutions of S and θ, respectively. But if $S^* > S_0$ occurs, then the investment for setup cost reduction for vendor becomes a negative quantity which does not make any sense. In this situation we reconsider $S^* = S_0$ and no investment is considered for setup cost reduction. Similarly for the case when $\theta^* > \theta_0$, we take $\theta^* = \theta_0$. We now describe the following algorithm to obtain the optimal solution of our model.

4.2.2 Solution algorithm SM 1

Step 1 Set $m = 1$.

Step 2 For each $L_i, i = 1, 2, ...n$; perform Step 2a−2f.

Step 2a Set $S_{i1} = 0$, $k_{i1} = 0$, $\theta_{i1} = 0$ (implies $\psi(k_{i1}) = 0.39894$).

Step 2b Substitute $\psi(k_{i1})$ into equation (30) and evaluate Q_{i1}.

Step 2c Utilize Q_{i1} to determine the value of $\Phi(k_{i2})$ from equation (31).

Step 2d For the value of $\Phi(k_{i2})$, find k_{i2} from the normal table and hence evaluate $\psi(k_{i2})$.

Step 2e Utilize Q_{i1} to obtain S_{i2} and θ_{i2} from equation (32) and (33).

Step 2f Repeat 2b to 2e until no changes occur in the values of Q_i, k_i, S_i, and θ_i as well as denote these values by the point $(Q_i^*, k_i^*, S_i^*, \theta_i^*)$.

Step 3 Comparison between S^* and S_0, and θ^* and θ_0.

Step 3a If $S_i^* < S_0$ and $\theta_i^* < \theta_0$ then $(Q_i^*, k_i^*, S_i^*, \theta_i^*)$ is the optimal solution. Go to Step 4.

Step 3b If $S_i^* > S_0$ and $\theta_i^* < \theta_0$ then set $S_i^* = S_0$ and utilize (30), (31), and (33) to determine new optimal value of (Q, k, θ) denoted by (Q_i', k_i', θ') by substituting S by S_0 and using the same procedure stated as Step 2. Then go to Step 4.

Step 3c If $S_i^* < S_0$ and $\theta_i^* > \theta_0$ then set $\theta_i^* = \theta_0$ and utilize (30), (31), and (32) to determine the new optimal value of (Q, k, S) denoted by (Q_i'', k_i'', S_i'') by substituting θ by θ_0 and using the same procedure stated as in Step 2. Then go to Step 4.

Step 4 Find $JATC(Q_i^*, k_i^*, S_i^*, \theta_i^*, L_i, m)$ and $Min_{i=1,2,...,n} JATC(Q_i^*, k_i^*, S_i^*, \theta_i^*, L_i, m)$.

Step 4a If $JATC(Q_i^*, k_i^*, S_i^*, \theta_i^*, L_i, m) = Min_{i=1,2,...,n} JATC(Q_i^*, k_i^*, S_i^*, \theta_i^*, L_i, m)$, then $JATC(Q_i^*, k_i^*, S_i^*, \theta_i^*, L_i, m)$ is the optimal solution for fixed m.

Step 5 Set $m = m + 1$.
If $JATC(Q_m^*, k_m^*, S_m^*, \theta_m^*, L_m, m) \leq JATC(Q_{m-1}^*, k_{m-1}^*, S_{m-1}^*, \theta_{m-1}^*, L_{m-1}, m-1)$, repeat Step 2, Step 3, and Step 4. Otherwise go to Step 6.

Step 6 Set $JATC(Q_m^*, k_m^*, S_m^*, \theta_m^*, L_m, m) = JATC(Q_{m-1}^*, k_{m-1}^*, S_{m-1}^*, \theta_{m-1}^*, L_{m-1}, m-1)$.
Then $(Q^*, k^*, L^*, S^*, \theta^*, m^*)$ is the optimal solution and the optimal reorder point can be obtained from $R^* = DL^* + k^*\sigma\sqrt{L^*}$, where R^* denotes the optimal solution for R, the reorder point.

4.2.3 Distribution free approach

Most of the time, the exact distribution of the lead time demand is very difficult to obtain. Managers need to pay a lot of money to collect the necessary information about lead time demand distribution. In this situation, min-max distribution free approach is very useful to solve the model, where the lead time demand does not follow any specific probability distribution except mean and standard deviation. We consider any distribution function (d.f.) F for the lead time demand in the class G of d.f.'s with mean DL and standard deviation $\sigma\sqrt{L}$. The value of expected shortage cannot be determined exactly as the lead time demand distribution is unavailable. The min-max distribution free approach is used to determine the most unfavorable distribution function function F in G for each (Q, R, L, S, θ, m) such that the expected joint total cost for vendor and buyer is maximized and then to minimize it over (Q, R, L, S, θ, m). Thus, the problem can be stated as

$$\begin{aligned} \text{Min Max}_{F \in G} \quad & JATC(Q, R, S, \theta, L, m) \\ \text{subject to} \quad & 0 < S \leq S_0 \\ & 0 < \theta \leq \theta_0 \end{aligned} \tag{34}$$

The following proposition is used by Gallego and Moon (1993) to approximate the value of $E(X - R)^+$.

4.2.4 Proposition 2

For any $F \in G$, the following inequality always holds.

$$E(X-R)^{+} \leq \frac{1}{2}\left[\sqrt{\sigma^2 L + (R-DL)^2} - (R-DL)\right] \tag{35}$$

Moreover the upper bound of the above equation is tight.

From model (24) and the inequality (35), the model (34) is reduced to

$$\begin{aligned}
\text{Min} \quad JATC_f(Q,k,S,\theta,L,m) &= \alpha\left[B(\ln S_0 - \ln S) + b(\ln\theta_0 - \ln\theta)\right] \\
&+ \frac{D}{Q}\Big[A + \frac{S}{m} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2}-k) \\
&+ C(L)\Big] + \frac{Q}{2}H(m) + r_b C_b k\sigma\sqrt{L} + \frac{sDmQ\theta}{2} \\
\text{subject to} \quad & 0 < S \leq S_0 \\
& 0 < \theta \leq \theta_0 \\
\text{where} \quad H(m) &= r_b C_b + r_v C_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]
\end{aligned} \tag{36}$$

Taking partial derivatives of the objective function with respect to $Q,\ k,\ S,$ and θ, then equating to zero we obtain

$$Q = \left\{\frac{2D\left[A + \frac{S}{m} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2}-k) + C(L)\right]}{H(m) + sDm\theta}\right\}^{\frac{1}{2}} \tag{37}$$

$$\frac{k}{\sqrt{1+k^2}} = 1 - \frac{2r_b C_b Q}{D\pi} \tag{38}$$

$$S = \frac{\alpha B Q m}{D} \tag{39}$$

$$\theta = \frac{2\alpha b}{sDmQ} \tag{40}$$

4.2.5 Proposition 3

If we denote $Q^{**}, k^{**},\ S^{**}$, and θ^{**} as optimal values of $Q,\ k,\ S,$ and θ then for fixed $L \in [L_i, L_{i-1}]$ and m, the Hessian matrix for $JATC_f(Q,k,S,\theta,L,m)$ is positive definite at the point $(Q^{**}, k^{**}, S^{**}, \theta^{**})$ obtained from equations (37)$-$(40) if the constraints $0 < S \leq S_0$ and $0 < \theta \leq \theta_0$ are relaxed.

Proof Similar proof as Proposition 1.

For fixed Q^{**}, k^{**}, S^{**}, θ^{**}, and L^{**} the optimal value of the objective function $JATC_f(Q, k, S, \theta, L, m)$ will be obtained when

$$JATC(m^{**} - 1) \geq JATC(m^{**}) \leq JATC(m^{**} + 1)$$

where m^{**} denotes the optimal value of m. The constraints $0 < S^{**} \leq S_0$ and $0 < \theta^{**} \leq \theta_0$ will be satisfied also. An algorithm to obtain the optimal value for distribution free model is illustrated below.

4.2.6 Solution algorithm SM 2

Step 1 Set $m = 1$.

Step 2 For each $L_i, i = 1, 2, ..., n$; perform Step 2a$-$2e.

Step 2a Set $S_{i1} = 0$, $k_{i1} = 0$, $\theta_{i1} = 0$.

Step 2b Substitute S_{i1}, k_{i1}, θ_{i1} into equation (37) and evaluate Q_{i1}.

Step 2c Utilize Q_{i1} to determine the value of k_{i2} from equation (38).

Step 2d Utilize Q_{i1} to obtain S_{i2} and θ_{i2} from equation (39) and (40).

Step 2e Repeat 2b to 2d until no changes occur in the values of Q_i, k_i, S_i, and θ_i denote these values by the point $(Q_i^{**}, k_i^{**}, S_i^{**}, \theta_i^{*}*)$.

Step 3 Comparison between S^{**} and S_0, as well as θ^{**} and θ_0.

Step 3a If $S_i^{**} < S_0$ and $\theta_i^{**} < \theta_0$ then $(Q_i^{**}, k_i^{**}, S_i^{**}, \theta_i^{**})$ is the optimal solution. Go to Step 4.

Step 3b If $S_i^{**} > S_0$ and $\theta_i^{**} < \theta_0$ then set $S_i^{**} = S_0$ and utilize (37), (38) and (40) to determine new optimal value of (Q, k, θ) denoted by $(Q_i^{*'}, k_i^{*'}, \theta^{*'})$ by substituting S by S_0 and using the same procedure stated as Step 2. Then go to Step 4.

Step 3c If $S_i^{**} < S_0$ and $\theta_i^{**} > \theta_0$ then set $\theta_i^{**} = \theta_0$ and utilize (37), (38), and (39) to determine the new optimal value of (Q, k, S) denoted by $(Q_i^{*''}, k_i^{*''}, S_i^{*''})$ by substituting θ by θ_0 and using the same procedure stated as in Step 2. Then go to Step 4.

Step 4 Find $JATC(Q_i^{**}, k_i^{**}, S_i^{**}, \theta_i^{**}, L_i, m)$ and $Min_{i=1,2,\ldots,n} JATC(Q_i^{**}, k_i^{**}, S_i^{**}, \theta_i^{**}, L_i, m)$.

Step 4a If $JATC(Q_i^{**}, k_i^{**}, S_i^{**}, \theta_i^{**}, L_i, m) = Min_{i=1,2,\ldots,n} JATC(Q_i^{**}, k_i^{**}, S_i^{**}, \theta_i^{**}, L_i, m)$, then, $JATC(Q_i^{**}, k_i^{**}, S_i^{**}, \theta_i^{**}, L_i, m)$ is the optimal solution when m is fixed.

Step 5 Set $m = m + 1$.

If $JATC(Q_m^{**}, k_m^{**}, S_m^{**}, \theta_m^{**}, L_m, m) \leq JATC(Q_{m-1}^{**}, k_{m-1}^{**}, S_{m-1}^{**}, \theta_{m-1}^{**}, L_{m-1}, m-1)$, repeat Step 2, Step 3, and Step 4. Otherwise go to Step 6.

Step 6 Set $JATC(Q_m^{**}, k_m^{**}, S_m^{**}, \theta_m^{**}, L_m, m) = JATC(Q_{m-1}^{**}, k_{m-1}^{**}, S_{m-1}^{**}, \theta_{m-1}^{**}, L_{m-1}, m-1)$.

Then, $(Q^{**}, k^{**}, L^{**}, S^{**}, \theta^{**}, m^{**})$ is the optimal solution and the optimal reorder point can be obtained from $R^{**} = DL^{**} + k^{**}\sigma\sqrt{L^{**}}$, where R^{**} denotes the optimal solution for R, the reorder point.

4.3 Numerical experiments

Example 1

This example deals with a model having the lead time demand, following a normal distribution. According to Ouyang *et al.* (2004), we consider $D = 600$ units/year, $A = \$200$/order, $C_b = \$100$/unit, $\pi = \$50$/unit, $\sigma = 7$ units/week, $P = 2000$ units/year, $S_0 = \$1500$/setup, $\theta_0 = 0.0002$, $C_v = \$70$/unit, $r_b = \$0.2$/unit/year, $r_v = \$0.2$/unit/year, $\alpha = 0.1$/dollar/year, $B = 18000$, and $b = 400$. The lead time has three components and Table 4.1 shows the data for the lead time. The results are given in Table 4.2 and Table 4.3.

Table 4.1

Lead time data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table 4.2

Solution of Example 1 for **variable setup cost** (for normal distribution case)

L	m	Q	k	S	θ	R	$JTEC$
28	1	192.25	1.14	576.76	9.24×10^{-6}	62	7161.33
28	2	142.09	1.31	852.54	6.25×10^{-6}	64	6848.75[a]
28	3	115.43	1.42	1038.87	5.13×10^{-6}	66	6856.84

[a] indicates the minimum joint total cost.

Table 4.3

Solution of Example 1 for **fixed setup cost** (for normal distribution case)

L	m	Q	k	S	θ	R	$JTEC$
28	1	266.67	0.92	1500	6.61×10^{-6}	59	7688.14
28	2	166.74	1.22	1500	5.33×10^{-6}	63	6995.86
28	3	126.12	1.38	1500	4.69×10^{-6}	65	6902.41[b]
28	3	103.72	1.48	1500	4.28×10^{-6}	67	6967.86

[b] indicates the minimum joint total cost.

Applying algorithm 1, we obtain the above results for Example 1. The optimal ordering quantity $Q^* = 142$ units, lead time $L^* = 28$ days, optimal setup cost for the vendor $S^* = \$853$, optimal *out-of-control* probability $\theta^* = 6.25 \times 10^{-6}$, the number of lots delivered from the vendor to the buyer $m^* = 3$, the reorder point $R^* = 64$ units, the minimum joint total cost is

\$6849/year.

Example 2

The data for Example 2 are the same as Example 1 in the distribution free case. Applying algorithm SM 2, we obtain the results as follows. The optimal ordering quantity $Q^{**} = 155$ units, the lead time $L^{**} = 28$ days, the optimal setup cost for the vendor $S^{**} = \$928$, the number of lots delivered from the vendor to the buyer $m^{**} = 2$, the optimal *out-of-control* probability $\theta^{**} = 5.74 \times 10^{-6}$, the reorder point $R^{**} = 64$ units, and the minimum joint cost is \$7164/year. Solutions for Example 2 are summarized in Table 4.4 and Table 4.5.

Table 4.4

Solution of Example 2 for **variable setup cost** (for distribution free case)

L	m	Q	k	S	θ	R	$JTEC$
28	1	207.83	1.05	623.50	8.55×10^{-6}	61	7401.25
28	2	154.62	1.31	927.75	5.74×10^{-6}	64	7164.34[c]
28	3	125.99	1.50	1133.89	4.70×10^{-6}	67	7236.62

[c] indicates the minimum joint total cost.

Table 4.5

Solution of Example 2 for **fixed setup cost** (for distribution free case)

L	m	Q	k	S	θ	R	$JTEC$
21	1	275.76	0.82	1500	6.44×10^{-6}	45	7848.76
28	2	174.87	1.19	1500	5.08×10^{-6}	63	7266.90
28	3	133.00	1.44	1500	4.43×10^{-6}	66	7256.76[d]
28	2	110.81	1.63	1500	4.01×10^{-6}	69	7392.18

[d] indicates the minimum joint total cost.

4.3.1 Sensitivity analysis

For sensitivity analysis, we change each key parameter by −50%, −25%, +25%, and +50%. Each parameter is changed one at a time keeping other parameters fixed. The effect of changes

Table 4.6

Sensitivity analysis for normal distribution case

Parameters	Changes(in %)	EAC^N	Parameters	Changes(in %)	EAC^N
	-50%	-5.82		-50%	-15.65
	-25%	-2.82		-25%	-7.39
A	$+25\%$	$+2.68$	r_b	$+25\%$	$+6.73$
	$+50\%$	$+5.23$		$+50\%$	$+12.95$
	-50%	-15.65		-50%	-8.76
	-25%	-7.39		-25%	-4.19
C_b	$+25\%$	$+6.73$	r_v	$+25\%$	$+3.88$
	$+50\%$	$+12.95$		$+50\%$	$+7.51$
	-50%	-8.76		-50%	-18.22
	-25%	-4.19		-25%	-7.56
C_v	$+25\%$	$+3.88$	S_0	$+25\%$	$+5.86$
	$+50\%$	$+7.51$		$+50\%$	$+10.66$

of the key parameters are illustrated in Table 4.7 and Table 4.8.

The percentage change in the joint total cost indicates that some parameters like C_b, r_b, and S_0 are more sensitive than that of A, C_v, and r_v to the optimal cost. From sensitivity analysis it is seen that when key parameters are increased total system cost also gets increased. Results of the sensitivity analysis are same for the distribution free case. However, percentage changes in the joint total cost with the change of key parameters are different in two cases.

Table 4.7

Sensitivity analysis for distribution free case

Parameters	Changes(in %)	EAC^N	Parameters	Changes(in %)	EAC^N
A	-50%	-5.06	r_b	-50%	-15.99
	-25%	-2.47		-25%	-7.53
	$+25\%$	$+2.36$		$+25\%$	$+6.85$
	$+50\%$	$+4.64$		$+50\%$	$+13.16$
C_b	-50%	-15.99	r_v	-50%	-9.09
	-25%	-7.53		-25%	-4.35
	$+25\%$	$+6.85$		$+25\%$	$+4.04$
	$+50\%$	$+13.16$		$+50\%$	$+7.82$
C_v	-50%	-9.09	S_0	-50%	-17.41
	-25%	-4.35		-25%	-7.23
	$+25\%$	$+4.04$		$+25\%$	$+5.61$
	$+50\%$	$+7.82$		$+50\%$	$+10.19$

4.3.2 Evaluation of expected value of additional information (EVAI)

We compare results of the distribution free case to the normal distribution case. From Table 4.2 and Table 4.4, we obtain $(Q^*, R^*, S^*, \theta^*, L^*, m^*) = (142, 64, 853, 6.25 \times 10^{-6}, 28, 2)$ and $(Q^{**}, R^{**}, S^{**}, \theta^{**}, L^{**}, m^{**}) = (155, 64, 928, 5.74 \times 10^{-6}, 28, 2)$. The added cost will be $JATC(Q^{**}, R^{**}, S^{**}, \theta^{**}, L^{**}, m^{**}) - JATC(Q^*, R^*, S^*, \theta^*, L^*, m^*) = \$6863 - \$6849 = \$14$ which is less than 1% of the joint total cost for the distribution free case. This amount is said to be the expected value of additional information (EVAI) for the buyer. This is the largest amount that a buyer should incur to obtain the knowledge of the lead time demand distribution.

4.4 Managerial insights

This chapter consists of an integrated vendor-buyer supply chain model with cost reduction and quality improvement. The managerial insights of this chapter are as follows.

- Manager can reduce the setup cost of vendor and improve quality of products by a capital investment. Reduction of setup cost results the reduction of total supply chain cost also.
- By incurring a lead time crashing cost retailer can reduce the delivery lead time which leads to a better service level.
- The distribution free approach is applied to obtain the optimal decisions if managers are not willing to pay fund to collect market information.
- EVAI suggests the manager whether to invest any fund or not to collect demand information.

4.5 Concluding remarks

This study considered an single-vendor single-buyer supply chain model with controllable lead time. This research improved the manufacturing quality of products and reduced the vendor's setup cost by using an investment function. From numerical results, it was seen that the total system cost is reduced for the variable setup cost rather than fixed setup cost. The initial *out-of-control* probability was also reduced after using the investment function for quality improvement. The EVAI was less than 1% of the total system cost which suggests managers to pay funds for collecting market information. The model can be extended by assuming multi-item.

4.6 Appendix of Chapter 4

Appendix A

Proof of Proposition 1.

For given value of L and m, the Hessian matrix H is as follows

$$H = \begin{bmatrix} \frac{\partial^2 JATC(\cdot)}{\partial Q^2} & \frac{\partial^2 JATC(\cdot)}{\partial Q \partial k} & \frac{\partial^2 JATC(\cdot)}{\partial Q \partial S} & \frac{\partial^2 JATC(\cdot)}{\partial Q \partial \theta} \\ \frac{\partial^2 JATC(\cdot)}{\partial k \partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial k^2} & \frac{\partial^2 JATC(\cdot)}{\partial k \partial S} & \frac{\partial^2 JATC(\cdot)}{\partial k \partial \theta} \\ \frac{\partial^2 JATC(\cdot)}{\partial S \partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial S \partial k} & \frac{\partial^2 JATC(\cdot)}{\partial S^2} & \frac{\partial^2 JATC(\cdot)}{\partial S \partial \theta} \\ \frac{\partial^2 JATC(\cdot)}{\partial \theta \partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial \theta \partial k} & \frac{\partial^2 JATC(\cdot)}{\partial \theta \partial S} & \frac{\partial^2 JATC(\cdot)}{\partial \theta^2} \end{bmatrix}$$

Now

$$\begin{aligned}
\frac{\partial^2 JATC(\cdot)}{\partial Q^2} &= \frac{2D}{Q^3}\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right] \\
\frac{\partial^2 JATC(\cdot)}{\partial Q \partial k} &= -\frac{D}{Q^2}\pi\sigma\sqrt{L}[\Phi(k) - 1] \\
\frac{\partial^2 JATC(\cdot)}{\partial Q \partial S} &= -\frac{D}{mQ^2} \\
\frac{\partial^2 JATC(\cdot)}{\partial k \partial Q} &= -\frac{D}{Q^2}\pi\sigma\sqrt{L}[\Phi(k) - 1] \\
\frac{\partial^2 JATC(\cdot)}{\partial k^2} &= \frac{D}{Q}\pi\sigma\sqrt{L}\phi(k) \\
\frac{\partial^2 JATC(\cdot)}{\partial k \partial S} &= 0 \\
\frac{\partial^2 JATC(\cdot)}{\partial S \partial Q} &= -\frac{D}{Q^2 m}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 JATC(\cdot)}{\partial S \partial k} &= 0 \\
\frac{\partial^2 JATC(\cdot)}{\partial S^2} &= \frac{\alpha B}{S^2} \\
\frac{\partial^2 JATC(\cdot)}{\partial \theta^2} &= \frac{\alpha b}{\theta^2} \\
\frac{\partial^2 JATC(\cdot)}{\partial Q \partial \theta} &= \frac{sDm}{2} \\
\frac{\partial^2 JATC(\cdot)}{\partial S \partial \theta} &= 0 \\
\frac{\partial^2 JATC(\cdot)}{\partial k \partial \theta} &= 0
\end{aligned}
$$

The first order principal minor of H is

$$
|H_{11}|_{(Q^*,k^*,S^*)} = \left| \frac{\partial^2 JATC(\cdot)}{\partial Q^2} \right|_{(Q^*,k^*,S^*)} = \frac{2D}{Q^{*3}} \left[A + \frac{S^*}{m} + \pi\sigma\sqrt{L}\psi(k^*) + C(L) \right] > 0
$$

The second order principal minor of H is as follows

$$
\begin{aligned}
|H_{22}|_{(Q^*,k^*,S^*)} &= \frac{\partial^2 JATC(\cdot)}{\partial Q^2} \frac{\partial^2 JATC(\cdot)}{\partial k^2} - \left(\frac{\partial^2 JATC(\cdot)}{\partial Q \partial k} \right)^2 \\
&= \frac{2D}{Q^{*4}} \pi\sigma\sqrt{L}\psi(k^*) \left[A + \frac{S^*}{m} + \pi\sigma\sqrt{L}\psi(k^*) + C(L) \right] - \frac{D^2}{Q^{*4}} \pi^2\sigma^2 L \left[\Psi(k^*) - 1 \right]^2 \\
&= \frac{2D^2}{Q^{*4}} \pi\sigma\sqrt{L}\psi(k^*) \left[A + \frac{S^*}{m} + C(L) \right] \\
&+ \frac{D^2}{Q^{*4}} \pi^2\sigma^2 L \left[2\psi(k^*)\phi(k^*) - (\Phi(k^*) - 1)^2 \right] > 0
\end{aligned}
$$

as $\phi(k)$, $\psi(k) > 0$ and $2\phi(k)\psi(k) - (\Phi(k)-1)^2 > 0$ for all $k > 0$ (Ouyang *et al.*, 2004).

The third order principal minor of H is

$$\begin{aligned}
|H_{33}|_{(Q^*,k^*,S^*)} &= \begin{vmatrix} \frac{\partial^2 JATC(\cdot)}{\partial Q^2} & \frac{\partial^2 JATC(\cdot)}{\partial Q\partial k} & \frac{\partial^2 JATC(\cdot)}{\partial Q\partial S} \\ \frac{\partial^2 JATC(\cdot)}{\partial k\partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial k^2} & \frac{\partial^2 JATC(\cdot)}{\partial k\partial S} \\ \frac{\partial^2 JATC(\cdot)}{\partial S\partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial S\partial k} & \frac{\partial^2 JATC(\cdot)}{\partial S^2} \end{vmatrix} \\
&= \frac{\alpha B}{S^{*2}}|H_{22}| - \frac{D^3}{Q^{*5}m^2}\pi\sigma\sqrt{L}\phi(k^*) \\
&> \frac{2D^2}{Q^{*4}}\pi^2\sigma^2 L\phi(k^*) + \frac{2D^2}{Q^{*4}}\pi\sigma\sqrt{L}\psi(k^*)\left[A + \frac{S^*}{m} + C(L)\right] - \frac{D^3}{Q^{*5}m^2}\pi\sigma\sqrt{L}\phi(k^*) \\
&= \frac{D^2}{Q^{*4}}\pi\sigma\sqrt{L}\left[2\pi\sigma\sqrt{L}\phi(k^*) + 2\psi(k^*)\left\{A + \frac{S^*}{m} + C(L)\right\} - \frac{D}{Qm^2}\phi(k^*)\right] > 0 \\
\text{as} \qquad & 2\pi\sigma\sqrt{L} + 2\psi(k^*)A + \frac{S^*}{m} + C(L) > \frac{D}{Q^*m^2}
\end{aligned}$$

The forth order principal minor of H is

$$\begin{aligned}
|H_{44}|_{(Q^*,k^*,S^*)} &= \begin{vmatrix} \frac{\partial^2 JATC(\cdot)}{\partial Q^2} & \frac{\partial^2 JATC(\cdot)}{\partial Q\partial k} & \frac{\partial^2 JATC(\cdot)}{\partial Q\partial S} & \frac{\partial^2 JATC(\cdot)}{\partial Q\partial \theta} \\ \frac{\partial^2 JATC(\cdot)}{\partial k\partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial k^2} & \frac{\partial^2 JATC(\cdot)}{\partial k\partial S} & \frac{\partial^2 JATC(\cdot)}{\partial k\partial \theta} \\ \frac{\partial^2 JATC(\cdot)}{\partial S\partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial S\partial k} & \frac{\partial^2 JATC(\cdot)}{\partial S^2} & \frac{\partial^2 JATC(\cdot)}{\partial S\partial \theta} \\ \frac{\partial^2 JATC(\cdot)}{\partial \theta\partial Q} & \frac{\partial^2 JATC(\cdot)}{\partial \theta\partial k} & \frac{\partial^2 JATC(\cdot)}{\partial \theta\partial S} & \frac{\partial^2 JATC(\cdot)}{\partial \theta^2} \end{vmatrix} \\
&= \frac{\alpha b}{\theta^{*2}}|H_{33}| - \frac{sDm}{2}\left\{\frac{\alpha B}{S^{*2}} \times \frac{D\pi\sigma\sqrt{L}\phi(k^*)}{Q^*} \times \frac{sDm}{2}\right\} \\
&= \frac{\alpha b}{\theta^{*2}}\Big[\frac{\alpha B}{S^{*2}}\Big\{\frac{2D^2}{Q^{*4}}\pi\sigma\sqrt{L}\psi(k^*)\left[A + \frac{S^*}{m} + C(L)\right] + \frac{D^2}{Q^{*4}}\pi^2\sigma^2\sqrt{L} \\
&\quad [2\psi(k^*)\phi(k^*) - (\Phi(k^*)-1)^2]\Big\} - \frac{D^3}{Q^{*5}m^2}\pi\sigma\sqrt{L}\phi(k^*)\Big] \\
&- \left(\frac{sDm}{2}\right)^2 \times \frac{\alpha B}{S^{*2}} \times \frac{D}{Q^*}\pi\sigma\sqrt{L}\phi(k^*)
\end{aligned}$$

This is enough to show that

$$\frac{2\alpha b}{Q^{*3}\theta^{*2}}\psi(k^*) - \frac{s^2 m^2 D}{2}\phi(k^*) > 0 \tag{41}$$

$$\text{and } \left[A + \frac{S^*}{m} + C(L)\right] + \frac{D^2\pi^2\sigma^2 L}{Q^4}[2\psi(k^*)\phi(k^*) - (\Phi(k^*) - 1)^2]$$

$$-\frac{D^3}{Q^{*5}m^2}\pi\sigma\sqrt{L}\phi(k^*) > 0 \tag{42}$$

$$\text{i.e. } \left[A + \frac{S^*}{m} + C(L) + \pi\sigma\sqrt{L}\right] - \frac{D}{Q^* m^2}\phi(k^*) > 0 \tag{43}$$

Inequality (22) holds because $\theta^* << 1, Q^*; \psi(k^*) > \phi(k^*)$ such that

$$\frac{2\alpha b}{\theta^{*2}Q^3}\psi(k^*) > \frac{s^2 D m^2}{2}\phi(k^*)$$

Inequality (24) holds because

$$A + \frac{S^*}{m} > \frac{D}{Q^* m^2}\phi(k^*) \text{ as } \phi(k^*) < 1$$

Chapter 5
Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction

5 Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction

For quality improvement purposes, any manufacturing unit has to change certain parts of equipments. Any such changes in the assembly−line manufacturing system or production process involves a cost known as setup cost. Minimizing the setup cost and improving the product quality is of prime importance in today's competitive business arena. This chapter develops the effect of setup cost reduction and quality improvement in a two-echelon supply chain model with deterioration under lot−splitting policy. The objective is to minimize the expected total cost of the entire supply chain model (SCM) by simultaneously optimizing setup cost, process quality, number of deliveries, and lot size.

5.1 Literature review

Dealing between two parties (vendor-buyer or manufacturer-retailer) there are various policies that confirms how a product will be delivered, namely the SSMD (Single-setup-multi-delivery) and the SSSD (Single-setup-single-delivery). Choosing a suitable delivery policy is an important criteria. An integrated production inventory model with single-vendor single-buyer was extended by Hill (1997) as a generalized policy. Cárdenas-Barrón (2007) presented a note on optimized inventory decisions in a multi-echelon multi-customer supply chain. Cárdenas-Barrón (2011) discussed the variation of inventory models with two backorder costs using analytical geometry and algebra. Teng *et al.* (2011) considered the economic lot size of the integrated vendor-buyer inventory model without derivatives and with a closed form optimal solution. Asghari *et al.* (2015) reversed a logistic network design with incentive-dependent return. Watanable and Kusukawa (2015) evaluated an optimal ordering policy in dual-sourcing

B. Sarkar, A. Majumder, M. Sarkar, B. K. Dey, G. Roy, Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction, Journal of Industrial and Management Optimization. Accepted.

supply chain considering supply disruptions and demand information. Wisittipanich and Hengmeechai (2015) discussed about a multi-objective differential evolution for just-in-time door assignment and truck scheduling in multi-door cross docking problems. Park (2015) invented a partial backordering inventory model where purchasing of products have an important role. Kusukawa and Alozawa (2015) approached an optimal operation for green supply chain with quality of recyclable parts and contract for recycling activity. Sarkar and Moon (2014) developed an improved quality and reduced setup cost with variable backorder costs in a production process with imperfect quality.

The effect of degradation of items in the inventory model was first studied by authors Ghare and Schrader (1963). An EOQ model with deterioration with Weibull distribution was later discussed by Covert and Philip (1973). Misra (1975) and Shah (1977) respectively proposed an optimal production lotsize model and an order-level lot size model for a system with deteriorating inventory. Economic ordering policy with deterioration over infinite time horizon was developed by Goyal (1987). A literature survey on continuously deteriorating inventory model was done by Raafat (1991). Inventory models with different types of deteriorating rates was extended in this direction by some researchers like Goyal (1988), Goswami and Chaudhuri (1991), Skouri and Papachristos (2003), Skouri *et al.* (2009), Sarkar (2012), Sarkar *et al.* (2013), Sarkar and Sarkar (2013), Sarkar and Sarkar (2013), Sarkar (2013).

5.2 Model formulation

In this section, assumptions to develop the mathematical model, model description with some lemma, and solution methodology are described.

5.2.1 Assumptions

Following assumptions are considered to develop the model.

1. Two-echelon supply chain model is considered with a buyer and a supplier for single-type

of products.

2. For saving buyer's holding cost, a SSMD policy is utilized for transportation of products between vendor and buyer.

3. As SSMD policy is used to save the holding cost of buyer, thus buyer pays transportation costs. For SSMD policy, it is assumed that there are some constant transportation costs and some variable costs. Both constant and variable transportation cost are paid by buyer.

4. Information of the demand and the inventory position of the buyer are given to the supplier. Production rate is always greater than demand, i.e., $P > D$ such that there are no shortages.

5. The model assumes a SSMD policy, which indicates that the products are produced within a single-setup which is generally in long-run. Thus, during long-run, any time the production of defective items may occur.

6. The vendor uses autonomation policy (automatically detects the defective item by machine, no human inspector is needed to inspect the defectiveness of items) to detect the imperfect production. As a result, if the system moves to *out-of-control* state from *in-control* state, it will continue production of defective items until the whole lot is produced.

7. Two investments are considered to reduce setup cost and to improve quality of products.

8. A constant rate of deterioration is considered for products.

A SSMD policy for a supplier is developed in this model and the average total cost for the buyer and the supplier which is minimized. In the proposed model the buyer's ordered quantity is manufactured at a time and the manufactured products are delivered in an equal amount over multiple deliveries after a fixed interval of time. The splitting of the ordered quantity into multiple lots is in accordance with Just−in−time implementation. The total time span T is split into two parts say, T_1, the production time for the supplier and T_2, the non-production

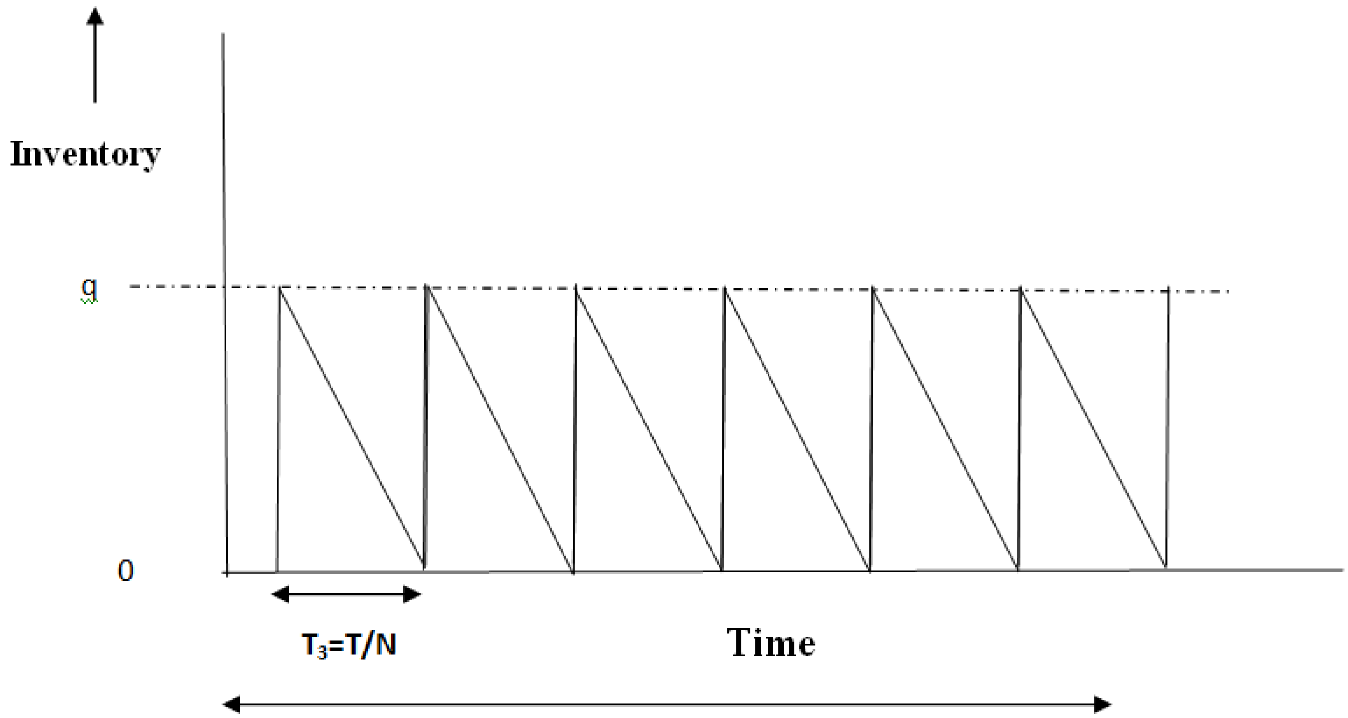

Figure 8 Buyer's inventory position

time for the supplier. T_3 is assumed as the time between two successive deliveries. Now, the individual inventory costs the buyer and the supplier are calculated as follows:

Buyer's cost function

The buyer's cost function is comprised of the following relevant costs.

1. Ordering cost $= \frac{A}{T}$
2. Holding cost $= \frac{H_b A_b}{T}$, where A_b is the area over which the inventory holds for buyer.
3. Deterioration cost $= \frac{C_d d A_b}{T}$
4. Transportation cost and handling cost $= \frac{(mF+Vmq)}{T}$

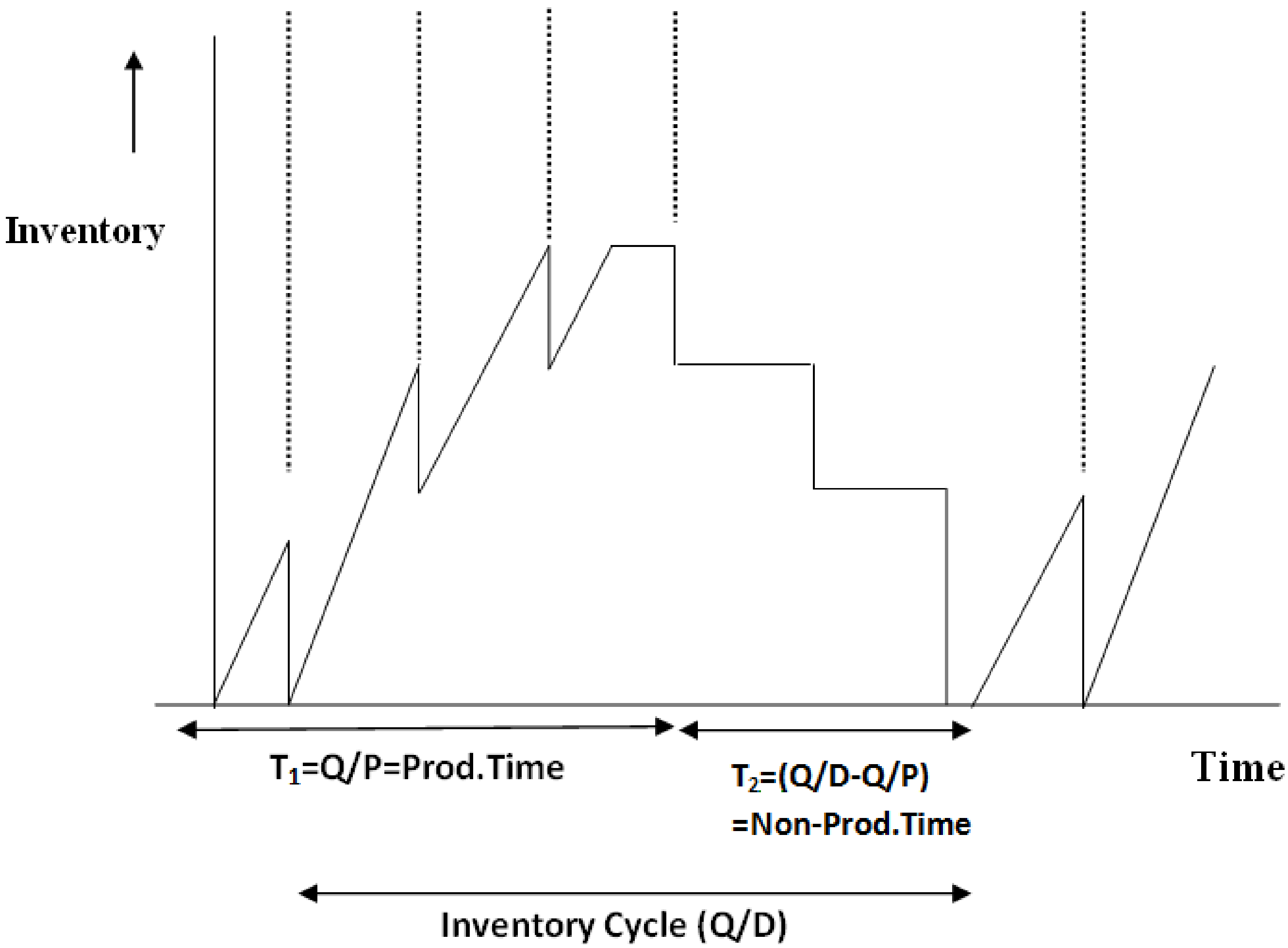

Figure 9 Supplier's inventory position

From Figure 8 and Figure 9, we obtain

$$\frac{1}{T} = \frac{D}{mq} + \frac{d}{2m}, \tag{44}$$

$$\text{and } \frac{A_b}{T} = \frac{q}{2}. \qquad \text{[See Appendix C]} \tag{45}$$

Using above equations, the buyer's total cost is obtained as

$$TC_b = \Big(\frac{D}{mq} + \frac{d}{2m}\Big)(A + mF + Vmq) + \frac{q}{2}[H_b + C_d d] \tag{46}$$

Supplier's cost function

Now the supplier's cost function is comprised of the following relevant costs.

1. Setup cost $= \frac{S}{T}$
2. Holding cost $= \frac{H_s A_s}{T}$, where A_s is the area over which the inventory holds for supplier.
3. Deterioration cost $= \frac{C_d d A_s}{T}$

Let y be the number of deteriorating items of the supplier. y can be expressed as $y = dA_s$. $y + dqT/2$ denotes the total number of deteriorating units for the entire supply chain. With the following expressions $Q = mq + y$ and $t_1 = \frac{Q}{P}$ and assuming the initial and the total inventory for the supply chain, one can obtain

$$y + \frac{dqT}{2} = \frac{dT}{2P}\{2Dq + (mq + y)(P - D)\}.$$

Hence,

$$\begin{aligned} A_s &= \frac{y}{d} \\ &= qT\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right) \end{aligned} \tag{47}$$

Considering an investment for quality improvement and setup cost reduction, the capital investment function is assumed as a logarithmic function as suggested by Porteus (1986). $I_\theta(\theta)$, investment to reduce the *out-of-control* probability θ is given by

$$I_\theta(\theta) = b\ln\left(\frac{\theta_0}{\theta}\right) \qquad \text{for} \qquad 0 < \theta \le \theta_0$$

It is to be noted that lower value of the probability θ gives higher value of quality level, where θ_0 is the initial probability that the production process may go to *out-of-control* state and $b = 1/\delta$, where δ is the decrease of percentage in θ/dollar increase in $I_\theta(\theta)$.

Now $I_S(S)$, the investment for setup cost reduction is expressed as

$$I_S(S) \quad = \quad B \ln\left(\frac{S_0}{S}\right) \qquad \text{for} \qquad 0 < S \leq S_0$$

where S_0 is the initial setup cost, $B = 1/\Delta$, Δ is a decrease of percentage in S/ dollar increase in $I_S(S)$.

Thus, the total investment in quality improvement and setup cost reduction is obtained as

$$I(\theta, S) \quad = \quad I_\theta(\theta) + I_S(S) = G - b\ln\theta - B\ln S,$$

where $G = b\ln\theta_0 + B\ln S_0$

This model considers a possible relationship between lot size and quality by incorporating a quality-related cost. In an imperfect production process, there is a certain probability θ that a system may go to *out-of-control* state. θ is provided and considered to be very small and close to zero. Once the process goes to *out-of-control* state it starts producing defective items and continues to do so unless the entire lot is produced. In such a situation, the expected defective units in a production lot size Q is approximated to be $\frac{Q^2\theta}{2}$ (for more details [See Appendix A]). Again it is considered s as the cost of replacing a defective item. Thus, the expected annual defective cost is $\frac{sD\theta}{2}\left[mq + \frac{2mdq^2}{2D+dq}\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right)\right]$ (for more details [See Appendix B]).

Therefore, the supplier's total cost function is obtained as

$$\begin{aligned} TC_s \quad &= \quad \Big(\frac{D}{mq} + \frac{d}{2m}\Big)S + q(H_s + C_d d)\Big(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\Big) \\ &+ \quad \alpha(G - b\ln\theta - B\ln S) + \frac{sD\theta}{2}\left[mq + \frac{2mdq^2}{2D+dq}\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right)\right] \end{aligned} \tag{48}$$

The integrated inventory cost for the entire SCM is obtained as $TC_b + TC_s$

$$\begin{aligned} TC(\theta, m, q, S) \quad &= \quad \Big(\frac{D}{mq} + \frac{d}{2m}\Big)(A + S + mF + Vmq) + \frac{q}{2}\Big[(H_B + C_d d) \\ &+ \quad (H_s + C_d d)\Big(\frac{(2-m)D}{P} + m - 1\Big)\Big] + \alpha\Big(G - b\ln\theta - B\ln S\Big) \\ &+ \quad sD\frac{\theta}{2}\left[mq + \frac{2mdq^2}{2D+dq}\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right)\right] \end{aligned} \tag{49}$$

for $0 < \theta \leq \theta_0$ and $0 < S \leq S_0$, α being the fractional cost of capital investment (e.g., the rate of interest).

5.2.2 Lemma 1

If θ^*, m^*, q^*, S^* are the optimal values of θ, m, q, S, then $TC(\theta, m, q, S)$ is global minimum solution at θ^*, m^*, q^*, S^* if the constraints $0 < \theta \leq \theta_0$ and $0 < S \leq S_0$ are relaxed.

Proof

For the global minimum solution of the total cost function the principal minor should be positive definite. For that purpose, the principal minor has to be greater than zero at the point where the first order partial derivatives with respect to θ,m,q,S equal to zero.

Differentiating TC partially with respect to θ,m,q,S, respectively, one can obtain

$$\frac{\partial TC}{\partial \theta} = -\frac{\alpha b}{\theta} + \frac{sD}{2}\left[mq + \frac{2mdq^2}{2D+dq}\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right)\right] \tag{50}$$

$$\begin{aligned}
\frac{\partial TC}{\partial m} &= -\left(\frac{D}{m^2 q} + \frac{d}{2m^2}\right)(A + S + mF + Vmq) + \left(\frac{D}{mq} + \frac{d}{2m}\right)(F + Vq) \\
&+ \frac{q}{2}\left[(H_s + C_d d)\left(1 - \frac{D}{P}\right)\right] + \frac{sD\theta}{2}\left[q + \frac{2dq^2}{2D+dq}\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right) + \frac{2mdq^2\left(1 - \frac{D}{P}\right)}{2D+dq}\right] \\
\frac{\partial TC}{\partial q} &= -\frac{D}{mq^2}(A + S + mF + Vmq) + \left(\frac{D}{mq} + \frac{d}{2m}\right)Vm \\
&+ \frac{1}{2}\left[(H_B + C_d d) + (H_s + C_d d)\{\frac{(2-m)D}{P} + m - 1\}\right] \\
&+ \frac{sD\theta}{2}\left[m + \left(\frac{4mdq}{2D+dq} - \frac{2md^2q^2}{(2D+dq)^2}\right)\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right)\right] \qquad (51) \\
\frac{\partial TC}{\partial S} &= \left(\frac{D}{mq} + \frac{d}{2m}\right) - \alpha\left(\frac{B}{S}\right) \qquad (52)
\end{aligned}$$

To find optimal values of decision variables, the values of $\frac{\partial TC}{\partial \theta}, \frac{\partial TC}{\partial m}, \frac{\partial TC}{\partial q}, \frac{\partial TC}{\partial S}$ have to be set to zero. One has the following expressions.

$$\theta = \frac{2b\alpha}{sD\left[mq + \frac{2mdq^2}{2D+dq}\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right)\right]} \tag{53}$$

$$m = \sqrt{\frac{\phi_2 - m^3\phi_3}{\phi_1}} \tag{54}$$

$$q = \sqrt{\frac{\rho_1}{\rho_4 + m\rho_2\{1 + \frac{2dq\rho_3(4D+dq)}{(2D+dq)^2}\}}} \tag{55}$$

$$S = \frac{2\alpha Bmq}{2D+dq} \tag{56}$$

where,

$$\begin{aligned}
\phi_1 &= \frac{q}{2}\left[(H_s + C_d d)\left(1 - \frac{D}{P}\right)\right] + \frac{sD^2\theta q}{2D+dq}(1 + \frac{dq}{P}) \\
\phi_2 &= \frac{(2D+dq)(A+S)}{2q} \\
\phi_3 &= \frac{3}{2}\left(1 - \frac{D}{P}\right)\left(\frac{sD\theta dq^2}{2D+dq}\right) \\
\rho_1 &= \frac{D}{m}(A + S + mF) \\
\rho_2 &= \frac{sD\theta}{2} \\
\rho_3 &= \left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right) \\
\rho_4 &= \frac{1}{2}\left[(H_B + C_d d) + (H_s + C_d d)\{\frac{(2-m)D}{P} + m - 1\}\right] + \frac{dV}{2}
\end{aligned}$$

Now, the Hessian matrix H are calculated as follows:

$$H = \begin{bmatrix} \frac{\partial^2 TC}{\partial \theta^2} & \frac{\partial^2 TC}{\partial \theta \partial m} & \frac{\partial^2 TC}{\partial \theta \partial q} & \frac{\partial^2 TC}{\partial \theta \partial S} \\ \frac{\partial^2 TC}{\partial m \partial \theta} & \frac{\partial^2 TC}{\partial m^2} & \frac{\partial^2 TC}{\partial m \partial q} & \frac{\partial^2 TC}{\partial m \partial S} \\ \frac{\partial^2 TC}{\partial q \partial \theta} & \frac{\partial^2 TC}{\partial q \partial m} & \frac{\partial^2 TC}{\partial q^2} & \frac{\partial^2 TC}{\partial q \partial S} \\ \frac{\partial^2 TC}{\partial S \partial \theta} & \frac{\partial^2 TC}{\partial S \partial m} & \frac{\partial^2 TC}{\partial S \partial q} & \frac{\partial^2 TC}{\partial S^2} \end{bmatrix} \tag{57}$$

Where $TC = TC(\theta, m, q, S)$.

Now, we evaluate principal minors of H.

The first principal minor of H is

$$|H_{11}| = \left|\alpha \frac{b}{\theta^2}\right| > 0$$

The second principal minor of H is

$$\begin{aligned}
|H_{22}| &= \begin{vmatrix} \alpha\frac{b}{\theta^2} & x_3 \\ x_3 & \frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+S)+\frac{sD\theta dq^2\left(1-\frac{D}{P}\right)}{2(2D+dq)} \end{vmatrix} \\
&= \frac{\alpha b}{\theta^2}\left[\frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+S)+\frac{sD\theta dq^2\left(1-\frac{D}{P}\right)}{2(2D+dq)}\right] - x_3^2 > 0 \qquad (58)
\end{aligned}$$

as θ is very small and it is in the denominator with square power. Thus, the 1st term of the expression is a large positive quantity than the 2nd term of the expression, even though it is in power two. Hence, for small value of θ, i.e, the small value of *out-of-control* probability, $H_{22} > 0$.

The third principal minor of H is

$$\begin{aligned}
|H_{33}| &= \begin{vmatrix} \frac{\alpha b}{\theta^2} & x_3 & x_2 \\ x_3 & x_4 & \xi_1 \\ x_2 & \xi_1 & x_1 \end{vmatrix} \\
&= \frac{\alpha b}{\theta^2}(x_4x_1 - {\xi_1}^2) - x_3(x_3x_1 - x_2\xi_1) + x_2(x_3\xi_1 - x_2x_4)
\end{aligned}$$

$$\begin{aligned}
H_{33} &= x_2(x_3\xi_1 - x_2x_4) - \xi_1\left(\frac{\alpha b}{\theta^2}\xi_1 + H_{22}\right) \\
&> 2x_2x_3\xi_1 - x_2^2x_4 - \frac{\alpha b}{\theta^2}\xi_1^2 \\
&= \left[(x_2x_3)^2 + 2x_2x_3\xi_1 + \xi_1^2\right] - \left[x_2^2(x_3^2 + x_4) + (1 + \frac{\alpha b}{\theta^2})\xi_1^2\right] \\
&= (x_2x_3 + \xi_1)^2 - \left[x_2^2(x_3^2 + x_4) + (1 + \frac{\alpha b}{\theta^2})\xi_1^2\right] \\
&> 0
\end{aligned}$$

[See Appendix E]

The forth principal minor of H is

$$
\begin{aligned}
|H_{44}| &= \begin{vmatrix} \frac{\alpha b}{\theta^2} & x_3 & x_2 & 0 \\ x_3 & x_4 & \xi_1 & x_5 \\ x_2 & \xi_1 & x_1 & x_6 \\ 0 & x_5 & x_6 & \frac{\alpha B}{S^2} \end{vmatrix} \\
&= x_5 \begin{vmatrix} \frac{\alpha b}{\theta^2} & x_3 & x_2 \\ x_2 & \xi_1 & x_1 \\ 0 & x_5 & x_6 \end{vmatrix} - x_6 \begin{vmatrix} \frac{\alpha b}{\theta^2} & x_3 & x_2 \\ x_3 & x4 & \xi_1 \\ 0 & x_5 & x_6 \end{vmatrix} + \frac{\alpha B}{S^2} H_{33}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\alpha b}{\theta^2}[2\xi_1 x_5 x_6 - x_1 x_5^2 - x_4 x_6^2] + [x_2^2 x_5^2 - 2x_3 x_2 x_6 x_5 + x_3^2 x_6^2] + \frac{\alpha B}{S^2} H_{33} \\
&= \frac{\alpha B}{S^2} H_{33} + (x_2 x_5 - x_3 x_6)^2 - \frac{\alpha b}{\theta^2}[x_1 x_5^2 + x_4 x_6^2 - 2\xi_1 x_5 x_6] \\
&> (x_2 x_5 - x_3 x_6)^2 - \frac{\alpha b}{\theta^2}[\xi_1 x_5^2 + \xi_1 x_6^2 - 2\xi_1 x_5 x_6] \\
&> (x_2 x_5 - x_3 x_6)^2 - \frac{\alpha b}{\theta^2}\xi_1[x_5^2 + x_6^2 - 2x_5 x_6] \\
&> (x_2 x_5 - x_3 x_6)^2 - \frac{\alpha b}{\theta^2}\xi_1(x_5 - x_6)^2 \\
&> 0 \qquad \text{[See Appendix F]} \qquad (59)
\end{aligned}
$$

Thus, the total cost function has the global optimum solution at optimum values of decision variables if the conditions are satisfied.

5.3 Numerical results

The buyer is staying in urban areas whereas the supplier is staying in rural areas, thus the holding cost of the buyer is huge comparing with supplier. This is the reason to use more holding cost for numerical experiment. This type of model can be managed easily by the SSMD policy. The numerical data is taken as follows: $A = \$10$/ order, $P = 100$ units/year, $S_0 = \$100$/batch, $H_b = \$4000$/unit/year, $H_s = \$6$/unit/year, $D = 40$ units, $d = 0.02$, $F = \$50$/delivery,

$V = \$1$/unit, $C_d = \$50$/unit and we consider the numerical data $b = 10$, $B = 4800$, $\theta_0 = 0.02$, $\alpha = \$0.1$/year, $s = \$10$/defective unit.

Then, the optimal solution is $TC = \$6297.31$/year, $m = 2$,$S = \$28.18$/order, $q = 1.17$ units, $\theta = 0.0021$.

5.3.1 Special case 1

A special case arises when the rate of deterioration is considered to be zero. From the following Table 5.1, we see that the single-supplier-single delivery (SSSD) policy is less favorable over SSMD policy as the cost increases for SSSD policy. Results are given in Table 5.1.

Table 5.1

Study for non-deterioration case

Total cost	Lot size	Number of deliveries	Setup cost	θ
6297.31	1.17	2	28.17	0.0021

5.3.2 Special case 2

A special case arises when the model follows a single-supplier-single delivery (SSSD) policy instead of SSMD policy then the total cost of the supply chain model is increased and it is given in Table 5.2. The values indicate that SSMD is more beneficial than SSSD for this model.

Table 5.2

Study for SSSD case

Total cost	Lot size	Number of deliveries	Setup cost	θ
6342.83	1.22	1	14.65	0.0041

5.3.3 Sensitivity analysis

This section performs sensitivity analysis of this model. This analysis gives clear idea about the behaviour of parameters over the cost function. It can also be found out which parameter is more sensitive to the cost. The analysis is given in Table 5.3.

Table 5.3

Sensitivity analysis for key parameters

Parameters	Changes of parameters (in %)	TC (in %)	Parameters	Changes of parameters (in %)	TC (in %)
	-50%	-5.29		-50%	-0.01
	-25%	-2.19		-25%	-0.007
S_0	+25%	1.70	d	+25%	0.007
	+50%	3.09		+50%	0.01
	-50%	-7.47		-50%	-0.007
	-25%	-3.31		-25%	-0.003
s	+25%	3.73	C_d	+25%	0.003
	+50%	7.47		+50%	0.007
	-50%	-1.67		-50%	-0.02
	-25%	-0.83		-25%	-0.01
A	+25%	0.82	H_s	+25%	0.01
	+50%	1.64		+50%	0.02
	-50%	-18.73		-50%	-0.32
	-25%	-8.74		-25%	-0.16
F	+25%	7.88	V	+25%	0.16
	+50%	15.12		+50%	0.32

From Table 5.3, the sensitivity of key parameters can be observed easily.

• The negative sensitiveness of setup cost parameter S_0 is more sensitive than the positive sensitiveness parameters. But it is more sensitive in negative percentage change than positive percentage change. From this observation, it can be found that if setup cost is reduced, total cost is reduced.

• The parameter s is almost same sensitive with respect to the total cost. Negative percentage change and positive percentage change are almost same. Decreasing this cost, total cost can be reduced effectively. In reverse, if this cost is very high, the total cost will be increased gradually.

• The parameter A which stands for ordering cost for buyer is less sensitive with respect to other parameters within the cost function. The positive and negative percentage change are almost same. If ordering cost is increased, then total cost is increased and vice versa.

• Transportation cost parameter F is more sensitive in negative percentage change than positive percentage change. This parameter is most sensitive parameter comparing to others.

• Sensitivity of this parameter is almost same in negative percentage change than the positive percentage change. When d increases, total cost increases and when d decreases, total cost decreases.

• The effect of this parameter C_d, namely, deterioration cost, is like deterioration rate d. The negative percentage change and positive percentage change is exactly same. Increased deterioration cost implies decreased total cost and decreased deterioration cost imply increased total cost.

• This parameter H_s is less sensitive with respect to the other parameters. The negative percentage change is much effective for total cost compare to other parameters. If holding cost for supplier reduces, total cost increased reasonably, but when supplier's holding cost is increased, total cost reduction is not reasonably decreased.

• The parameter V is not rationally sensitive for total cost. The negative percentage change and positive percentage change are same for the variable cost/unit for handling and receiving of order. When this cost increases, total cost increases and vice versa.

5.4 Managerial insights

A two echelon supply chain model was considered in this chapter with single-buyer and single-supplier under lot-splitting policy. The managerial insights of this chapter are as follows.

- A capital investment has the ability to reduce setup cost and probability of system failure. This study suggests managers to invest fund for setup cost reduction and for improving quality of products.
- For long term agreement, supplier should split the order quantity into a number of small sub-lots to save the delivery lead time.
- The total supply chain cost is reduced with reduced setup cost and improved quality under deterioration of products when lot-splitting policy is adopted to deliver products.

5.5 Concluding remarks

The objective of this study was to minimize the cost of the total supply chain while simultaneously optimizing lot size, number of deliveries, setup cost, and process quality. Two logarithmic investment functions for quality improvement and setup cost reduction, respectively were incorporated in this model. Quality improvement and setup cost reduction played a very significant role in improving efficiency of businesses and organizations from every sphere by reducing redundancy in costs and enhancing productivity thereby accounting for the flexibility of today's diverse business environment. Any adverse event would have a direct consequence on the business and customers leading to wastage of time and resource. An accurate expertise on the approaches of industries and organizations to implement these changes for a sustainable quality improvement is therefore critical. This model proved the global optimization solution of decision variables. The model saved almost $7000 per year which is large enough for any business industry to adopt the policies suggested by this model. A constant demand rate is one of the limitations of this model. Further research could be done by considering a general investment function with variable demand. The possible extension of this model would be incorporated with delay-in-payments and time-varying deterioration.

5.6 Appendices of Chapter 5

Appendix A

From Porteus (1986) the expected units of defective items in a lot size Q is

$$Q - \frac{\acute{\theta}(1-\acute{\theta}^Q)}{\theta}$$

As $\acute{\theta} = 1 - \theta$ is approximately 1, we use the Taylor series expansion of $\acute{\theta}^Q$ and obtain

$$\acute{\theta}^Q = e^{(\ln\acute{\theta})Q} \cong 1 + (\ln\acute{\theta})Q + \frac{[(\ln\acute{\theta})Q]^2}{2}$$

Hence we have the number of defective items

$$\begin{aligned}
&= Q - \frac{\acute{\theta}(1-\acute{\theta}^Q)}{\theta} \\
&= Q - \frac{1 - 1 - (\ln\acute{\theta})Q - \frac{(\ln\acute{\theta})^2 Q^2}{2}}{\theta} \\
&= Q - \frac{\frac{\theta}{\acute{\theta}}Q - \frac{\theta^2}{2\acute{\theta}^2}Q^2}{\theta} \\
&= Q - \frac{\theta Q - \frac{\theta^2 Q^2}{2}}{\theta} \\
&= \frac{\theta Q^2}{2}
\end{aligned}$$

Appendix B

The expected annual defective cost is

$$
\begin{aligned}
&= sD - \frac{sD\acute{\theta}(1-\acute{\theta}^Q)}{\theta Q} \\
&= sD - \frac{sD\left(1 - 1 - (\ln\acute{\theta})Q - \frac{(\ln\acute{\theta})^2Q^2}{2}\right)}{\theta Q} \\
&\quad \left[\text{since } \acute{\theta} = 1-\theta \cong 1 \text{ and } \acute{\theta}^Q = e^{(\ln\acute{\theta})Q} \cong 1 + (\ln\acute{\theta})Q + \frac{[(\ln\acute{\theta})Q]^2}{2}\right] \\
&= sD - \frac{sD\left(\frac{\theta}{\acute{\theta}}Q - \frac{\theta^2}{2\acute{\theta}^2}Q^2\right)}{\theta Q} \\
&= sD - \frac{sD\left(\theta Q - \frac{\theta^2Q^2}{2}\right)}{\theta Q} \\
&= sD - sD\left(1 - \frac{\theta Q}{2}\right) \\
&= \frac{sD\theta Q}{2} \\
&= \frac{sD\theta}{2}\left[mq + \frac{2mdq^2}{2D+dq}\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right)\right]
\end{aligned}
$$

Appendix C

The delivery lot size q, expressed as $q = x + DT_3$ is divided into two components: x and DT_3. x being the number of deteriorating units during T_3 and DT_3 for consumption.

For smaller rate of deterioration square and higher powers of it can be neglected. Hence, during time interval T_3

$$x = \frac{T_3dq}{2}$$

and

$$q = \frac{T}{m}\left(D + \frac{dq}{2}\right) \text{ as } \frac{T}{m} = T_3$$

Now

$$q = x + DT_3$$

implies

$$\frac{1}{T} = \frac{D}{mq} + \frac{d}{2m}$$

i.e.,

$$\frac{q}{2} = \frac{mq}{dT} - \frac{D}{d}$$

Again, the total deterioration of the buyer is obtained as

$$dA_b = mq - DT$$

which implies

$$A_b = \frac{(mq - DT)}{d}$$

i.e.,

$$\frac{A_b}{T} = \frac{q}{2}$$

Appendix D

Differentiating (46) partially with respect to θ,m,q,C, respectively, one can obtain

$$\begin{aligned}
\frac{\partial^2 TC}{\partial C \partial \theta} &= 0 \\
\frac{\partial^2 TC}{\partial C \partial m} &= -\left(\frac{D}{m^2 q} + \frac{d}{2m^2}\right) \\
\frac{\partial^2 TC}{\partial C \partial q} &= -\frac{D}{mq^2} \\
\frac{\partial^2 TC}{\partial C^2} &= \alpha \frac{B}{C^2}
\end{aligned}$$

Differentiating (47) partially with respect to θ,m,q,C, respectively, one has

$$\begin{aligned}
\frac{\partial^2 TC}{\partial \theta^2} &= \alpha \frac{b}{\theta^2} \\
\frac{\partial^2 TC}{\partial \theta \partial m} &= \frac{sD}{2}\left[q + \frac{2dq^2}{2D + dq}\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right) + \frac{2mdq^2\left(1 - \frac{D}{P}\right)}{2D + dq}\right] = x_3 \\
\frac{\partial^2 TC}{\partial \theta \partial q} &= \frac{sD}{2}\left[m + \left(\frac{4mdq}{2D + dq} - \frac{2md^2q^2}{(2D + dq)^2}\right)\left(\frac{D}{P} + \frac{m-1}{2} - \frac{Dm}{2P}\right)\right] = x_2 \\
\frac{\partial^2 TC}{\partial \theta \partial C} &= 0
\end{aligned}$$

Differentiating (48) partially with respect to θ,m,q,C, respectively, the partial derivatives are obtained as follows:

$$\begin{aligned}
\frac{\partial^2 TC}{\partial m \partial\theta} &= \frac{sD}{2}\left[q + \frac{2dq^2}{2D+dq}\left(\frac{D}{P}+\frac{m-1}{2}-\frac{Dm}{2P}\right)+\frac{2mdq^2\left(1-\frac{D}{P}\right)}{2D+dq}\right] = x_3 \\
\frac{\partial^2 TC}{\partial m^2} &= \frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+C) + \frac{sD\theta dq^2\left(1-\frac{D}{P}\right)}{2(2D+dq)} = x_4 \\
\frac{\partial^2 TC}{\partial m \partial q} &= \frac{D}{m^2q^2}(A+C+mF+VmF) + \frac{1}{2}\left(1-\frac{D}{P}\right)(H_s + C_d d) \\
&+ \frac{sD\theta}{2}\left[1+\left(\frac{4dq}{2D+dq}-\frac{2dq^2}{(2D+dq)^2}\right)\left(\frac{D}{P}+\frac{m-1}{2}-\frac{Dm}{2P}\right)+\frac{2mdq\left(1-\frac{D}{P}\right)}{2D+dq}\right] = \xi_1 \\
\frac{\partial^2 TC}{\partial m \partial C} &= -\left(\frac{D}{m^2 q}+\frac{d}{2m^2}\right)
\end{aligned}$$

Differentiating (49) partially with respect to θ,m,q,C, respectively, the partial derivatives are obtained as follows:

$$\begin{aligned}
\frac{\partial^2 TC}{\partial q \partial\theta} &= \frac{sD}{2}\left[m+\left(\frac{4mdq}{2D+dq}-\frac{2md^2q^2}{(2D+dq)^2}\right)\left(\frac{D}{P}+\frac{m-1}{2}-\frac{Dm}{2P}\right)\right] = x_2 \\
\frac{\partial^2 TC}{\partial q \partial m} &= \frac{D}{m^2q^2}(A+C+mF+VmF) + \frac{1}{2}\left(1-\frac{D}{P}\right)(H_s + C_d d) \\
&+ \frac{sD\theta}{2}\left[1+\left(\frac{4dq}{2D+dq}-\frac{2dq^2}{(2D+dq)^2}\right)\left(\frac{D}{P}+\frac{m-1}{2}-\frac{Dm}{2P}\right)+\frac{2mdq\left(1-\frac{D}{P}\right)}{2D+dq}\right] = \xi_1 \\
\frac{\partial^2 TC}{\partial q^2} &= \frac{2D}{mq^3}(A+C+mF) + \frac{8sD^3\theta md}{(2D+dq)^3} = x_1 \\
\frac{\partial^2 TC}{\partial q \partial C} &= -\frac{D}{mq^2}
\end{aligned}$$

Appendix E

The third principal minor of H is

$$\begin{aligned}
|H_{33}| &= \begin{vmatrix} \alpha\frac{b}{\theta^2} & 0 & \frac{sD}{2} \\ 0 & \frac{2}{m^3}(\frac{D}{q}+\frac{d}{2})(A+C) & \frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_d d) \\ \frac{sD}{2} & \frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_d d) & \frac{2D}{mq^3}(A+C+mF) \end{vmatrix} \\
&= \tfrac{\alpha b}{\theta^2}\Bigg\{ \frac{2D}{mq^3}(A+C+mF)\frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+C) \\
&- \left(\frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_d d)\right)^2\Bigg\} - \left(\frac{sD}{2}\right)^2\frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+C)
\end{aligned}$$

which can be written in the form

$$\frac{2}{m^3}\Big(\frac{D}{q}+\frac{d}{2}\Big)(A+C)\Big\{\frac{\alpha b}{\theta^2}\frac{2D}{m(q^3)}(A+C+mF)-\Big(\frac{sD}{2}\Big)^2\Big\}$$
$$-\Big(\frac{\sqrt{\alpha b}}{\theta}\Big[\frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_d d)\Big]\Big)^2$$

is of the form $x*y-z^2$. In order to prove $x*y-z^2>0$, we show that $x>z$ and $y>z$, where

$$\begin{aligned}
x &= \frac{2}{m^3}\Big(\frac{D}{q}+\frac{d}{2}\Big)(A+C)\\
y &= \frac{\alpha b}{\theta^2}\frac{2D}{mq^3}(A+C+mF)-\frac{s^2D^2}{4}\\
z &= \frac{\sqrt{\alpha b}}{\theta}\left(\frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_d d)\right)
\end{aligned}$$

First we show that $x-z>0$.

$$\begin{aligned}
&\frac{\alpha b}{\theta^2}\Big(\frac{2D}{mq^3}(A+C+mF)\frac{2}{m^3}(\frac{D}{q}+\frac{d}{2})(A+C)\\
&-\left(\frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_d d)\right)^2\Big)\\
&-\left(\frac{SD}{2}\right)^2\frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+C)\\
&=\frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+C)\left(\frac{2\alpha bD}{mq^3\theta^2}(A+C+mF)-\frac{S^2D^2}{4}\right)\\
&-\frac{\alpha b}{\theta^2}\left(\frac{D}{m^2q^2}(A+C)+\frac{(P-D)}{2P}(H_s+C_d d)\right)^2
\end{aligned}$$

i.e. $$\frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+C)>\frac{\sqrt{\alpha b}}{\theta}\left(\frac{D}{mq}(A+C)+\frac{1}{2}\left(1-\frac{D}{P}\right)(H_s+C_d d)\right)$$

$\Rightarrow$ $$\frac{2}{m^3}\left(\frac{D}{q}+\frac{d}{2}\right)(A+C)+\frac{\sqrt{\alpha b}}{2\theta}\left(1-\frac{D}{P}\right)(H_s+C_d d)>\frac{\sqrt{\alpha b}}{\theta}\left(\frac{D}{mq}(A+C)\right)$$

It is always true for any positive optimum value of decision variable.

Now we show that $y-z>0$.

$$\Big\{\frac{\sqrt{\alpha b}}{\theta^2}\frac{2D}{mq^3}(A+C+mF)\Big\}-\frac{SD}{4}-\Big\{\frac{\sqrt{\alpha}b}{\theta}\Big(\frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_d d)\Big)\Big\}>0$$

Substituting the value of m^2 the above expression takes the form

$$\frac{\sqrt{\alpha}b}{\theta}\sqrt{\frac{(H_s+C_d d)(P-D)}{P(2D+qd)}}\Big[\frac{\sqrt{\alpha}b}{\theta}\frac{2D}{q^2}\frac{(A+C+mF)}{\sqrt{A+C}}-\frac{4D+qd}{2}\sqrt{\frac{(H_s+C_d d)(P-D)}{P(2D+qd)}}\Big]$$

Then $y-z>0$ if and only if $\frac{\sqrt{\alpha}b}{\theta}\frac{2D}{q^2}\frac{(A+C+mF)}{\sqrt{A+C}} - \frac{4D+qd}{2}\sqrt{\frac{(H_s+C_dd)(P-D)}{P(2D+qd)}} > 0$.

Appendix F

The forth principal minor of H is

$$|H_{44}| = \begin{vmatrix} \alpha\frac{b}{\theta^2} & 0 & \frac{sD}{2} & 0 \\ 0 & \frac{2}{m^3}(\frac{D}{q}+\frac{d}{2})(A+C) & \frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_dd) & -(\frac{D}{m^2q}+\frac{d}{2m^2}) \\ \frac{sD}{2} & \frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_dd) & \frac{2D}{mq^3}(A+C+mF) & -\frac{D}{mq^2} \\ 0 & -(\frac{D}{m^2q}+\frac{d}{2m^2}) & -\frac{D}{mq^2} & \alpha\frac{B}{C^2} \end{vmatrix}$$

$$\begin{aligned} &= \alpha\frac{b}{\theta^2}\Big[\frac{2}{m^3}\Big(\frac{D}{q}+\frac{d}{2}\Big)(A+C)\Big(\frac{2BD\alpha}{mq^3C^2}(A+C+mF)-\frac{D^2}{m^2q^4}\Big)-\Big(\frac{D}{m^2q^2}(A+C)\Big) \\ &+ \frac{P-D}{2P}(H_s+C_dd)\Big)\Big\{\frac{\alpha B}{C^2}\Big[\frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_dd)\Big]-\frac{D}{mq^2}\Big(\frac{D}{m^2}+\frac{d}{2m^2}\Big)\Big\} \\ &- \Big(\frac{D}{m^2q}+\frac{d}{2m^2}\Big)\Big\{-\frac{D}{mq^2}\Big[\frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_dd)\Big] \\ &+ \Big(\frac{D}{m^2q}+\frac{d}{2m^2}\Big)\Big(\frac{2D(A+C+mF)}{mq^3}\Big)\Big\}\Big]+\Big(\frac{sD}{2}\Big)^2\Big[\Big(\frac{D}{m^2q}+\frac{d}{2m^2}\Big)^2-\frac{\alpha B}{C^2}\frac{2}{m}\Big(\frac{D}{qm^2} \\ &+ \frac{d}{2m^2}\Big)(A+C)\Big] \end{aligned}$$

To show that the fourth principal minor of H is greater than 0, we consider parts of it and solve them separately. Let us first show that $\left(\frac{2BD\alpha}{mq^3C^2}(A+C+mF)-\frac{D^2}{m^2q^4}\right)>0$.

$$\frac{D}{mq^3}\Big\{\frac{2B\alpha(A+C+mF)}{C^2}-\frac{D}{mq}\Big\}$$

which is greater than zero if and only if $\left\{\frac{2B\alpha(A+C+mF)}{C^2}-\frac{D}{mq}\right\}>0$.......

Now we show that $\left\{-\frac{\alpha B}{C^2}\left[\frac{D}{m^2q^2}(A+C)+\frac{P-D}{2P}(H_s+C_dd)\right]+\frac{D}{mq^2}\left(\frac{D}{m^2}+\frac{d}{2m^2}\right)\right\}>0$.

Substituting the value of m^2 in the above expression, one can obtain

$$\begin{aligned} &\frac{D(H_s+C_dd)(P-D)}{mP(A+C)2q}-\frac{\alpha B(P-D)(H_s+C_dd)}{C^2P}\Big\{\frac{D}{2D+qd}+\frac{1}{2}\Big\} \\ = &\frac{(H_s+C_dd)(P-D)}{2P}\Big\{\frac{D}{mq(A+C)}-\frac{\alpha B(4D+qd)}{C^2(4D+2qd)}\Big\} \end{aligned}$$

which is grater than 0 if and only if $\left\{\frac{D}{mq(A+C)}-\frac{\alpha B(4D+qd)}{C^2(4D+2qd)}\right\}>0$......

Now, we show that $\frac{D}{mq^2}\left\{\frac{D(A+C)}{m^2q^2}+\frac{(P-D)(H_s+C_dd)}{2P}\right\}-\frac{1}{m^2}\left\{\left(\frac{D}{q}+\frac{d}{2}\right)\left(\frac{2D(A+C+mF)}{mq^3}\right)\right\}>0$.

Substituting the value of m^2 the above expression takes form

$$\frac{(H_s + C_d d)(P-D)D}{Pmq^2}\Big[\frac{D}{2D+dq} - \Big(\frac{1}{2} + \frac{mF}{A+C}\Big)\Big]$$

which is greater than 0 if and only if $\frac{D}{2D+dq} - \left(\frac{1}{2} + \frac{mF}{A+C}\right) > 0$.

Now we show that $\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)^2 - \frac{\alpha B}{C^2}\frac{2}{m}\left(\frac{D}{qm^2} + \frac{d}{2m^2}\right)(A+C) > 0$

$$\Big(\frac{D}{m^2q} + \frac{d}{2m^2}\Big)\Big\{\frac{D}{m^2q} + \frac{d}{2m^2} - \frac{2\alpha B(A+C)}{C^2m}\Big\}$$

which is greater than 0 if and only if $\frac{1}{m^2}\left(\frac{D}{q} + \frac{d}{2}\right) - \frac{2\alpha B}{C^2m}(A+C) > 0$.

$$|H_{44}| = -\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)\begin{vmatrix} \alpha\frac{b}{\theta^2} & 0 & \frac{sD}{2} \\ \frac{sD}{2} & \frac{D}{m^2q^2}(A+C) + \frac{P-D}{2P}(H_s + C_d d) & \frac{2D}{mq^3}(A+C+mF) \\ 0 & -(\frac{D}{m^2q} + \frac{d}{2m^2}) & -\frac{D}{mq^2} \end{vmatrix}$$

$$+ \frac{D}{mq^2}\begin{vmatrix} \alpha\frac{b}{\theta^2} & 0 & \frac{sD}{2} \\ 0 & \frac{2}{m^3}(\frac{D}{q} + \frac{d}{2})(A+C) & \frac{D}{m^2q^2}(A+C) + \frac{P-D}{2P}(H_s + C_d d) \\ 0 & -(\frac{D}{m^2q} + \frac{d}{2m^2}) & -\frac{D}{mq^2} \end{vmatrix} + \frac{\alpha B}{C^2}H_{33}$$

$= \left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)\left[\frac{\alpha b}{\theta^2}\left[\left(\frac{D^2}{m^3q^4}(A+C) + \frac{(P-D)D}{2Pmq^2}(H_s + C_d d)\right) - \frac{2D(A+C+mF)}{mq^3}\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)\right]\right.$
$\left. + \frac{SD}{2}\left(\frac{SD}{2}\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)\right)\right] + \frac{\alpha B}{C^2}H_{33} + \frac{D}{mq^2}\left[\frac{\alpha b}{\theta^2}\left(-\frac{2D}{m^4q^2}\left(\frac{D}{q} + \frac{d}{2}\right)(A+c) + \frac{D}{m^2q^2}\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)(A+\right.\right.$
$\left.\left. C)\left(\frac{P-D}{2P}\right)(H_s + C_d d)\right)\right]$

Thus, if we can show

$\frac{S^2D^2}{4m^2}\left(\frac{D}{q} + \frac{d}{2}\right) + \frac{\alpha b}{\theta^2}\left[\frac{D^2}{m^3q^4}(A+C) + \frac{(P-D)D}{2Pmq^2}(H_s + C_d d) - \frac{2D(A+C+mF)}{mq^3}\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)\right] > 0$

thus, $H_{44} > 0$

$\frac{D^2}{m^3q^4}(A+C) + \left(1 - \frac{D}{P}\right)\frac{D}{2mq^2}(H_s + C_d d) - \frac{2D}{mq^3}\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)(A+C) - \frac{2DmF}{mq^3}\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right)$
$= \frac{D^2}{m^3q^4}(A+C) - \frac{2D^2(A+C)}{m^3q^4} - \frac{2Dd}{2m^3q^3}(A+C) - \frac{2DF}{q^3}\left(\frac{D}{m^2q} + \frac{d}{2m^2}\right) + \left(1 - \frac{D}{P}\right)\frac{D}{2mq^2}(H_s + C_d d)$
$= \frac{D^2}{m^3q^4}(A+C) - \frac{Dd}{m^3q^3}(A+C) - \frac{2D^2F}{m^2q^4} - \frac{DdF}{m^2q^3} + \left(1 - \frac{D}{P}\right)\frac{D}{2mq^2}(H_s + C_d d)$
$= \frac{1}{2m^3q^4}\left[(H_s + C_d d)m^2q^2D\left(1 - \frac{P}{D}\right) - 2D^2(A+C) - 2Ddq(A+C) - 4D^2mF - 2mqDdF\right]$
$= \frac{1}{2m^3q^4}\left[(H_s + C_d d)m^2q^2D\left(1 - \frac{P}{D}\right) - 2D(A+C)(D-dq) - 2DmF(2D+dq)\right]$
$= \frac{1}{2m^3q^4}\left[(H_s + C_d d)m^2q^2D\left(1 - \frac{P}{D}\right) - 2D(D+dq)(A+C+mF) - 2D^2mF\right] - \frac{2D}{m^4q^2}\left(\frac{D}{q} + \frac{d}{2}\right)(A+$
$C) + \frac{D}{2m^4q^2}\left(\frac{D}{q} + \frac{d}{2}\right)(A+C)\left(1 - \frac{P}{D}\right)(H_s + C_d d)$
$= \frac{D^2\left(\frac{D}{q} + \frac{d}{2}\right)(A+C)}{2m^4q^2}\left[\left(1 - \frac{P}{D}\right)(H_s + C_d d) - 4\right]$

Chapter 6
Joint effect of price and demand on decision making in a supply chain management

6 Joint effect of price and demand on decision making in a supply chain management

This chapter deals with a manufacturer-retailer supply chain model with decentralized decisions. Both manufacturer and retailer make their decisions independently. Manufacturer's profit depends on decisions made by retailer. Depending on the nature of the purchasing cost of retailer, this study considers two cases. In first case, retailer's purchase cost fully depends on decisions made by retailer and in second case manufacturer determines the purchasing cost of the retailer independently. Single-setup single-delivery (SSSD) and single-setup multi-delivery (SSMD) policies are considered for first and second cases, respectively. Retailer obtains the optimum selling-price of product to maximize its profit. The customer's demand is price-sensitive, whereas the lead time demand is considered as stochastic and supposed to follow a normal distribution. The distribution free approach is considered for known mean and standard deviation.

In every suburban area, a significant number of small retailers exist, whereas there are a few number of multiplexes or shopping malls. People use to purchase products from small retailers or retailers. It basically happens due to the economic status of these regions. The cost of an item has huge importance to sell that particular product. The price is such sensitive that customers deny to purchase product, if the price becomes higher than the previous price. These retailers receive their orders from the manufacturer, but instead of joint collaboration they make their decisions independently. Thus, manufacturer's profit depends on the quantity ordered by the retailer. This research describes a model where manufacturer follows firstly SSSD policy in order to determine the purchase cost for retailer depending on decisions made by retailer. In the second case, SSMD policy is taken by manufacturer and it makes an agreement with retailer to improve this supply chain by shortening lead time. The manufacturer maximizes its profit even if the independent decisions are made by retailer. In this case, the purchasing cost for retailer is totally independent by decisions made by retailer.

6.1 Literature review

Selling-price plays an important role for demands of any product. Customers always purchase products of high quality and longevity, but with fare price. This is obvious that demand of any product should increase if any highly reliable product can be purchased with low cost. Accordingly, increasing selling-price can dwindle the demand of a particular product. Whitin (1955) developed concepts of economic price theory and inventory control. Lau and Lau (1988) extended the classical newsboy problem with stochastic price-demand relationship. Gallego and Ryzin (1994) investigated dynamic pricing of inventories, where demand is price-sensitive as well as stochastic and firm's objective is to maximize its expected profit. Abad (1996) formulated a dynamic and lot-sizing model for perishable items. Dutta and Paul (2001) analyzed an inventory system where demand rate was influenced by selling-price as well as stock level. Teng and Chang (2005) discussed an economic production quantity model for deteriorating products with price and stock dependent demand. Sana (2011) developed an economic order quantity model with perishable items and quadratic price-sensitive demand. Chen *et al.* (2012) investigated the pricing strategy for the manufacturer and warranty period dependant demand.

6.2 Model formulation

This section contains assumptions to formulate a mathematical model, entire model description, lemmas, and solution algorithms.

6.2.1 Assumptions

The following assumptions are considered to develop this model.

1. Due to the economic background of the people of suburban areas, increasing selling-price is a important factor of decreasing demand. Thus, we assume that demand is dependent on selling-price of retailer with the relation $D(p) = a - bp - cp^2$; $a, b, c > 0$ (Sana, 2011).

2. Continuous review policy is considered i.e., retailer places an order when the inventory level reaches to the reorder point.

3. The reorder point $r=$ expected lead time demand+safety stock.

4. Lead time has n mutually independent component with normal duration $\bar{b}_i$, minimum duration $\bar{a}_i$, and lead time crashing cost $\bar{c}_i$ for ith component and $\bar{c}_1 \leq \bar{c}_2 \leq ... \leq \bar{c}_n$.

5. The lead time components are crashed one at a time. The crashing costs of lead time gradually increase from the first component then the second and so on. We consider $L_0 \equiv \sum_{j=1}^{n} \bar{b}_j$ and L_i is the length of the lead time having components $1, 2, ..., i$ crashed to their minimum duration. $L_i = L_0 - \sum_{j=1}^{i}(\bar{b}_j - \bar{a}_j),\ i = 1, 2, ..., n$. The lead time crashing cost per cycle is given by $C(L) = \bar{c}_i(L_{i-1} - L) + \sum_{j=1}^{i-1} \bar{c}_j(\bar{b}_j - \bar{a}_j)$, $L \in [L_i, L_{i-1}]$. (Ouyang *et al*, 2004).

6. Shortages are considered and fully backlogged.

Demand of the customer is price-sensitive i.e., dependent on the selling-price of products. We consider the quadratic price-dependent demand function as $D(p) = a - bp - cp^2$ (Sana, 2011) where a is the customer's annual demand without price dependency, b and c are ratios of price which influences the annual demand. The customer's demand decreases quadratically with the increase of selling-price of product. The inventory level of retailer gradually decreases with time, which is equal to demand rate i.e.,

$$\frac{dQ}{dt} = -D(p) = a - bp - cp^2$$

which gives $Q = D(p)t = (a - bp - cp^2)t$

The system is continuously reviewed by retailer. When the inventory level fall down to the reorder point r, an order is placed immediately by retailer such that the expected inventory level just before and after receipt of the order are $r - D(p)L$ and $Q + r - D(p)L$, respectively. Thus, the average inventory of the retailer over the cycle is $Q/2 + r - D(p)L$. Total expected profit for retailer is

$$
\begin{aligned}
TEP_b(Q,r,L,p) &= \text{Total revenue} - \text{Ordering cost} - \text{Holding cost} \\
&- \text{Material cost} - \text{Shortage cost} - \text{Lead time crashing cost} \\
&= pD(p) - \frac{A}{t} - r_bC_b\left(\frac{Q}{2} + r - D(p)L\right) \\
&- C_bD(p) - \frac{\pi}{t}E(X-r)^+ - \frac{C(L)}{t}
\end{aligned}
$$

The above equation can be written as

$$
\begin{aligned}
TEP_b(Q,r,L,p) &= (p-C_b)D(p) - \frac{AD(p)}{Q} - r_bC_b\left(\frac{Q}{2} + r - D(p)L\right) \\
&- \frac{\pi D(p)}{Q}E(X-r)^+ - \frac{D(p)C(L)}{Q}
\end{aligned} \tag{60}
$$

Expected shortage can be calculated as

$E(X-r)^+ = \int_r^\infty (x-r)f(x)dx = \sigma\sqrt{L}\psi(k),$ where $\psi(k) = \phi(k) - k[1-\Phi(k)]$ and

$\phi(k) =$ standard normal probability density function

$\Phi(k) =$ cumulative density function of normal distribution

In order to optimize the order quantity Q and the safety factor k, we consider the total cost equation for retailer

$$
TEC_b = \frac{AD(p)}{Q} + r_bC_b\left(\frac{Q}{2} + r - D(p)L\right) + C_bD(p) + \frac{\pi D(p)}{Q}\sigma\sqrt{L}\psi(k) + \frac{D(p)C(L)}{Q} \tag{61}
$$

As the revenue for the retailer $pD(p)$ is independent of the order quantity and safety factor thus, optimizing the order quantity Q and the safety factor k from (61) is equivalent to obtaining Q and k from (60). Therefore, taking partial derivatives of (61) with respect to Q and k and equating to zero, we obtain

$$
Q_b = \left\{\frac{2D(p)[A + \pi\sigma\sqrt{L}\psi(k) + C(L)]}{r_bC_b}\right\}^{1/2} \tag{62}
$$

$$
\Phi(k) = 1 - \frac{r_bC_bQ_b}{\pi D(p)} \tag{63}
$$

The second order partial derivative of (61) with respect to L is negative

$$\frac{\partial^2 ATC_b}{\partial L^2} = -\frac{r_b C_b k\sigma}{4L^{3/2}} - \frac{\pi D(p)\sigma\psi(k)}{4QL^{3/2}}$$

Thus, the optimum value of L can be obtained at the end point of the interval $[L_i, L_{i-1}]$.

Now, to determine the optimal selling-price of retailer, we consider equation (60) i.e., the expected profit of retailer. Putting the value of $D(p)$ and taking partial derivative of retailer's profit equation with respect to the selling-price, we obtain

$$\frac{\partial TEP_b}{\partial p} = (a - 2bp - 3cp^2) + C_b(b + 2cp) + (b + 2cp)\left(\frac{A}{Q} + \frac{\pi\sigma\sqrt{L}\psi(k)}{Q} + \frac{C(L)}{Q}\right)$$

$$\begin{aligned} \frac{\partial TEP_b}{\partial p} &= 0 \text{ implies} \\ p^* &= \frac{\pm\sqrt{B_1^2 - 4A_1C_1} - B_1}{2A_1} \end{aligned}$$

We assume

$$p_1^* = \frac{\sqrt{B_1^2 - 4A_1C_1} - B_1}{2A_1} \tag{64}$$

$$p_2^* = \frac{-\sqrt{B_1^2 - 4A_1C_1} - B_1}{2A_1} \tag{65}$$

$$\begin{aligned} \text{where } A_1 &= -3c \\ B_1 &= 2\left\{cC_b - b + \frac{\alpha c}{Q}\right\} \\ C_1 &= C_b b + \frac{b\alpha}{Q} + a \\ \alpha &= A + \pi\sigma\sqrt{L}\psi(k) + C(L) \text{ [See Appendix A]} \end{aligned} \tag{66}$$

6.2.2 Lemma 1

For all real and positive values of parameters, p^* is always real and positive if $p^* = \frac{-\sqrt{B_1^2 - 4A_1C_1} - B_1}{2A_1}$ is chosen, otherwise $p^* > 0$ only if $b > c(C_b + \alpha/Q)$ holds.

Proof

From (66), $A_1 = -3c$, $B_1^2 - 4A_1C_1 = B_1^2 + 12cC_1 > 0$ for $c > 0$.

Therefore, the value of p^* is always real for $c > 0$.

Case 1: $p^* = p_1^* = \frac{-\sqrt{B_1^2-4A_1C_1}-B_1}{2A_1}$ is chosen.

If $b < c(C_b + \alpha/Q)$ then, $B_1 > 0$ gives $p_1^* = \frac{\sqrt{B_1^2+12cC_1}+B_1}{6c} > 0$.

If $b > c(C_b+\alpha/Q)$ then, $B_1 < 0$. Let $B_1 = -B$, $B \in R$ which results $p_1^* = \frac{\sqrt{B^2+12cC_1}-B}{6c} > 0$ as $\sqrt{B^2 + 12cC_1} \leq \sqrt{B^2} + \sqrt{12cC_1} = B + \sqrt{12cC_1} > B$.

Case 2: $p^* = p_2^* = \frac{\sqrt{B_1^2-4A_1C_1}-B_1}{2A_1}$ is chosen.

If $b < c(C_b + \alpha/Q)$ then, $B_1 > 0$ gives $p_2^* = \frac{\sqrt{B_1^2+12cC_1}-B_1}{-6c} < 0$ as $\sqrt{B_1^2 + 12cC_1} > B_1$.

If $b > c(C_b + \alpha/Q)$ then, $B_1 < 0$ which gives $p_2^* = \frac{\sqrt{B^2+12cC_1}+B}{6c} > 0$, assuming $B_1 = -B$, $B \in R$.

6.2.3 Lemma 2

If (Q_b^*, k^*, p^*) are the optimal values for (Q, k, p) then for fixed $L \in [L_i, L_{i-1}]$, the optimal value of $TEP_b(Q, k, p, L)$ attains its global maximum at the point (Q_b^*, k^*, p^*) if the following inequality holds $[\pi\sigma\sqrt{L}\psi(k^*) + 2A + C(L)] > \pi\sigma\sqrt{L}(1 - \Phi(k^*))Q^* > 0$

Proof

See Appendix B.

Now, we consider that manufacturer follows SSSD policy to deliver the order quantity to retailer. In this case, manufacturer's total profit equation is

$$\begin{aligned} TEP_v(p,Q) &= \text{Total revenue} - \text{Setup cost} - \text{Holding cost} - \text{Material cost} && (67)\\ &= C_bD(p) - \frac{SD(p)}{Q} - r_vC_v\frac{QD(p)}{2P} - C_vD(p) && (68)\end{aligned}$$

The optimal lot size for manufacturer can be obtained by taking partial derivative with respect to Q and equating it to zero. The optimal lot size for manufacturer is

$$Q_v = \left\{\frac{2SP}{r_vC_v}\right\}^{1/2} \tag{69}$$

Clearly, the order quantity for retailer must be equal to the lot size produced by manufacturer. In order to determine the purchasing cost for retailer, we equate $Q_b^* = Q_v^*$, which gives

$$C_b^* = \frac{r_v C_v D(p)\alpha}{r_b P S} \tag{70}$$

The manufacturer's total profit can be obtained by substituting values of the decision variables of the retailer's in the manufacturer's profit function i.e.,

$$TEP_v(Q_b^*, C_b^*, p^*) = C_b^* D(p^*) - \frac{SD(p^*)}{Q^*} - \frac{r_v C_v Q^* D(p^*)}{2P} - C_v D(p^*) \tag{71}$$

Now, we investigate if manufacturer follows the SSMD policy to deliver the lot to retailer. This policy suggests manufacturer to produce integer multiple of retailer's ordering quantity in single-setup to reduce the setup cost. In this case, manufacturer determines the purchase cost for retailer of its own. The total profit equation for manufacturer is

$$TEP_v(Q, C_b, p, m) = C_b D(p) - \frac{SD(p)}{mQ} - \frac{r_v C_v}{2}\left[m\left(1 - \frac{D(p)}{P}\right) - 1 + \frac{2D(p)}{P}\right] Q - C_v D(p) \tag{72}$$

Again, substituting values of decision variables of retailer in manufacturer's total profit equation for SSMD policy, the maximum profit for the manufacturer can be obtained, when the following inequality holds

$$TEP_v(Q_b^*, C_b^*, p^*, m-1) \leq TEP_v(Q_b^*, C_b^*, p^*, m) \geq TEP_v(Q_b^*, C_b^*, p^*, m+1)$$

The following algorithm is used to obtain solutions of the both SSSD and SSMD model with normally distributed lead time demand.

6.2.4 Solution algorithm 1

Step 1 Set $m = 1$ and for each L_i, $i = 0, 1, ..., n$ perform the following Step 1a-2e.

Step 1a Set $k_{i1} = 0$ and obtain $\psi(k_{i1}), \phi(k_{i1})$, and $\Phi(k_{i1})$.

Step 1b Compute Q_{bi1} from (62).

Step 1c Compute $\Phi(k_{i2})$ from (63) and hence obtain k_{i2} from normal table.

Step 2 If $b < c(C_b + \alpha/Q)$ then,

Step 2a Compute p_{i1} from (65) and C_{bi1} from (70).

Step 2b Repeat Step 1-2a until no changes occur in the values of Q_{bi}, k_i, p_i, and C_{bi}.

Step 2c Compute $TEP_b(Q_i, k_i, p_i, C_{bi})$.

Step 2d Obtain $max_{i=0,1,\ldots,n} TEP_b(Q_i, k_i, p_i, C_{bi})$, the maximum profit for the retailer and the optimal reorder point is $r^* = D(p^*)L^* + k^*\sigma\sqrt{L^*}$.

Step 2e For SSSD policy, obtain $TEP_v(Q_i^*, p_i^*, C_{bi}^*)$ from (70).

Step 2f For SSMD policy set $m = m + 1$ and obtain $TEP_v(Q_i^*, p_i^*, m)$.

Step 2g If $TEP_v(Q_i^*, p_i^*, m) \leq TEP_v(Q_i^*, p_i^*, m-1)$ go to Step 2f otherwise go to Step 2h.

Step 2h Set $TEP_v(Q_i^*, p_i^*, m^*) = TEP_v(Q_i^*, p_i^*, m-1)$, the maximum profit for the manufacturer.

Step 3 If $b > c(C_b + \alpha/Q)$, compute p_{i1}^1 from (64) and p_{i1}^2 from (65).

Step 3a Repeat Step 1-1c and Step 3 until no changes occur in the values of Q_{bi}, k_i, p_{i1}^1, and p_{i1}^2.

Step 3b For both p_{i1}^1, and p_{i1}^2 execute Step 2a-2d.

Step 3c Set $max_{i=0,1,\ldots,n} TEP_b(Q_i, k_i, p_i, C_{bi})$= $max\{max_{i=0,1,\ldots,n} TEP_b(Q_i, k_i, p_i^1, C_{bi}), max_{i=0,1,\ldots,n} TEP_b(Q_i, k_i, p_i^2, C_{bi})\}$.

6.2.5 Distribution free approach

The idea of distribution free approach comes into light when difficulties occur for finding the information about demand information during lead time. This procedure is applicable when no information about the lead time demand distribution is available except known mean and standard deviation. We consider a distribution function (d.f.) F for the lead time demand in the class G (say) of d.f.'s with the finite mean $D(p)L$ and standard deviation $\sigma\sqrt{L}$. This approach suggests to figure out the least favorable d.f. F in G for each of the decision variables of retailer such that the value of the retailer's total profit should be minimum. The following Lemma is used to estimate the expected shortage.

6.2.6 Lemma 3

For any $F \in G$ the following inequality always holds

$$E(X-r)^+ \leq \frac{1}{2}\sigma\sqrt{L}[\sqrt{1+k^2}-k].$$

Moreover the upper bound of the above inequality is tight (Gallego and Moon (1993)).

Using the above inequality, retailer's total cost equation can be written as

$$\begin{aligned} TEC_b^f(Q,k,p,L) &= \frac{AD(p)}{Q} + r_bC_b\left(\frac{Q}{2}+r-D(p)L\right) + C_bD(p) \\ &+ \frac{\pi D(p)}{Q}\sigma\sqrt{L}(\sqrt{1+k^2}-k)/2 + \frac{D(p)C(L)}{Q} \end{aligned} \tag{73}$$

Using the inequality of Lemma 3, the above cost equation attends the worst possible distribution of lead time demand i.e., maximum cost and accordingly it makes the total profit equation for retailer attains its lowest profit. Thus, the total profit equation for retailer

$$\begin{aligned} TEP_b^f(Q,k,p,L) &= (p-C_b)D(p) - \Big[\frac{AD(p)}{Q} + r_bC_b\left(\frac{Q}{2}+r-D(p)L\right) + C_bD(p) \\ &+ \frac{\pi D(p)}{Q}\sigma\sqrt{L}(\sqrt{1+k^2}-k)/2 + \frac{D(p)C(L)}{Q}\Big] \end{aligned} \tag{74}$$

has its minimum value for each (Q,k,p,L). Therefore, the problem reduces to obtain the maximum value of the above equation for each of decision variables (Q,k,p,L).

According to the normal distribution case, using the similar method, optimal values of Q

and k can be written as

$$Q_b^{f*} = \left\{ \frac{2D(p)[A + \pi\sigma\sqrt{L}[\sqrt{1+k^2} - k]/2 + C(L)]}{r_b C_b} \right\}^{1/2} \quad (75)$$

$$\frac{k}{\sqrt{1+k^2}} = 1 - \frac{2Q r_b C_b}{\pi D(p)} \quad (76)$$

Taking partial differential of the retailer's profit equation with respect to the selling-price and equating to zero, the optimal selling-price for distribution free case is

$$p^{f*} = \frac{\pm\sqrt{B_2^2 - 4A_2C_2} - B_2}{2A_2} \text{ [See Appendix C]} \quad (77)$$

6.2.7 Lemma 4

For all real and positive values of parameters, p^* is always real and positive if $p^{f*} = \frac{-\sqrt{B_2^2-4A_2C_2}-B_2}{2A_2}$ is chosen otherwise $p^{f*} > 0$ only if $b > c(C_b + \beta/Q)$ holds.

Proof

The proof similar as Lemma 1.

We assume

$$p_1^{f*} = \frac{\sqrt{B_2^2 - 4A_2C_2} - B_2}{2A_2} \quad (78)$$

$$p_2^{f*} = \frac{-\sqrt{B_2^2 - 4A_2C_2} - B_2}{2A_2} \quad (79)$$

Now, for SSSD policy the purchase cost for the retailer and the manufacturer's profit equation can be calculated as

$$C_b^{f*} = \frac{r_v C_v D(p)\beta}{r_b PS} \quad (80)$$

$$\text{and } TEP_v(Q_b^{f*}, C_b^{f*}, p^{f*}) = C_b^{f*} D(p^{f*}) - \frac{SD(p^{f*})}{Q^{f*}} - \frac{r_v C_v Q^{f*} D(p^{f*})}{2P} - C_v D(p^{f*}) \quad (81)$$

Similarly, for SSMD policy the total profit for manufacturer is

$$\begin{aligned} TEP_v(Q^{f*}, C_b^{f*}, p^{f*}, m) &= C_b^{f*} D(p^{f*}) - \frac{SD(p^{f*})}{mQ^{f*}} \\ &- \frac{r_v C_v}{2}\left[m\left(1 - \frac{D(p^{f*})}{P}\right) - 1 + \frac{2D(p^{f*})}{P}\right] Q^{f*} - C_v D(p^{f*}) \end{aligned} \quad (82)$$

The optimal profit for the manufacturer can be obtained as

$$TEP_v(Q_b^{f*}, C_b^{f*}, p^{f*}, m-1) \leq TEP_v(Q_b^{f*}, C_b^{f*}, p^{f*}, m) \geq TEP_v(Q_b^{f*}, C_b^{f*}, p^{f*}, m+1)$$

6.2.8 Solution algorithm 2

The following algorithm is used to solve the model for distribution free case

Step 1 Set $m = 1$ and for each L_i, $i = 0, 1, ..., n$ perform the following step 1a-2e.

Step 1a Set $k_{i1} = 0$.

Step 1b Compute Q_{bi1}^f from (75) and k_{i2} from (76).

Step 2 If $b < c(C_b + \alpha/Q)$ then,

Step 2a Compute p_{i1}^f from (79).

Step 2b compute C_{bi1}^f from (80).

Step 2c Repeat Step 1-2b until no changes occur in the values of Q_{bi}^f, k_i, p_i^f, and C_{bi}^f.

Step 2d Compute $TEP_b(Q_i^f, k_i, p_i^f, C_{bi}^f)$.

Step 2e Obtain $max_{i=0,1,...,n} TEP_b(Q_i^f, k_i, p_i^f, C_{bi}^f)$, the maximum profit for the retailer and the optimal reorder point is $r^* = D(p^{f*})L^* + k^*\sigma\sqrt{L^*}$.

Step 2f For SSSD policy, obtain $TEP_v(Q_i^{f*}, p_i^{f*}, C_{bi}^{f*})$ from (81).

Step 2g For SSMD policy set $m = m + 1$ and obtain $TEP_v(Q_i^{f*}, p_i^{f*}, m)$.

Step 2h If $TEP_v(Q_i^{f*}, p_i^{f*}, m) \leq TEP_v(Q_i^{f*}, p_i^{f*}, m-1)$ go to Step 2g otherwise go to Step 2i.

Step 2i Set $TEP_v(Q_i^{f*}, p_i^{f*}, m^*) = TEP_v(Q_i^{f*}, p_i^{f*}, m-1)$, the maximum profit for the manufacturer.

Step 3 If $b > c(C_b + \alpha/Q)$, compute p_{i1}^{1f} from (78) and p_{i1}^{2f} from (79).

Table 6.1

Lead time data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i ($/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Step 3a Repeat Step 1-1b and Step 3 until no changes occur in the values of $Q_{bi}^f, k_i, p_{i1}^{1f}$, and p_{i1}^{2f}.

Step 3b For both p_{i1}^{1f}, and p_{i1}^{2f} execute Step 2b-2e.

Step 3c Set $max_{i=0,1,\ldots,n} TEP_b(Q_i^f, k_i, p_i^f, C_{bi}^f)=$
$max\{max_{i=0,1,\ldots,n} TEP_b(Q_i^f, k_i, p_i^{1f}, C_{bi}^f), max_{i=0,1,\ldots,n} TEP_b(Q_i^f, k_i, p_i^{2f}, C_{bi}^f)\}$.

6.3 Numerical results

Example 1

We consider the following example to illustrate the SSSD model with X, having normal distribution. The values of parameters are $a = 1500$ units, $P = 2000$ units, $A = \$750$/order, $S = \$800$/setup, $r_v = 0.8$/unit/unit time, $r_b = 0.12$/unit/unit time, $C_v = \$19$/unit, $\pi = \$40$/unit, $\sigma = 12$ units/week, $b = 5$, $c = 0.1$. The lead time data is given below.

Using the above parameter values the SSMD policy is not satisfied. To validate this policy the setup cost for manufacturer must be high. Thus, we use the following parameter values $a = 1000$ units, $P = 2000$ units, $A = \$4900$/order, $S = \$5000$/setup, $r_v = \$0.7$, $r_b = \$0.35$, $C_v = \$12$/unit, $\pi = \$90$/unit, $\sigma = 4$ units/week, $b = 1$, $c = 0.01$. The lead time data are same.

Table 6.2

Variable purchase cost for SSSD policy

L	$D(p)$	Q	k	p	C_b	TEP_b	TEP_v
8	613.84	458.89	1.355	72.39	39.11	18067.86	10207.69
6	614.38	458.85	1.356	72.36	39.05	18131.43	10181.03
4	612.01	458.82	1.350	72.49	39.33	17980.58	10304.13

Table 6.3

Fixed purchase cost for SSSD policy

L	C_b	$D(p)$	Q	k	p	TEP_b	TEP_v
4	25	729.67	623.51	1.521	66.26	28125.77	1712.97
4	30	690.21	555.00	1.458	68.40	24377.60	5141.67
4	35	649.07	499.63	1.399	70.57	20850.21	8113.57
4	40	606.22	452.96	1.343	72.79	17548.79	10616.51
4	45	561.56	412.31	1.287	75.05	14479.96	12631.09
4	50	514.98	375.89	1.229	75.35	11651.53	14132.90
4	55	466.38	342.44	1.169	79.70	9092.29	15093.12
4	60	415.56	310.962	1.104	82.09	6752.13	15477.82
4	65	362.27	280.60	1.032	84.55	4702.33	15245.53
4	70	306.08	250.46	0.947	87.09	2936.16	14341.13
3	75	245.47	221.84	0.829	89.76	1471.81	12654.38

Table 6.4

Fixed purchase cost for SSMD policy

m	L	C_b	$D(p)$	Q	k	p	TEP_b	TEP_v
1	4	70	478.66	443.38	0.668	183.74	43448.78	21918.74
2	4	75	467.95	423.81	0.631	186.02	40693.83	24940.25
2	4	80	457.04	405.83	0.594	188.32	38009.85	26558.71
2	4	85	445.93	389.18	0.558	190.64	35396.54	28053.96
2	4	90	434.62	373.67	0.552	192.98	32853.84	29423.13
2	4	95	423.09	359.13	0.486	195.34	30318.89	30663.05
2	4	100	411.34	345.43	0.449	197.72	27980.97	31770.29
2	4	110	387.30	318.77	0.379	202.53	23475.30	33578.79
2	4	120	362.12	259.55	0.265	207.46	19182.89	34808.70
2	4	130	335.83	273.93	0.221	212.52	15188.51	35412.80
2	4	140	308.28	253.42	0.131	217.72	11501.79	35354.28
2	4	150	279.24	233.60	0.030	223.09	8135.02	34565.44

Example 2

We consider same parametric values for SSSD and SSMD policy as assumed in Example 1 for distribution free case. Solutions are summarized in Table 6.5.

The following table indicates the optimal results for retailer and manufacturer for SSSD policy.

• Both manufacturer and retailer can earn profit if the manufacturer determines the purchasing cost for retailer depending on retailer's decisions.

• From Table 6.3 and Table 6.6, it can be found when manufacturer increases the purchasing

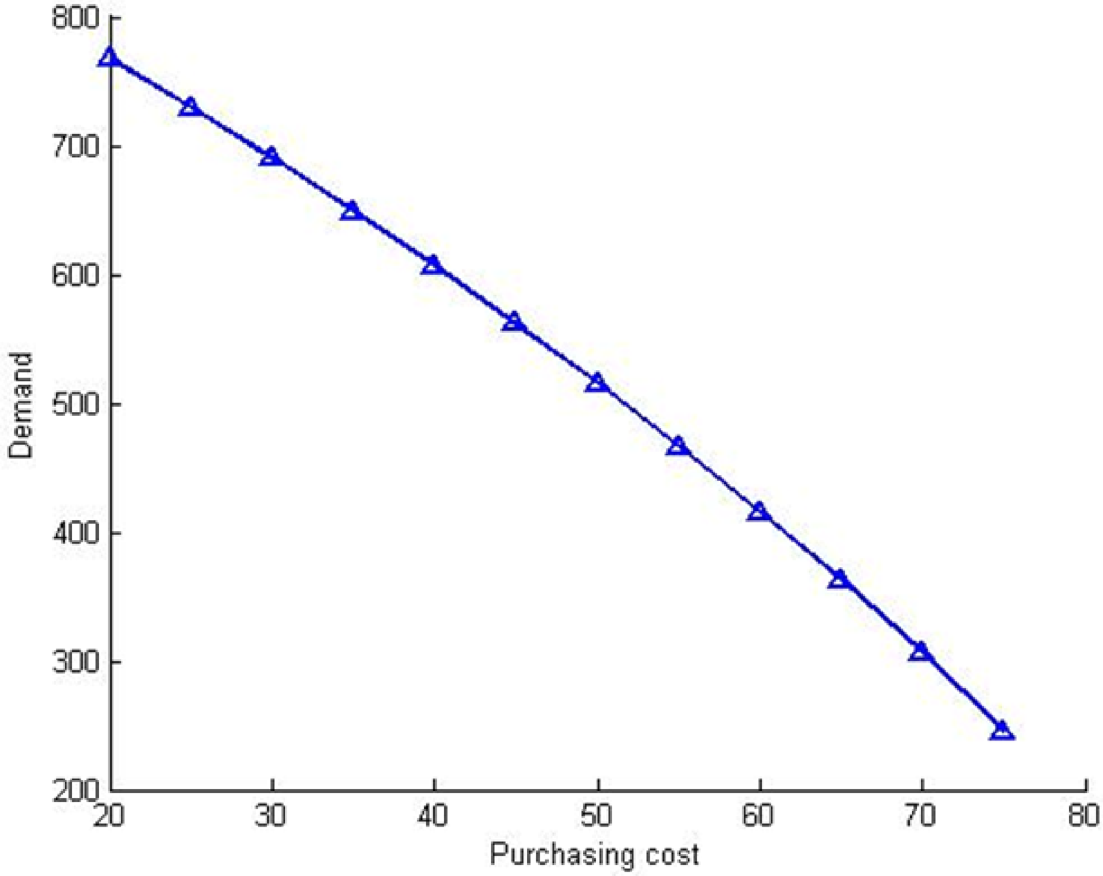

Figure 10 Purchasing cost versus demand

Table 6.5

Variable purchasing cost for SSSD policy

L	$D(p)$	Q	k	p	C_b	TEP_b	TEP_v
8	566.86	458.82	1.268	74.78	44.23	14657.33	12321.37
6	574.13	458.82	1.295	74.42	43.46	15183.46	12038.75
4	579.69	458.83	1.317	74.14	42.86	15606.21	11811.51
3	576.02	458.82	1.302	74.32	43.25	15371.75	11962.68

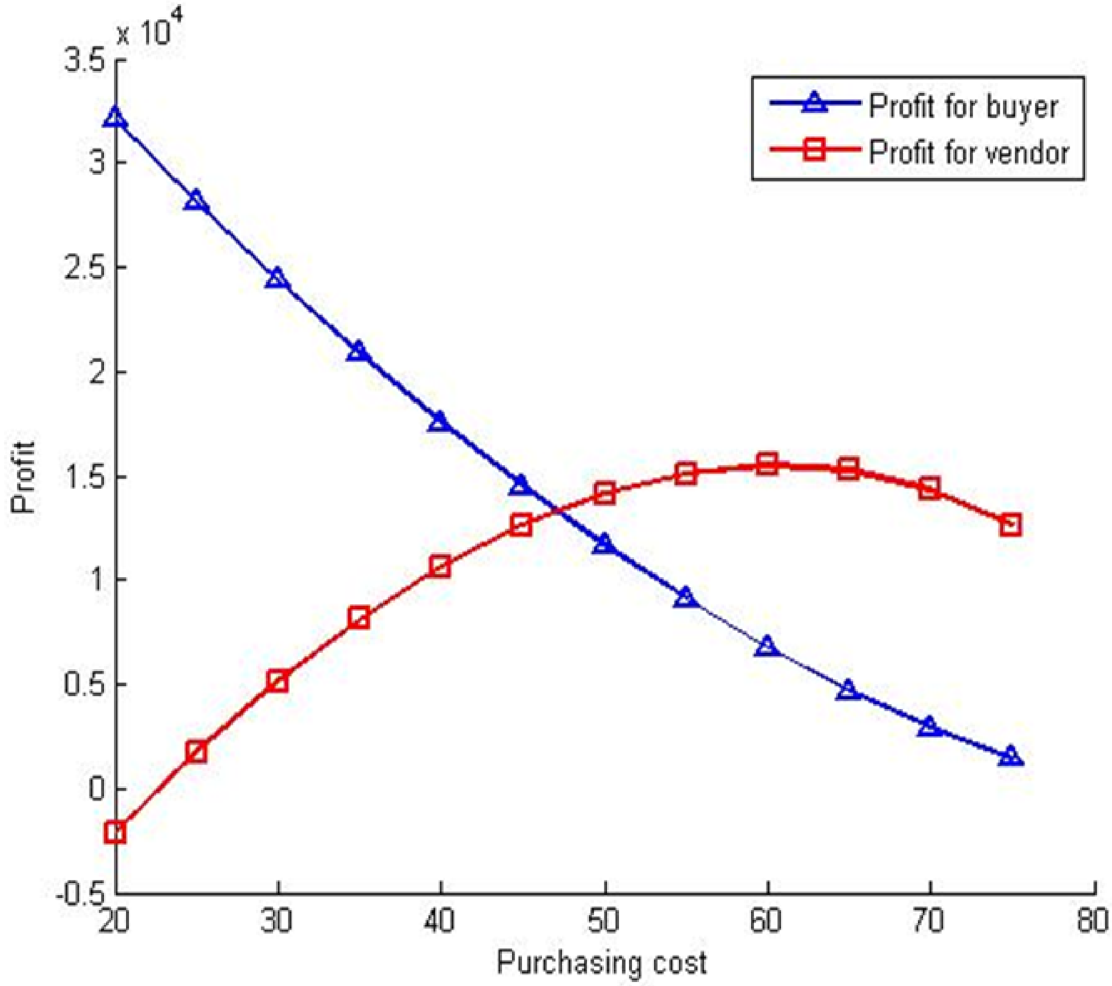

Figure 11 Purchasing cost versus profit by single-setup single-delivery policy

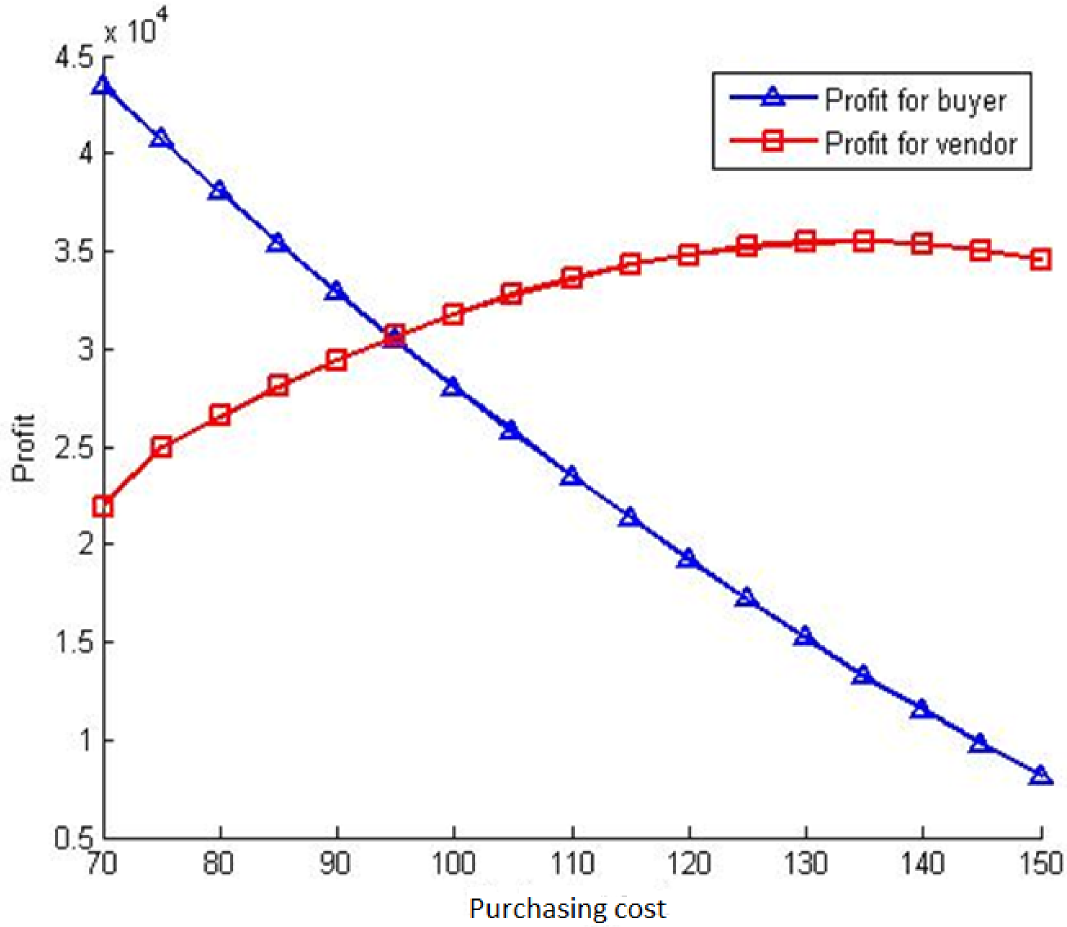

Figure 12 Purchasing cost versus profit by single-setup multiple-delivery policy

Table 6.6

Fixed purchasing cost for SSSD policy

L	C_b	$D(p)$	Q	k	p	TEP_b	TEP_v
4	25	729.07	662.19	1.714	66.29	27995.04	1659.03
4	30	689.46	590.76	1.585	68.74	24236.79	5102.62
3	35	648.06	537.38	1.465	70.63	20703.15	8080.82
3	40	605.03	487.68	1.364	72.85	17398.48	10591.86
3	45	560.16	444.27	1.271	75.12	14328.63	12609.85
3	50	513.36	405.25	1.181	77.43	11501.38	14110.09
3	60	413.33	335.32	1.002	82.20	6611.37	15433.84
3	65	359.64	302.37	0.907	84.67	4570.24	15178.82
3	70	302.92	269.44	0.803	87.23	2815.95	14239.23
3	75	242.15	235.11	0.681	89.91	1366.22	12520.49

Table 6.7

Fixed purchasing cost for SSMD policy

m	L	C_b	$D(p)$	Q	k	p	TEP_b	TEP_v
2	4	70	478.55	445.45	0.567	183.76	43443.88	23199.43
2	4	80	456.93	407.57	0.497	188.34	38009.19	26556.43
2	4	90	434.50	375.10	0.431	193.00	32857.44	29420.11
2	4	100	411.23	346.56	0.367	197.74	27988.68	31766.35
2	4	110	387.34	320.93	0.304	202.58	23405.80	33566.84
2	3	120	361.81	297.64	0.238	207.52	19114.23	34786.78
3	3	130	335.51	275.71	0.171	212.58	15121.41	35440.11
3	3	140	307.94	254.89	0.099	217.78	11436.40	35426.96
3	3	150	278.90	234.76	0.017	223.15	8071.37	34673.71

Table 6.8

Optimal results for SSSD policy

Case	L	C_b	$D(p)$	Q	k	p	TEP_b	TEP_v
Normal	6	39.11	614.38	458.85	1.356	72.36	18131.43	10181.03
Distribution free	4	579.69	458.83	1.317	74.14	42.86	15606.21	11811.51

cost, the selling-price of product becomes higher which results downfall of demand. In this case, retailer's profit decreases and manufacturer's profit increases. But, after a certain amount of the purchasing cost, manufacturer also face decrease in profit. This happens because of a major fall in demand of that particular product due to high selling-price.

• Manufacturer should fix the purchasing cost at such an amount that the difference between the profit of both parties should minimum. Table 6.3 shows when manufacturer fixes the purchase cost at \$45, the difference between the profits at both ends is minimum.

• From Table 6.4, when SSMD policy is followed by manufacturer, the fixed purchasing cost is \$95 for which the difference between profits is the lowest. The crucial thing is that, by SSSD policy, for same profit for retailer (\$30381.89), manufacturer can increase its profit upto \$30663.05 instead of \$28907.09.

• Retailer's total profit for distribution free case is less than that of normal distribution case. This is obvious because the most unfavorable distribution function is used for the distribution free case.

6.3.1 Sensitivity analysis

Key parameters A, S, r_b, and r_v are varied over -20% to +20% and changes on total profits of buyer and vendor are analyzed individually. The sensitivity of total profit with changes of key parameters are given in Table 6.9.

Table 6.9

Sensitivity analysis

Parameters	Changes (in %)	TEP_b	TEP_v	Parameters	Changes (in %)	TEP_b	TEP_v
	-20%	$+21.20$	-27.52		-20%	-16.90	$+19.63$
	-10%	$+9.98$	-12.50		-10%	-8.84	$+11.10$
A	$+10\%$	-9.01	$+10.44$	r_v	$+10\%$	$+9.83$	-13.19
	$+20\%$	-17.08	$+18.97$		$+20\%$	$+20.90$	-29.01
	-20%	-21.12	$+22.23$		-20%	$+15.37$	-22.89
	-10%	-9.76	$+12.23$		-10%	$+8.00$	-11.66
S	$+10\%$	$+8.89$	-11.88	r_b	$+10\%$	-8.81	$+12.00$
	$+20\%$	$+17.07$	-23.42		$+20\%$	-18.69	$+23.61$

6.4 Managerial insights

This chapter deals with decentralized supply chain model with single-manufacturer and single-retailer under variable price dependent demand. The managerial insights are given as follows.

• In a decentralized decision making system, purchasing cost should be determined by manufacturer depending on retailers decision. This technique provides a profitable business for each party.

• Retailer should always estimate the selling-price of product according to economic condition of the locality at which the business is running. A hike in selling-price results decrease in customer demand and manager could experience significantly low profit.

6.5 Concluding remarks

This chapter developed a manufacturer-retailer decentralized supply chain model. Manufacturer's profit was dependent on retailer's decisions. When manufacturer determined the purchasing cost depending on the retailer's decisions, it was found that both manufacturer and retailer were gainer. If the manufacturer determined retailer's purchasing cost independently, without taking retailer's decisions into consideration, then either manufacturer or retailer or both of them would face low profit. In that case, manufacturer fixed purchasing cost such that the difference between the total profit of manufacturer and retailer would be minimum. Otherwise, uncontrolled increase of purchasing cost might result increase of selling-price of retailer which decreased the annual demand and hence the revenue also. Thus, the retailer might face a significant loss. Furthermore, for fixed purchase cost, the manufacturer can also follow SSMD policy to increase its profit.

6.6 Appendices of Chapter 6

Appendix A

$$\begin{aligned}
\frac{\partial TEP_b}{\partial p} &= (a - 2bp - 3cp^2) + C_b(b + 2cp) + (b + 2cp)(A + \pi\sigma\sqrt{L} + C(L))/Q \\
\text{Now, } \frac{\partial TEP_b}{\partial p} &= 0 \text{ gives}
\end{aligned}$$

$$\begin{aligned}
& - \quad 3cp^2 + 2cC_b p - 2bp + 2cp\frac{A + \pi\sigma\sqrt{L}\psi(k) + C(L)}{Q} \\
& + \quad bC_b + \frac{b}{Q}(A + \pi\sigma\sqrt{L}\psi(k) + C(L)) + a = 0 \\
\text{Let} & \qquad \alpha = A + \pi\sigma\sqrt{L}\psi(k) + C(L) \text{ then,} \\
& - \quad 3cp^2 + (2cC_b - 2b + 2c\alpha/Q)p + bC_b + b\alpha/Q + a = 0 \\
\text{Hence,} & \qquad A_1 p^2 + B_1 p + C_1 = 0
\end{aligned}$$

$$\begin{aligned}
\text{where } A_1 &= -3c \\
B_1 &= 2\left\{cC_b - b + \frac{\alpha c}{Q}\right\} \\
C_1 &= C_b b + \frac{b\alpha}{Q} + a
\end{aligned}$$

Appendix B

For a fixed value of L, the Hessian matrix for the function $TEP_b(Q,k,p)$ is given by

$$H = \begin{bmatrix} \frac{\partial^2 TEP_b}{\partial Q^2} & \frac{\partial^2 TEP_b}{\partial Q \partial k} & \frac{\partial^2 TEP_b}{\partial Q \partial p} \\ \frac{\partial^2 TEP_b}{\partial k \partial Q} & \frac{\partial^2 TEP_b}{\partial k^2} & \frac{\partial^2 TEP_b}{\partial k \partial p} \\ \frac{\partial^2 TEP_b}{\partial p \partial Q} & \frac{\partial^2 TEP_b}{\partial p \partial k} & \frac{\partial^2 TEP_b}{\partial p^2} \end{bmatrix}$$

The first order principal minor of H is

$$|H_{11}|_{(Q^*,k^*,p^*)} = -\frac{2AD(p*)}{Q^{*2}} - \frac{\pi D(p^*)}{Q^{*3}}\sigma\sqrt{L}\psi(k^*) - \frac{D(p^*)C(L)}{Q^{*3}} < 0$$

The second order principal minor of H is

$$\begin{aligned} |H_{22}|_{(Q^*,k^*,p^*)} &= \left[\frac{2AD(p^*)}{Q^{*3}} + \frac{\pi D(p^*)}{Q^{*3}}\sigma\sqrt{L}\psi(k^*) + \frac{D(p^*)C(L)}{Q^{*3}}\right] \\ & \left[\frac{\pi D(p^*)}{Q^{*2}}\sigma\sqrt{L}(1-\Phi(k^*))\right] - \frac{\pi^2 D^2(p^*)}{Q^4}\sigma^2 L(\Phi(k^*)-1)^2 \end{aligned}$$

We need to show the above equation positive. Thus, it is enough to show that

$$[\pi\sigma\sqrt{L}\psi(k^*) + 2A + C(L)]/Q^* - \pi\sigma\sqrt{L}(1-\Phi(k^*)) > 0$$

The above inequality holds only if

$$[\pi\sigma\sqrt{L}\psi(k^*) + 2A + C(L)] > \pi\sigma\sqrt{L}(1-\Phi(k^*))Q^* > 0$$

which is the condition for the lemma to be held true.

The second order principal minor of H is

$$\begin{aligned} |H_{33}|_{(Q^*,k^*,p^*)} &= [-(2b+6cp) + 2c(\alpha + C_b)] \times |H_{22}| \\ &- \frac{\pi\sigma\sqrt{L}}{Q^*}(\Phi(k^*)-1)^2\frac{D(p^*)}{Q^{*4}}\alpha\pi\sigma\sqrt{L}[(b+2cp^*) - (b+2cp^*)] \\ &- \frac{(b+2cp^*)^2}{Q^{*5}}\alpha[\pi\sigma\sqrt{L}(\Phi(k^*)-1)^2 - \alpha\phi(k^*)]\pi\sigma\sqrt{L}D(p^*) \\ &= [2c(\alpha + C_b) - (2b+6cp)] \times |H_{22}| \\ &- \frac{(b+2cp^*)^2}{Q^{*5}}\alpha[\pi\sigma\sqrt{L}(\Phi(k^*)-1)^2 - \alpha\phi(k^*)]\pi\sigma\sqrt{L}D(p^*) > 0 \end{aligned}$$

Principal minors of the Hessian matrix are of alternative sign which indicates that the Hessian matrix is negative definite. Hence, the function $TEP_b(Q^*, k^*, p^*)$ attains its global maximum at the point (Q^*, k^*, p^*).

Appendix C

$$\begin{aligned}
\text{where } A_2 &= -3c \\
B_2 &= 2\left\{cC_b - b + \frac{\beta c}{Q}\right\} \\
C_2 &= C_b b + \frac{b\beta}{Q} + a \\
\beta &= A + \pi\sigma\sqrt{L}\sqrt{1+k^2} - k]/2 + C(L)
\end{aligned}$$

Chapter 7
Distribution free newsvendor model with consignment policy and retailer's royalty reduction

7 Distribution free newsvendor model with consignment policy and retailer's royalty reduction

This chapter deals with a single-period newsvendor problem with consignment policy. The consignment policy is an agreement between any two parties say the consignor and the consignee. Both parties carry some parts of holding cost instead of one. An improved policy for paying the fixed fee to the consignee is introduced. This study considers no specific probability distribution for customer's demand except an educated mean and standard deviation. The solution of this model is obtained by using distribution free approach. A comparison between the traditional supply chain policy and the consignment policy is established. The price sensitivity on mean demand is analyzed. Some numerical examples and graphical representations are given for both traditional and consignment policy. The organization has to decide how much to stock the product in inventory before observing the customer's demand. In case of demand uncertainty, stock owner sometimes faces overstock or understock situation. As products are perishable, overage products begin to deteriorate. At this point of view, there is a significant economic importance of this kind of problem.

7.1 Literature review

Arrow (1951) developed the formulation of newsvendor problem. Since then, this type of problem has been gaining the attention of researchers throughout the world. Today, in supply chain and inventory management problems, newsvendor problem plays an important role for short-life cycle products like agricultural, dairy products (Cárdenas-Barrón 2001, 2009; Pal *et al.*, 2015).

Consignment policy is one of the most efficient policies in supply chain management. In general, two parties are considered namely consignor and consignee. The upstream party (e.g., manufacturer) is referred to the consignor and the consignee denotes the downstream party (e.g., retailer). Consignment stock is an inventory which is retained by the downstream party but upstream party holds the ownership of this inventory. No fund transfer is to be taken place until and unless an item is sold from the retailer's stock. The retailer receives per unit commission per each sold item, which is deducted from the revenue and the rest part is transferred to the manufacturer. The manufacturer also agrees to pay a fixed fee to the retailer. As the retailer's space is used to stock the inventory, the holding cost is divided between the both parties. According to Chen and Liu (2008), the inventory carrying cost for the retailer is reduced to 8% to 18% in case of consignment contract from 20% to 36% in case of traditional system.

In make to order policy, to satisfy customer's demand, manufacturer must produce goods in case of insufficient inventory and deliver these to the retailer (Mahajan *et al.*, 2002). According to Valentini and Zavanella (2003), inventory carrying or holding cost mainly consists of two components as financial component and operational component. In traditional policy, total carrying cost has to be incurred by the retailer. Thus, manufacturer does not carry any inventory cost after delivering the product to the retailer. On the other hand, in case of consignment policy, the financial component is incurred by the manufacturer and the operational component is carried by the vendor (Chen and Liu, 2008). Zhang *et al.*, (2010) introduced the channel coordination in a consignment contract. Ru and Wang (2010) developed a single-period supplier-retailer supply chain model with consignment contract regarding the issue which party should have the right to control the inventory. Adida and Ratisoontorn (2011) investigated how competition among retailers affects the supply chain decisions and profits under different consignment contracts.

Corbett (2001) developed a stochastic supply chain model with with consignment stock, cycle and safety stock along with asymmetric information. Gerchak and Khmelnitsky (2003) examined a consignment policy where there is no scope to verify the retailer's sales reports. Braglia and Zavanella (2003) worked on an industrial strategy for supply chain and inventory management with consignment stock. Yi and Sarkar (2013) developed an integrated inventory model under consignment stock policy with the buyer's space limitation and controllable lead time. Sarker (2014) worked on a critical review to study the consignment stocking policy on supply chain management. Hu *et al.* (2014) studied the impact of consumer return policies with inventory control under consignment contracts. Wang *et al.* (2012) modeled a consignment inventory system with deteriorating items, when buyer has warehouse capacity constraint.

The idea of a newsvendor problem comes into the light following the problem how many newspaper to stock into the inventory by the newsvendor (Gallego, 1995). The newspaper becomes obsolete if it is not sold on the day of the publication of the paper. Thus, it is very tough to make the inventory decisions for the vendor under uncertain demand. The same thing happens for any perishable item and this types of model can also be applied to them. In general, some specific probability distribution (e.g., uniform or normal) is considered for demand uncertainty. To know this distribution, the manager should use a lot of fund. There should be a convenient method to reduce this cost if the managerial decisions can be made without considering any specific probability distribution.

7.2 Model formulation

Entire model description with assumptions, solution methodology with algorithm are elaborated in this section.

7.2.1 Assumptions

Following assumptions and notation are used to develop the model.

1. A single period newsvendor model is considered.

2. No specific probability distribution is considered for the customer's demand. But the mean and standard deviation of the demand are known.

3. In traditional policy, total inventory carrying cost is carried by the retailer, while in consignment policy, the financial and the operational part of the inventory holding cost are divided into the manufacturer and the retailer, respectively.

4. The manufacturer pays a commission to the retailer per unit item is sold as well as a fixed fee.

5. During stockout, for each item the manufacturer or retailer has to face a goodwill loss.

7.2.2 Traditional policy

In traditional policy, the total inventory carrying/holding cost is incurred by the retailer. The retailer purchases the total lot from the manufacturer at a wholesale price, retains the ownership of it, and sells the product to the customer at a selling-price. The manufacturer takes no responsibility of goods and incurs no cost for holding after delivering items to the retailer's end. The retailer's total profit for traditional system is

$$\pi_r^{TS} = \begin{Bmatrix} p\mu - wQ - h_r^{TS}(Q-D)^+; \; D \leq Q \\ (p-w)Q - s_r(D-Q)^+; \; Q < D \end{Bmatrix}$$

The retailer's expected profit can be written as

$$E(\pi_r^{TS}) = \begin{Bmatrix} p\mu - wQ - h_r^{TS}E(Q-D)^+; \; D \leq Q \\ (p-w)Q - s_rE(D-Q)^+; \; Q < D \end{Bmatrix} \tag{83}$$

Distribution free approach

As no assumption for the probability distribution of the random demand D is considered, a class Ω of cumulative distribution is assumed. We suppose any cumulative distribution function F of D with mean μ and standard deviation σ such that $F \in \Omega$.

7.2.3 Lemma 1

$$(i)\; E(Q-D)^+ \leq \frac{1}{2}[\sqrt{\sigma^2 + (Q-\mu)^2} - (\mu - Q)] \tag{84}$$

$$(ii)\; E(D-Q)^+ \leq \frac{1}{2}[\sqrt{\sigma^2 + (Q-\mu)^2} - (Q - \mu)] \tag{85}$$

Moreover the upper bound of the above inequality is tight (Gallego and Moon, 1993).

Proof

See appendix A.

Using (84) and (85), (83) can be written as

$$
\begin{aligned}
E(\pi_r^{TS}) &= p(\mu + Q) - wQ - h_r^{TS}\left\{\frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (\mu - Q)}{2}\right\} \\
&\quad - s_r\left\{\frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q-\mu)}{2}\right\} \\
&= p(\mu + Q) - wQ - \frac{h_r^{TS} + s_r}{2}[\sigma^2 + (Q-\mu)^2]^{1/2} - \frac{h_r^{TS} - s_r}{2}(Q-\mu)
\end{aligned} \tag{86}
$$

The revenue of the manufacturer is wQ and the cost incurred by the manufacturer is the manufacturing cost. The expected profit of the manufacturer is

$$E(\pi_m^{TS}) = wQ - cQ = (w-c)Q \tag{87}$$

In order to maximize the expected total profit of the retailer, we take the derivative of (86) with respect to Q.

$$\frac{\partial E(\pi_r^{TS})}{\partial Q} = p - w - \frac{h_r^{TS} + s_r}{2}[\sigma^2 + (Q-\mu)^2]^{-1/2}(Q-\mu)$$

Equating the above equation to zero, we obtain

$$Q_r^* = \mu + \frac{\sigma\Gamma}{\sqrt{1-\Gamma^2}} \text{ where } \Gamma = \frac{2(p-w) - (h_r^{TS} - s_r)}{h_r^{TS} + s_r} \text{ [See Appendix B]} \tag{88}$$

Using (88), (87) can be written as

$$E(\pi_m^{TS}) = (w-c)Q_r^* = (w-c)\left[\mu + \frac{\sigma\Gamma}{\sqrt{1-\Gamma^2}}\right] \tag{89}$$

7.2.4 Lemma 2

For real Q and $\mu > 0$, $E(\pi_r^{TS})$ is bounded above and the upper bound is

$$\frac{p[3\mu - Q - \sqrt{\sigma^2 + (Q-\mu)^2}]}{2}$$

Proof

From (88), we can see that for real Q, $\Gamma < 1$ must holds which implies $\Gamma^2 << 1$ i.e.,

$$
\begin{aligned}
& \frac{2(p-w) - (h_r^{TS} - s_r)}{h_r^{TS} + s_r} < 1 \\
\text{or,} \quad & h_r^{TS} > (p-w)
\end{aligned} \tag{90}
$$

Now using (90), (86) can be written as

$$\begin{aligned}
E(\pi_r^{TS}) &< p(\mu+Q) - wQ - (p-w)\left[\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right] \\
&- s_r\left[\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)}{2}\right] \\
&= p\left[\mu+Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right] \\
&- w\left[Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right] \\
&- s_r\left[\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)}{2}\right] \\
&\leq p\left[\mu+Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right] \text{ holds if and only if} \\
& Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2} > 0 \text{ and } \frac{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)}{2} > 0 \quad (91)
\end{aligned}$$

From the above conditions (91), we have

$$Q+\mu > \sqrt{\sigma^2+(Q-\mu)^2} \quad (92)$$

$$-Q+\mu > -\sqrt{\sigma^2+(Q-\mu)^2} \quad (93)$$

Solving the above equations we get $\mu > 0$. Thus, for $\mu > 0$,

$$E(\pi_r^{TS}) < p\left[\mu+Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right] = \frac{p[3\mu-Q-\sqrt{\sigma^2+(Q-\mu)^2}]}{2}$$

7.2.5 Consignment policy

The revenue for the retailer comes from the per unit commission and the fixed fee paid by the manufacturer. In this policy, the retailer does not carry the total inventory holding cost as the ownership of products is retained by the manufacturer. The operational part of the holding cost which includes storage space, material handling etc., is carried by the retailer. But, the financial component (opportunity cost while investing the capital, taxes etc.) is incurred by the manufacturer. Thus, the total cost faced by the retailer in this policy consists of the operational part of the holding cost and the goodwill loss during stockout.

The expected total profit of the retailer for consignment policy is

$$E(\pi_r^{CP}) = \left\{ \begin{array}{l} \alpha\mu - h_r^{CP} E(Q-D)^+ + A;\ D \leq Q \\ \alpha Q - s_r E(D-Q)^+ + A;\ Q < D \end{array} \right\} \tag{94}$$

Using (84) and (85), (94) can be written as

$$\begin{aligned} E(\pi_r^{CP}) &= \alpha(\mu+Q) - h_r^{CP} \left\{ \frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (\mu - Q)}{2} \right\} \\ &- s_r \left\{ \frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q-\mu)}{2} \right\} + A \\ &= \alpha(\mu+Q) - \frac{h_r^{CP} + s_r}{2} \sqrt{\sigma^2 + (Q-\mu)^2} \\ &+ \frac{h_r^{CP} - s_r}{2} (\mu - Q) + A \end{aligned} \tag{95}$$

In maximize the profit of the retailer, we take the derivative of (95) with respect to Q which gives

$$\frac{\partial E(\pi_r^{CP})}{\partial Q} = \alpha - \frac{(h_r^{CP} + s_r)(Q-\mu)}{2\sqrt{\sigma^2 + (Q-\mu)^2}} - \frac{h_r^{CP} - s_r}{2}$$

Now, equating the above equation to zero, we obtain

$$Q_r^{CP*} = \mu + \frac{\sigma \Gamma_{CP}}{\sqrt{1 - \Gamma_{CP}^2}} \text{ where } \Gamma = \frac{2\alpha - (h_r^{CP} - s_r)}{h_r^{CP} + s_r} \text{ [See Appendix B]} \tag{96}$$

Now, at the manufacturer's end, though the entire lot is delivered to the retailer's warehouse the manufacturer keeps the ownership of the merchandize. The financial part of the holding cost is carried by the manufacturer. The selling-price is fixed by the manufacturer and the retailer transfer the balance amount after deducting the per unit commission. This balance amount is the source of revenue for the manufacturer. No fund transfer is to be taken place if the item is unsold. Besides this the manufacturer also agreed to pay a fixed fee to the retailer. The expected total profit of the manufacturer is

$$\pi_m^{CP} = \left\{ \begin{array}{l} (p-\alpha)\mu - cQ - h_m^{CP} E(Q-D)^+ - A;\ D \leq Q \\ (p-\alpha-c)Q - s_m E(D-Q)^+ - A;\ D > Q \end{array} \right\} \tag{97}$$

Using (84) and (85), the above equation can be written as

$$\begin{aligned} E(\pi_m^{CP}) &= p(\mu+Q) - \alpha\mu - \alpha Q - cQ - h_m^{CP}\left\{\frac{[\sigma^2+(Q-\mu)^2]^{1/2}-(\mu-Q)}{2}\right\} \\ &- s_m\left\{\frac{[\sigma^2+(Q-\mu)^2]^{1/2}-(Q-\mu)}{2}\right\} - A \\ &= p(\mu+Q) - cQ - \alpha Q - \alpha\mu - \frac{h_m^{CP}+s_m}{2}\sqrt{\sigma^2+(Q-\mu)^2} \\ &+ \frac{h_m^{CP}-s_m}{2}(\mu-Q) - A \end{aligned} \tag{98}$$

The expected joint total profit for manufacturer and retailer under consignment policy is

$$E(\pi_J^{CP}) = E(\pi_r^{CP}+\pi_m^{CP}) = E(\pi_r^{CP}) + E(\pi_m^{CP}) \text{ i.e.,}$$

$$\begin{aligned} E(\pi_J^{CP}) &= \alpha(\mu+Q) - \frac{h_r^{CP}+s_r}{2}\sqrt{\sigma^2+(Q-\mu)^2} + \frac{h_r^{CP}-s_r}{2}(\mu-Q) + A \\ &+ p(\mu+Q) - cQ - \alpha Q - \alpha\mu - \frac{h_m^{CP}+s_m}{2}\sqrt{\sigma^2+(Q-\mu)^2} \\ &+ \frac{h_m^{CP}-s_m}{2}(\mu-Q) - A \\ &= (\alpha+p)(\mu+Q) - cQ - \alpha Q - \alpha\mu \\ &- \frac{(h_r^{CP}+h_m^{CP})+(s_r+s_m)}{2}\sqrt{\sigma^2+(Q-\mu)^2} \\ &+ \frac{(h_r^{CP}+h_m^{CP})-(s_r+s_m)}{2}(\mu-Q) \end{aligned} \tag{99}$$

In order to maximize the expected joint total profit, we take the derivative of (99) with respect to Q

$$\begin{aligned} \frac{\partial E(\pi_J^{CP})}{\partial Q} &= p - c - \frac{(h_r^{CP}+h_m^{CP})+(s_r+s_m)}{2\sqrt{\sigma^2+(Q-\mu)^2}} \\ &- \frac{(h_r^{CP}+h_m^{CP})-(s_r+s_m)}{2} \end{aligned} \tag{100}$$

Equating the above equation to zero, we obtain

$$Q_J^{CP*} = \mu + \frac{\sigma\lambda}{\sqrt{1-\lambda^2}} \text{ where } \lambda = \frac{2(p-c)-((h_r^{CP}+h_m^{CP})-(s_r+s_m))}{(h_r^{CP}+h_m^{CP})+(s_r+s_m)} \tag{101}$$

[See Appendix B]

Evaluation of per unit commission

In order to obtain the per unit commission α given by the manufacturer to the retailer, we equate $Q_r^{CP*} = Q_j^{CP*}$ and obtain the value of α as

$$\alpha = \frac{h_r - s_r}{2} + \frac{h_r + s_r}{2(H_{CP} + S_{CP})}(2(p-c) - (H_{CP} - S_{CP})) \tag{102}$$
$$\text{where } H_{CP} = (h_r^{CP} + h_m^{CP}) \text{ and } S_{CP} = (s_r + s_m)$$

Evaluation of the fixed fee paid by the manufacturer to the retailer

In consignment policy, retailers expects to earn at least as much as in traditional policy. This is the manufacturer's responsibility to ensure that the retailer's expected profit for CP reaches or exceeds the expected profit for TP. i.e.,

$$\begin{aligned} & \text{Max z} = E(\pi_m^{CP}) \\ \text{subject to} \quad & E(\pi_r^{CP}) \geq E(\pi_r^{TS}) \end{aligned} \tag{103}$$

$$\begin{aligned} A \leq\ & p(\mu + Q_r^{TS}) - wQ_r^{TS} - \frac{h_r^{TS} + s_r}{2}\sqrt{\sigma^2 + (Q_r^{TS} - \mu)^2} \\ & - \frac{h_r^{TS} - s_r}{2}(Q_r^{TS} - \mu) - \alpha(\mu + Q_r^{CP}) + \frac{h_r^{CP} + s_r}{2}\sqrt{\sigma^2 + (Q_r^{CP} - \mu)^2} \\ & + \frac{h_r^{CP} - s_r}{2}(Q_r^{CP} - \mu) \end{aligned} \tag{104}$$

A point should be noted that this fixed fee is not always positive. This fee is calculated comparing the retailer's expected profit under CP and TP. The responsibility for the manufacturer is to confirm the more profit of the retailer in CP than that of TP. If this happens, then replacing the inequality sign of (104) to equality, the fixed fee becomes negative; otherwise it is positive. A positive fixed fee is regarded as an 'admission fee' which is a slotting allowance paid by the manufacturer to the retailer and a negative fixed fee is referred to as a 'royalty' paid by the retailer to the manufacturer (Chen and Liu, 2008).

7.2.6 Proposed way to evaluate the fixed fee in CP

In CP the manufacturer holds the ownership of the product. The retailer just provides the warehouse space to hold the item. In return, the retailer is offered with a per unit commission on selling an item and a fixed fee. On the other hand, a part of holding cost has to carried out by the retailer. An awkward situation for the retailer comes into the way while making the consignment contract with the manufacturer. This is a difficult decision for the retailer to agree with per unit commission and the fixed fee. Specially, this fixed fee as it is not always positive which means the retailer has to give a royalty to the manufacturer. This may create an issue to sign a consignment contract as the per unit commission is the only source of revenue for the retailer.

This proposed method can reduce the royalty, which has to be paid by the retailer to the manufacturer in case of negative fixed fee. We convert the inequality sign of (104) to equality which shows

$$A = E(\pi_r^{TS}) - E(\pi_r^{CP})$$

The value of A will be negative if

$$E(\pi_r^{CP}) > E(\pi_r^{TS}) \tag{105}$$

Now, we obtain the ratio

$$r = \left|\frac{E(\pi_r^{TS})}{E(\pi_r^{CP})}\right| \tag{106}$$

If (105) holds, then $r < 1$ and

$$A_n = rA = \left|\frac{E(\pi_r^{TS})}{E(\pi_r^{CP})}\right| A < A \tag{107}$$

Thus, the royalty which is to be given by the retailer can be reduced in this way. The expected profit for the retailer and the manufacturer for this new fixed cost will be

$$\begin{aligned} E(\pi_{rn}^{CP}) &= \alpha(\mu + Q_r^{CP}) - h_r^{CP}\left\{\frac{[\sigma^2 + (Q_r^{CP} - \mu)^2]^{1/2} - (\mu - Q_r^{CP})}{2}\right\} \\ &- s_r\left\{\frac{[\sigma^2 + (Q_r^{CP} - \mu)^2]^{1/2} - (Q_r^{CP} - \mu)}{2}\right\} + A_n \\ &= \alpha(\mu + Q_r^{CP}) - \frac{h_r^{CP} + s_r}{2}\sqrt{\sigma^2 + (Q_r^{CP} - \mu)^2} \\ &+ \frac{h_r^{CP} - s_r}{2}(\mu - Q_r^{CP}) + A_n \end{aligned} \tag{108}$$

and

$$E(\pi_{mn}^{CP}) = p(\mu + Q_r^{CP}) - \alpha\mu - \alpha Q_r^{CP} - cQ_r^{CP} - h_m^{CP}\left\{\frac{[\sigma^2 + (Q_r^{CP} - \mu)^2]^{1/2} - (\mu - Q_r^{CP})}{2}\right\}$$
$$- s_m\left\{\frac{[\sigma^2 + (Q_r^{CP} - \mu)^2]^{1/2} - (Q_r^{CP} - \mu)}{2}\right\} - A_n$$
$$= p(\mu + Q_r^{CP}) - cQ_r^{CP} - \alpha Q_r^{CP} - \alpha\mu - \frac{h_m^{CP} + s_m}{2}\sqrt{\sigma^2 + (Q_r^{CP} - \mu)^2}$$
$$+ \frac{h_m^{CP} - s_m}{2}(\mu - Q_r^{CP}) - A_n, \text{ respectively.} \tag{109}$$

7.2.7 Solution algorithm

The following algorithm is used to obtain the optimal solution of the model.

For traditional policy

Step 1 Input all parameter values μ, σ, p, w, c, s_r^{TS}, s_r^{CP}, h_r^{TS}, h_r^{CP}, h_m^{CP}, and s_m^{CP}.

Step 2 Evaluate the order quantity for the retailer Q_r^{TS} from (88).

Step 3 Utilize the value of Q_r^{TS} to obtain the retailer's total expected profit $E(\pi_r^{TS})$ from (86) and the manufacturer's total expected profit $E(\pi_m^{TS})$ from (89).

For consignment policy

Step 4 Evaluate the value of per unit commission α for the retailer using (102).

Step 5 Obtain the value of the order quantity in consignment policy Q_r^{CP} for the retailer from (96).

Step 6 Evaluate the fixed cost A from (104).

Step 7 If $A > 0$, then evaluate the expected profit of the retailer $E(\pi_r^{CP})$ and the manufacturer $E(\pi_m^{CP})$ from (95) and (98), respectively.

Step 8 If $A < 0$, then execute the following steps

Table 7.1

Optimal solutions for traditional system

h_r	Q_r^{TS}	$E(\pi_r^{TS})$	$E(\pi_m^{TS})$	Joint profit
$0.63c$	236.07	854.25	1416.40	2270.65
$0.73c$	206.67	500.00	1240.00	1740.40

Step 8a Evaluate A_n from (107).

Step 8b Utilize the value of A_n to evaluate the new expected profit for the retailer and the manufacturer i.e., $E(\pi_{rn}^{CP})$ and $E(\pi_{mn}^{CP})$, respectively from (108) and (109).

7.3 Numerical experiments

Example 1

We use following parameter values for traditional policy. $\mu = 100$, $\sigma = 200$, $p = \$30$, $w = \$25$, $c = \$19$, and $s_r = \$20$. Now using the solution algorithm, the optimal solution for the traditional system model are summarized in Table 7.1.

Example 2

We use same parameter values as used in Example 1. Example 2 is used for consignment policy. The inventory holding cost is divided into both parties. The optimal solutions for consignment policy are summarized in Table 7.2. This solution is obtained for general consignment policy. The optimal solutions for the proposed method to evaluate the royalty are given in Table 7.3.

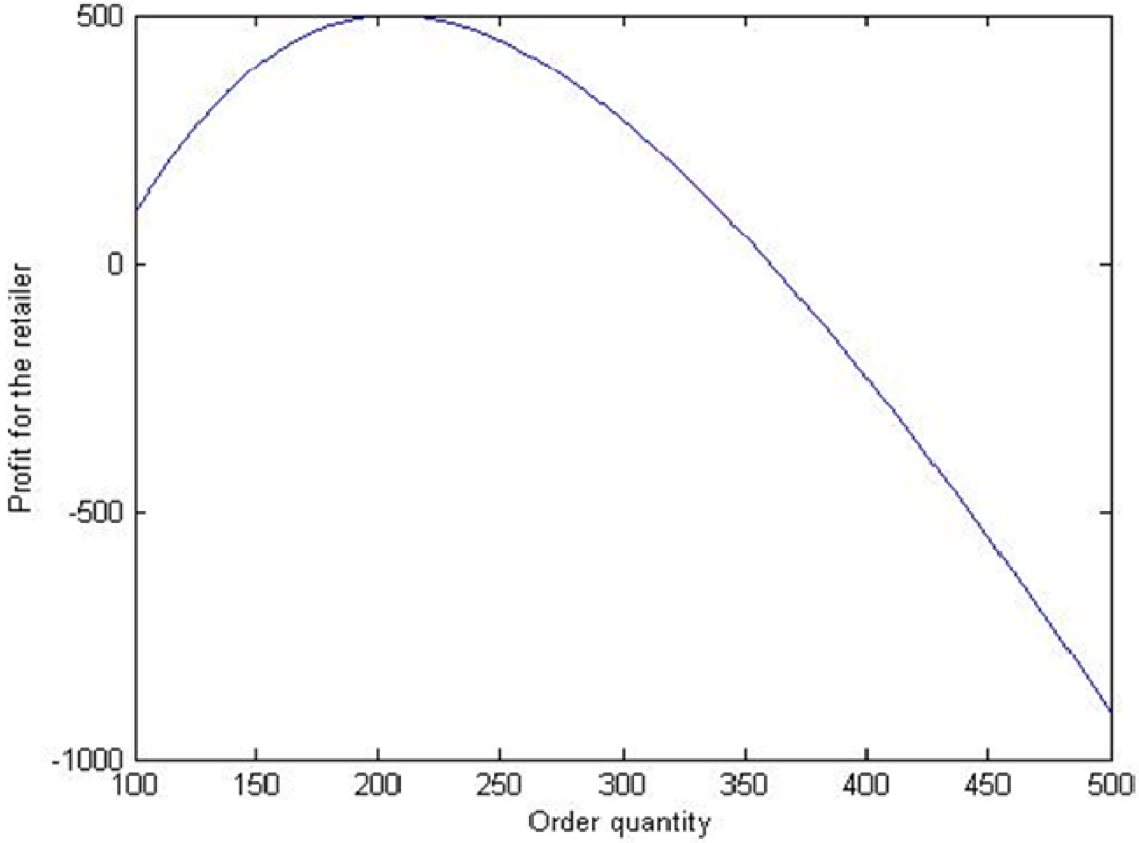

Figure 13 Graphical representation of order quantity versus profit for consignment policy

Table 7.2

Optimal solutions for consignment policy

(h_r, h_m)	(s_r, s_m)	α	Q_r^{CP}	$E(\pi_r^{CP})$	$E(\pi_m^{CP})$	Joint profit	A
(0.21c,0.42c)	(0,20)	3.87	638.82	854.25	2132.19	2986.44	218.44
(0.26c,0.47c)	(0,20)	4.56	390.35	500.00	1671.27	2171.27	-128.13

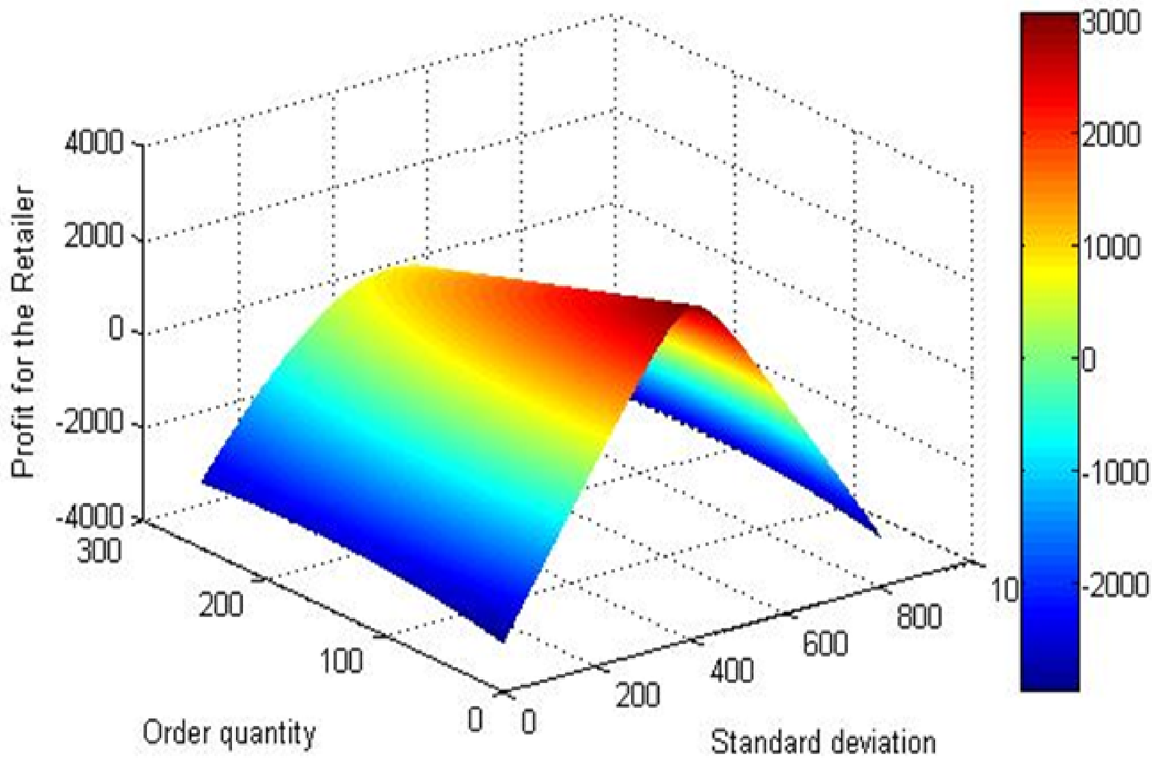

Figure 14 Graphical representation of order quantity, standard deviation versus profit for consignment policy

Table 7.3

Optimal solutions for the proposed method in consignment policy

(h_r, h_m)	(s_r, s_m)	α	Q_r^{CP}	$E(\pi_r^{CP})$	$E(\pi_m^{CP})$	Joint profit	A_n
(0.26c,0.47c)	(0,20)	4.56	390.35	526.14	1645.13	2171.27	-101.99

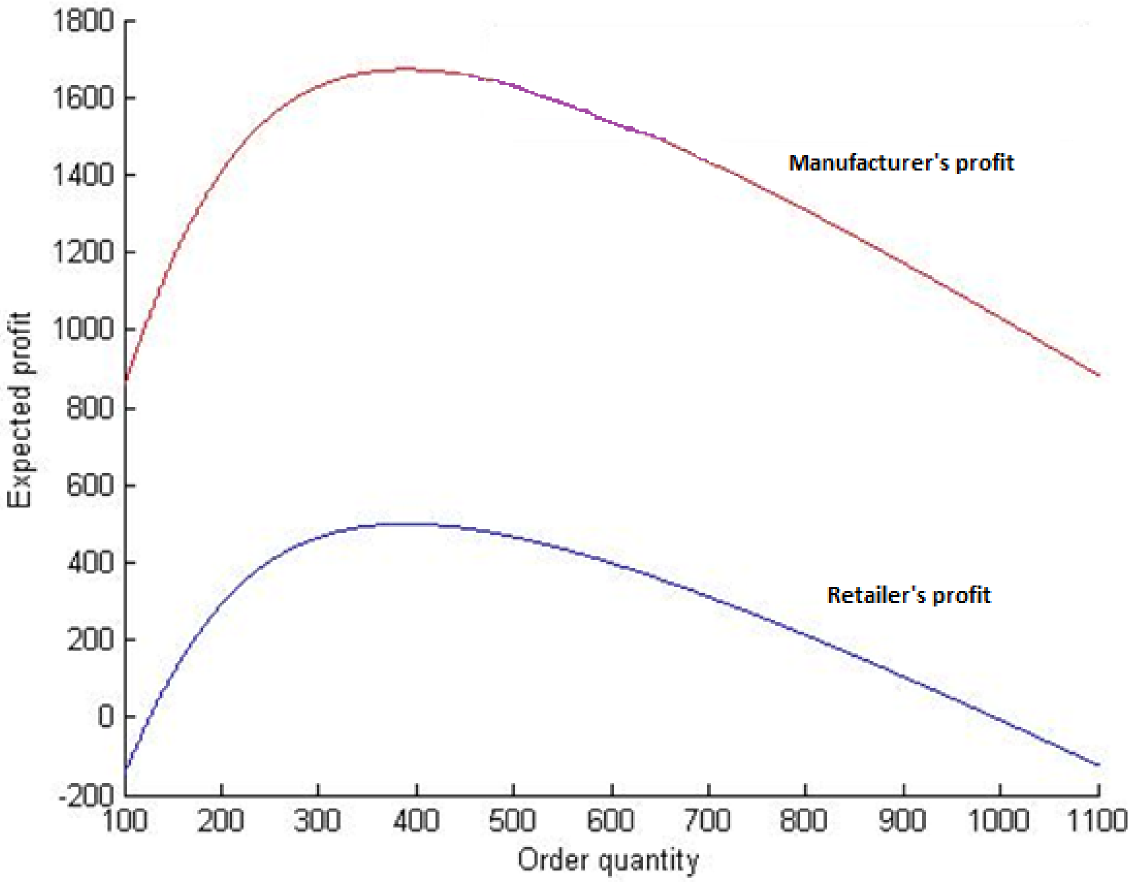

Figure 15 Graphical representation of order quantity versus expected profit for traditional policy

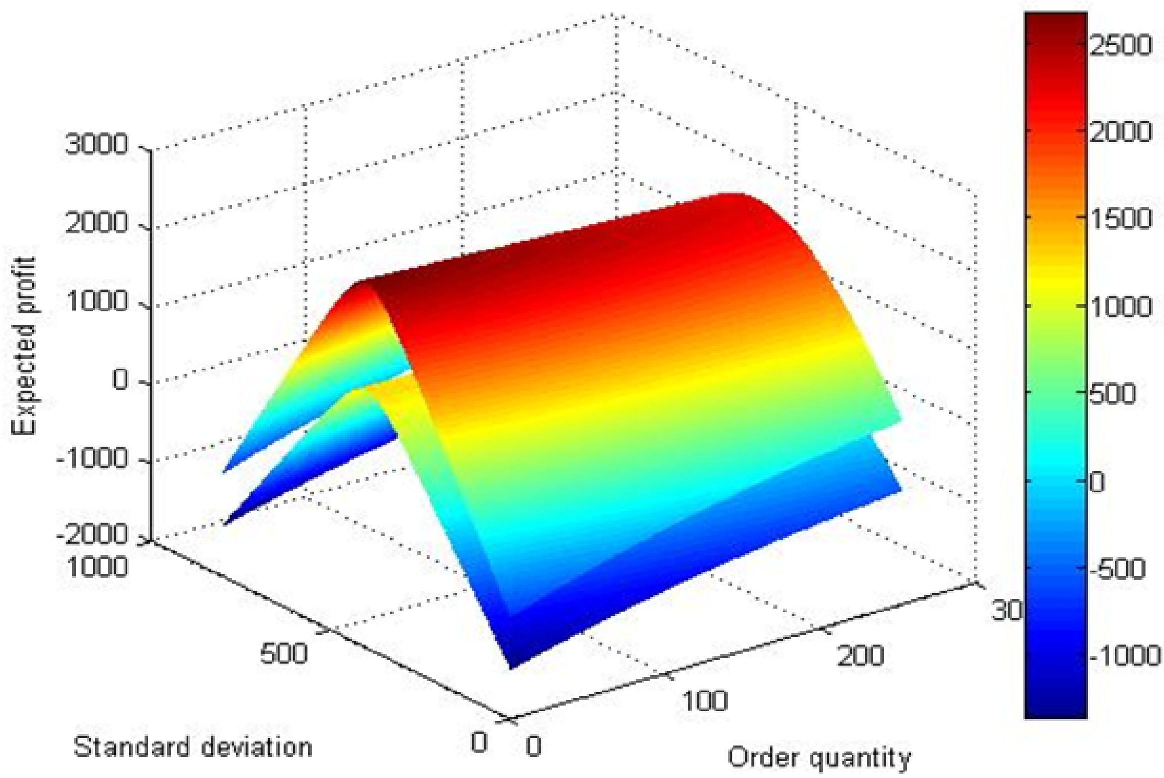

Figure 16 Graphical representation of order quantity, standard deviation versus profit for traditional policy

Table 7.4

Optimal solutions for price sensitivity on demand

β	α	Q_r^{CP}	$E(\pi_r^{CP})$	$E(\pi_m^{CP})$	Joint profit	A	A_n
0.1	4.56	387.35	465.48	1582.79	2048.27	-205.77	-135.29
0.2	4.56	384.35	430.08	1495.18	1925.26	-283.42	-143.34
0.3	4.56	381.35	423.74	1378.53	1802.27	-361.07	-122.32
0.4	4.56	378.35	451.05	1228.22	1679.27	-438.72	-67.66

7.3.1 Price sensitivity on demand

According to the variation of selling-price the demand varies following the relation $D = a - \beta p$, where a is annual demand and β is the fraction of price which influences the demand. In case of distribution free approach only an educated mean and standard deviation is known. There are no other information regarding the probability distribution of the demand. We consider that the mean demand μ is price-sensitive and follows the linear relationship of demand with the selling-price as $\mu = a - \beta p$. Example 3 summarizes the optimal results by replacing μ with $a - \beta p$.

Example 3

We use the same parameter values as used in Example 2 considering $a = 100$ and the holding costs as $(h_r, h_m) \equiv (0.26c, 0.47c)$. The optimal results are summarized in Table 7.4.

Table 7.1 shows the maximum expected profit for the vendor and the buyer with different holding cost assumption. Table 7.2 represents the optimal expected profit for both parties for consignment policy and changes of expected profit with different holding costs. Table 7.3 indicates the optimal results under the proposed method to reduce the royalty for the retailer. It is observed that the royalty can be reduced without interrupting the joint profit for the vendor and the buyer. Table 7.4 gives linear-price sensitivity on mean demand. The changes of optimal profit for the buyer and the vendor with changes in the price fraction β which influences the

Table 7.5

Sensitivity analysis for holding cost components

Parameters	Changes (in %)	Joint profit (in %)	Parameters	Changes (in %)	Joint profit (in %)
	-20%	40.31568		-20%	80.65674
	-15%	28.83955		-15%	57.18234
	-10%	17.79346		-10%	35.95507
	-5%	8.092732		-5%	16.94367
h_r^{CP}	$+5\%$	-6.70292	h_m^{CP}	$+5\%$	-14.9342
	$+10\%$	-12.1953		$+10\%$	-27.864
	$+15\%$	-16.6257		$+15\%$	-38.7594
	$+20\%$	-20.1179		$+20\%$	-47.5653

mean demand.

7.3.2 Sensitivity analysis

Table 7.5 represents the sensitivity analysis for holding costs for the vendor and the buyer in consignment policy. It is found that the manufacturer's holding cost is more sensitive than the retailer's holding cost to the joint optimal profit.

7.4 Managerial insights

This chapter highlights a comparison between traditional and consignment policy under a manufacturer-retailer supply chain system. The managerial insights are given as follows:

• A consignment contract saves fund in favor of retailer as the financial and operational part of holding cost is divided into retailer and manufacturer, respectively.

• Demand is considered as random, but free of any specific distribution with known mean and standard deviation. The distribution free approach is applied to solve the model which can save fund for collecting demand data from market.

• An effective procedure is established to reduce retailer's royalty to strengthen the negotiation between manufacturer and retailer in achieving a consignment contract.

7.5 Concluding remarks

In this study, a comparison between the traditional policy and the consignment policy for a newsvendor problem with the manufacturer and the retailer was shown under distribution free approach. The demand distribution is completely unknown except mean and standard deviation depending on which the entire model was developed. The optimal decisions was obtained for both traditional and consignment policy. It was observed that the joint profit for the consignment policy is greater than that of the traditional policy. To reduce the royalty for the retailer to the manufacturer a new methodology was provided. The royalty for the retailer was reduced without affecting the joint total profit for both parties. The price-sensitivity on demand was examined, which proved that increment of a fraction of price may result reduction of total expected profit.

7.6 Appendices of Chapter 7

Appendix A

$$
\begin{aligned}
(Q-D)^+ &= \frac{|Q-D|+(Q-D)}{2} \\
E|Q-D| &\leq \sqrt{E(Q-D)^2} \text{ (By Cauchy Schwartz inequality)} \\
&= \sqrt{E(Q^2-2QD+D^2)} \\
&= \sqrt{Q^2-2Q\mu+E(D^2)} \text{ (As } E(D)=\mu) \\
&= \sqrt{\sigma^2+(Q-\mu)^2} \text{ (As } \sigma^2=E(D^2)-\mu^2) \\
\text{Now, } E(Q-D)^+ &= \frac{E|Q-D|+E(Q-D)}{2} \\
&\leq \frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}
\end{aligned}
$$

Similarly, we can prove that

$$
E(D-Q)^+ \leq \frac{\sqrt{\sigma^2+(Q-\mu)^2-(Q-\mu)}}{2}
$$

Appendix B

Optimal order quantity for traditional policy

$$
\begin{aligned}
& \frac{\partial E(\pi_r^{TS})}{\partial Q} = 0 \\
\text{i.e.,} \quad & p-w-\frac{1}{2}\sqrt{\sigma^2+(Q-\mu)^2}(Q-\mu)(h_r^{TS}+s_r)-\frac{h_r^{TS}-s_r}{2}=0 \\
\text{i.e.,} \quad & \frac{(Q-\mu)}{\sqrt{\sigma^2+(Q-\mu)^2}} = \frac{2(p-w)-(h_r^{TS}-s_r)}{h_r^{TS}+s_r} \\
\text{or,} \quad & \frac{(Q-\mu)^2}{\sigma^2+(Q-\mu)^2} = \Gamma^2 \\
& \left[\Gamma = \frac{2(p-w)-(h_r^{TS}-s_r)}{h_r^{TS}+s_r}\right] \\
\text{or,} \quad & Q = \mu + \frac{\sigma\Gamma}{\sqrt{1-\Gamma^2}}
\end{aligned}
$$

Optimal order quantity for consignment policy

$$\frac{\partial E(\pi_r^{CP})}{\partial Q} = 0$$

$$\text{i.e.,} \qquad \alpha - \frac{(h_r^{CP} + s_r)(Q-\mu)}{\sqrt{\sigma^2 + (Q-\mu)^2}} - \frac{h_r^{CP} - s_r}{2} = 0$$

$$\text{i.e.,} \qquad \frac{(Q-\mu)^2}{\sigma^2 + (Q-\mu)^2} = \left[\frac{2\alpha - h_r^{CP} + s_r}{h_r^{CP} + s_r}\right]^2 = \Gamma_{CP}^2$$

$$\left[\frac{2\alpha - h_r^{CP} + s_r}{h_r^{CP} + s_r} = \Gamma_{CP}\right]$$

$$\text{i.e.,} \qquad Q_r^{CP} = \mu + \frac{\sigma \Gamma_{CP}}{\sqrt{1 - \Gamma_{CP}^2}}$$

Optimal order quantity for joint policy

$$\frac{\partial E(\pi_J^{CP})}{\partial Q} = 0$$

$$\text{i.e.,} \qquad p - c - \frac{1}{2}\frac{(h_r + h_m + s_r + s_m)(Q-\mu)}{\sqrt{\sigma^2 + (Q-\mu)^2}} - \frac{h_r + h_m - (s_r + s_m)}{2} = 0$$

$$i.e., \qquad \frac{(h_r + h_m + s_r + s_m)(Q-\mu)}{\sqrt{\sigma^2 + (Q-\mu)^2}} = 2(p-c) - (h_r + h_m - s_r - s_m)$$

$$i.e., \qquad \frac{(Q-\mu)^2}{\sigma^2 + (Q-\mu)^2} = \left[\frac{2(p-c) - (h_r + h_m - s_r - s_m)}{(h_r + h_m + s_r + s_m)}\right]^2 = \lambda^2$$

$$\left[\lambda = \frac{2(p-c) - (h_r + h_m - s_r - s_m)}{(h_r + h_m + s_r + s_m)}\right]$$

$$i.e., \; Q = \mu + \frac{\sigma\lambda}{\sqrt{1-\lambda^2}}$$

Chapter 8
A multi-retailer supply chain model with backorder and variable production cost

8 A multi-retailer supply chain model with backorder and variable production cost

This chapter considers an multi-retailer supply chain model, where a single-vendor manufactures goods in a batch production process and supplies to a set of buyers over multiple times. Instead of assuming a fixed production rate, variable production rate is considered by the vendor and also the production cost of the vendor is treated as a function of production rate. The continuous review inventory policy is applied by buyers to inspect the inventory level and a crashing cost is incurred by all buyers to reduce lead time. The lead time demand is normally distributed. The unsatisfied demand at buyer's end are partially backordered. A service-level constraint is incorporated corresponding to each buyer. A model is formulated to minimize the expected joint cost of the vendor-buyers supply chain system.

8.1 Literature review

Similar literatures used in previous chapters have been omitted in this chapter. Literatures related to specific contributions in this chapter are only discussed.

Banerjee and Burton (1994) developed a comparison between coordinated and independent replenishment policies in a single-vendor multi-buyer supply chain model. Banerjee and Banerjee (1994) developed a multi-buyer inventory model using electronic data interchange with order-up-to inventory control policy. Sarmah *et al.* (2008) considered a single supplier multi buyer coordinated supply chain model with trade credit policy. Hoque (2008) created three different single-vendor multi-buyer models by synchronizing the production flow with equal and unequal sized batch transfer for first two models and the last last model, respectively.

The production rate is assumed to be constant in classical supply chain model whereas, in many cases, the machine production rate may change (Khouja and Mehrez, 1994). Conard and McClamrock's (1987) analysis stated that 10% change in processing rate results 50% change in machine tool cost. Moreover, the possibility of failure of production process gradually increases with the increasing production rate. As a result, the product quality may deteriorate at some percentage. Thus, it is reasonable to consider the production rate as a decision variable. Unit production cost also depends on the production rate and treated to be one of the decision variables.

8.2 Model formulation

In this section assumptions, model development, and solution procedure with algorithm are described thoroughly.

8.2.1 Assumptions

Following assumptions are considered to develop the model.

1. This chapter assumes a single-vendor multi-buyer supply chain model for single-type of products.

2. To satisfy the demand of each buyer, vendor supplies a total of Q quantity such that $Q = \sum_{i=1}^{n} q_i$.

3. The vendor manufactures mQ quantity against the order of q_i quantity of buyer i but the shipment should be in quantity Q over m times. The shipment procedure follows the relation $q_i = d_i \frac{Q}{D}$ i.e., $\frac{q_i}{d_i} = \frac{Q}{D}$.

4. The inventory is continuously reviewed by each buyer. According to this policy, an order is placed whenever the level of inventory decreases to a particular inventory level (reorder point).

5. Production rate is a variable quantity, which varies within the range P_{min} ($P_{min} > D = \sum_{i=1}^{n} d_i$) and P_{max}.

6. The unit production cost of the vendor is a function of P.

7. Partial backorder is considered with backorder ratio β_i for ith retailer.

8. For ith retailer, we assume $L_{i,0} \equiv \sum_{j=1}^{n_i} b_{i,j}$. $L_{i,r}$ is the length of lead time having the components $1, 2, ..., r$ crashed to their minimum duration. Thus, $L_{i,r}$ can be expressed as $L_{i,r} = L_{i,0} - \sum_{j=1}^{r}(b_{i,j} - a_{i,j})$, $r = 1, 2, ..., n$; and the lead time crashing cost per cycle $C_i(L_i)$ is expressed as $C_i(L_i) = c_{i,r}(L_{i,r-1} - L_i) + \sum_{j=1}^{r-1} c_{i,j}(b_{i,j} - a_{i,j})$, $L \in [L_{i,r}, L_{i,r-1}]$.

9. The lead time crashing cost entirely belongs to the buyer's cost component.

10. The time horizon is infinite.

Total expected cost for buyers

At this stage, we derive all cost components for each buyer. The ordering cost for ith buyer is $\frac{A_{bi}d_i}{q_i}$ as the expected cycle time for each buyer is $\frac{q_i}{d_i}$. The inventory level is continuously reviewed by the each buyer. Thus, the ith buyer places an order q_i only when the level of inventory reaches down to a specified indicator say, reorder point r_i. The net inventory level for buyer i just before and after receipt of an order is $r_i - d_iL_i$ and $q_i + r_i - d_iL_i$, respectively. Therefore, the approximated average inventory for buyer i over the cycle is $\frac{q_i}{2} + r_i - d_iL_i$. Now, r_i can be expressed as $D_iL_i + k_i\sigma_i\sqrt{L_i}$, which makes the average inventory for ith buyer as $\frac{q_i}{2} + k_i\sigma_i\sqrt{L_i}$. Again, $(1 - \beta_i)$ is the fraction of demand, which is not backordered. Hence, the holding cost for buyer i per unit time is $h_{bi}[\frac{q_i}{2} + r_i - d_iL_i + (1 - \beta_i)E(X_i - r_i)^+]$. As π_{0i} and π_i are the marginal profit and stockout cost per unit item, respectively for buyer i, $\{\pi_i + \pi_{0i}(1 - \beta_i)\}\frac{d_i}{q_i}E(X_i - r_i)^+$ is the shortage cost per item per unit time. According to assumption, the expression of the lead time crashing cost/unit time is $R(L_i)\frac{d_i}{q_i}$ for buyer i.

Total expected cost for buyer i is

TEC_{bi} =Ordering cost+holding cost+shortage cost+Lead time crashing cost

Thus, TEC_{bi} leads to the following expression

$$\begin{aligned} TEC_{bi}(q_i, k_i, L_i) \quad &= \quad \Big[\frac{A_{bi}d_i}{q_i} + h_{bi}\Big\{\frac{q_i}{2} + k_i\sigma_i\sqrt{L_i} + (1 - \beta_i)E(X_i - r_i)^+\Big\} \\ &+ \quad \{\pi_i + \pi_{0i}(1 - \beta_i)\}\frac{d_i}{q_i}E(X_i - r_i)^+ + R(L_i)\frac{d_i}{q_i}\Big] \end{aligned} \tag{110}$$

X_i is the lead time demand for buyer i having a normal distribution with $d_i L_i$ and $\sigma_i\sqrt{L_i}$ as mean and standard deviation, respectively. Shortages occur only when $X_i > r_i$ for each ith buyer. The expected shortage at cycle end for ith buyer is $E(X_i - r_i)^+ = \int_{r_i}^{\infty}(x_i - r_i)dF(x) = \sigma_i\sqrt{L_i}\psi(k_i)$ where $\psi(k_i) = \phi(k_i) - k_i[1-\Phi(k_i)]$, ϕ stands for the standard normal probability density function, and $\Phi =$ stands for the cumulative distribution function of normal distribution.

According to assumption 3 and using $E(X_i - r_i)^+ = \sigma_i\sqrt{L_i}\psi(k_i)$, (110) becomes

$$\begin{aligned} TEC_{bi}(Q, k_i, L_i) &= \Big[\frac{A_{bi}D}{Q} + h_{bi}\left\{\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i} + (1-\beta_i)\sigma_i\sqrt{L_i}\psi(k_i)\right\} \\ &+ \{\pi_i + \pi_{0i}(1-\beta_i)\}\frac{D}{Q}\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i)\frac{D}{Q}\Big] \end{aligned} \tag{111}$$

Total expected cost for the vendor

The setup cost for vendor per unit time is $\frac{A_v D}{mQ}$.

The average inventory of the vendor is

$$\begin{aligned} &\left[\left\{mQ\left(\frac{Q}{P} + (m-1)\frac{Q}{D}\right) - \frac{m^2Q^2}{2P}\right\} - \left\{\frac{Q^2}{D}(1+2+...+(m-1))\right\}\right]\frac{D}{mQ} \\ = &\ \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \text{ Figure (7)} \end{aligned}$$

Therefore, the holding cost per unit time for vendor is

$$h_v\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]$$

The production cost for the vendor is assumed to be the function of P. The expression of unit production cost is $C(P) = \left(\frac{a_1}{P} + a_2 P\right)$ (Khouja and Mehrez, 1994). The production rate which minimizes the unit production cost is $P^* = \sqrt{\frac{a_2}{a_1}}$. Therefore, the total expected cost of vendor is expressed as

TEC_v=Setup cost+Holding cost+Material cost i.e.,

$$TEC_v(m, Q, P) = \frac{A_v D}{mQ} + \frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + D\left(\frac{a_1}{P} + a_2 P\right) \tag{112}$$

In order to obtain the centralized decisions for both vendor and buyer to minimize the entire supply chain cost, the total cost expression of both the ends must be combined together.

Therefore, the expected joint total cost for both vendor and buyers ($JTEC$) is obtained.

$$\begin{aligned} JTEC(Q, k_i, L_i, P, m) &= \sum_{i=1}^{n} \frac{D}{Q}\left[A_{bi} + \{\pi_i + \pi_{oi}(1-\beta_i)\}\sigma_i\sqrt{L_i}\psi(k_i) + \frac{A_v}{m} + R(L_i)\right] \\ &+ \sum_{i=1}^{n} h_{bi}\left[\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i} + (1-\beta_i)\sigma_i\sqrt{L_i}\psi(k_i)\right] \\ &+ \frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + D\left(\frac{a_1}{P} + a_2 P\right) \end{aligned} \tag{113}$$

Now, the aim is to obtain the optimal solution for all decision variables such that the joint expected total cost is minimized. The problem becomes an unconstrained minimization problem with five decision variables. Therefore, in order to obtain the optimal supply chain cost, we need to obtain derivatives of the objective function (total cost function) with respect to all decision variables and equate them with zero. Now, according to the assumption, m is an integer and therefore, it can be treated as discrete decision variable and differentiability of the cost function with respect to m is not possible. Thus, after taking derivatives with respect to Q, k_i, L_i, and P, we obtain

$$\begin{aligned} \frac{\partial JTEC(Q, k_i, L_i, P, m)}{\partial Q} &= \sum_{i=1}^{n} \frac{h_{bi}}{2D}d_i + \frac{h_v}{2}\left[m(1-\frac{D}{P}) - 1 + \frac{2D}{P}\right] \\ &- \sum_{i=1}^{n} \frac{D}{Q^2}[A_{bi} + \{\pi_i + \pi_{0i}(1-\beta_i)\}\sigma_i\sqrt{L_i}\psi(k_i) \\ &+ A_v/m + R(L_i)] \\ \frac{\partial JTEC(Q, k_i, L_i, P, m)}{\partial k_i} &= \frac{D}{Q}\{\pi_i + \pi_{0i}(1-\beta_i)\}\sigma_i\sqrt{L_i}(\Phi(k_i) - 1) + h_{bi}\sigma_i\sqrt{L_i} \\ &+ (1-\beta_i)\sigma_i\sqrt{L_i}(\Phi(k_i) - 1) \\ \frac{\partial JTEC(Q, k_i, L_i, P, m)}{\partial L_i} &= \frac{D}{2Q}\{\pi_i + \pi_{0i}(1-\beta_i)\}\sigma_i\psi(k_i)L_i^{-1/2} - \frac{Dc_{i,r}}{Q} \\ &+ (k_i\sigma_i + (1-\beta_i)\sigma_i\psi(k_i))\frac{h_{bi}L_i^{-1/2}}{2} \\ \frac{\partial JTEC(Q, k_i, L_i, P, m)}{\partial P} &= \frac{D}{P^2}\left[\frac{Q}{2}h_v(m-2) - a_1\right] + a_2 D \end{aligned} \tag{114}$$

Again, we note that the second order partial derivative of the joint total cost function with respect to L_i is

$$\begin{aligned} \frac{\partial^2 JTEC(Q,k_i,L_i,P,m)}{\partial L_i^2} &= -\frac{D}{4Q}\{\pi_i+\pi_{0i}(1-\beta_i)\}\sigma_i\psi(k_i)L_i^{-3/2} \\ &- (k_i\sigma_i+(1-\beta_i)\sigma_i\psi(k_i))\frac{h_{bi}L_i^{-3/2}}{4} \end{aligned} \tag{115}$$

which is a negative term for $0<\beta_i<1$ and all positive values of parameters and decision variables present in (115). Therefore, for fixed $Q,\ k_i,\ P$, and m, the function $JTEC(Q,k_i,L_i,P,m)$ is concave in L_i. Thus, for fixed $Q,\ k_i,\ P$, and m, the minimum value of $JTEC(Q,k_i,L_i,P,m)$ attains at the end point of the interval $[L_{i,j},L_{i,j-1}]$. Now for fixed positive integer m, and for any fixed value of L_i, values of Q, $\Phi(k_i)$, and P can be obtained by equating every individual equation of the system (114) to zero.

$$Q=\left\{\frac{2D\{A_v/m+\sum_{i=1}^{n}(A_{bi}+[\pi_i+\pi_{0i}(1-\beta_i)]\sigma_i\sqrt{L_i}\psi(k_i)+R(L_i))\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]}\right\}^{1/2} \tag{116}$$

$$\Phi(k_i)=1-\frac{h_{bi}}{\frac{D}{Q}(\pi_i+\pi_{0i}(1-\beta_i))+(1-\beta_i)} \tag{117}$$

$$P=\left\{\frac{2a_1-Qh_v(m-2)}{2a_2}\right\}^{1/2} \tag{118}$$

8.2.2 Solution algorithm

Step 1 Set $m=1$ and for all buyers $i=1,2,...,n$ assign values of all parameters and perform the following steps.

Step 2 For every combination of $L_{i,r}, r=1,2,...,N_i,\ i=1,2,...,n$ perform Step 2a−2e.

Step 2a Set $k_i^{j1}=0$ for each buyer i.

Step 2b Substitute k_i^{j1}, (i=1,2,...,n) into (128) and evaluate Q^{j1}.

Step 2c Utilize Q^{j1} to determine the value of $\Phi(k_i^{j2})$ for each i from (129).

Step 2d Using the value of $\Phi(k_i^{j2})$, obtain the value of k_i^{j2} from normal table.

Step 2e Repeat 2b to 2d until no changes occur in the values of Q^j and k_i^j denote these values by the point Q^{j*} and k_i^{j*}, respectively.

Step 3 Evaluate the value of P^{j*} from (130) using the value of Q^{j*}.

Step 4 Denote the latest updated values of Q^j, k_i^j, and P^j by Q^{j**}, k_i^{j**}, and P^{j**}.

Step 5 Obtain $JTEC(Q^{j**}, k_i^{j**}, P^{j**}, L_{i,r}, m)$ and $Min_{j=1,2,\ldots,N_i} JATC(Q^{j**}, k_i^{j**}, P^{j**}, L_{i,r}, m)$ for all i.

Step 6 Set $m = m + 1$.

If $JATC(Q_m^{**}, k_{im}^{**}, P_m^{**}, L_{i,m}, m) \leq JATC(Q_{m-1}^{**}, k_{m-1}^{**}, S_{m-1}^{**}, \theta_{m-1}^{**}, L_{m-1}, m-1)$, repeat Step 2 and Step 3. Otherwise go to Step 7.

Step 7 Set $JATC(Q_m^{**}, k_m^{**}, S_m^{**}, \theta_m^{**}, L_m, m) = JATC(Q_{m-1}^{**}, k_{m-1}^{**}, S_{m-1}^{**}, \theta_{m-1}^{**}, L_{m-1}, m-1)$. Then, $(Q^{**}, k^{**}, L^{**}, S^{**}, \theta^{**}, m^{**})$ is the optimal solution and the optimal reorder point can be obtained from $R^{**} = DL^{**} + k^{**}\sigma\sqrt{L^{**}}$, where R^{**} denotes the optimal solution for R, the reorder point.

8.3 Numerical experiments

Following parameter values are used to interpret the model numerically. $d_1 = 200$ units/week, $d_2 = 300$ units/week, $d_3 = 200$ units/week, $A_v = \$4000$/setup, $A_{b1} = \$100$/setup, $A_{b2} = \$150$/setup, $A_{b3} = \$100$/setup, $h_v = \$10$/unit/week, $h_{b1} = \$11$/unit/week, $h_{b2} = \$11$/unit/week, $h_{b3} = \$12$/unit/week, $\sigma_1 = 9$, $\sigma_2 = 10$, $\sigma_3 = 15$, $\pi_{01} = \$150$/unit, $\pi_{02} = \$140$/unit, $\pi_{03} = \$152$/unit, $\pi_1 = \$50$/unit, $\pi_2 = \$50$/unit, $\pi_3 = \$51$/unit. Table 8.1 for lead time data is given below.

From Table 8.2, 8.3, and 8.4, the optimal values of decision variables are obtained from different backorder ratios. Table 8.2, 8.3, and 8.4 represent optimal results for $\beta = 0.0$, 0.5, and

Table 8.1

Lead time data

Buyer i	Lead time component	Normal duration ($b_{i,r}$)	Minimum duration ($a_{i,r}$)	Unit crashing cost ($c_{i,r}$)
1	1	20	6	0.1
	2	20	6	1.2
	3	16	9	5.0
2	1	20	6	0.5
	2	16	9	1.3
	3	13	6	5.1
3	1	25	11	0.4
	2	20	6	2.5
	3	18	11	5.0

Table 8.2

Total optimal cost for $\beta_i = 0$, $i = 1, 2, 3$

m	L_1	L_2	L_3	Q	k_1	k_2	k_3	p	$C(p)$	TEC
3	3	4	4	469.943	1.789	1.765	1.755	1328.543	28.339	30287.838
3	4	4	4	466.458	1.792	1.769	1.759	1329.199	28.338	30270.744
3	4	3	4	469.925	1.789	1.765	1.755	1328.547	28.339	30282.853
3	4	4	3	471.373	1.787	1.764	1.754	1328.274	28.339	30276.958
3	4	3	3	474.778	1.784	1.760	1.751	1327.633	28.340	30288.206
3	3	4	3	474.797	1.784	1.760	1.751	1327.630	28.340	30293.196
3	3	3	4	473.367	1.785	1.762	1.752	1327.899	28.340	30299.351
3	3	3	3	478.160	1.781	1.757	1.747	1326.996	28.341	30303.874

Table 8.3

Total optimal cost for $\beta_i = 0.5$, $i = 1, 2, 3$

m	L_1	L_2	L_3	Q	k_1	k_2	k_3	p	$C(p)$	TEC
3	3	4	4	471.905	1.561	1.541	1.525	1328.174	28.340	30153.094
3	4	4	4	468.487	1.565	1.544	1.529	1328.817	28.339	30131.424
3	4	3	4	471.879	1.561	1.541	1.525	1328.179	28.340	30148.556
3	4	4	3	473.274	1.560	1.539	1.524	1327.916	28.340	30146.095
3	4	3	3	476.604	1.556	1.536	1.520	1327.289	28.341	30162.393
3	3	4	3	476.631	1.556	1.536	1.520	1327.284	28.341	30166.934
3	3	3	4	475.255	1.558	1.537	1.522	1327.543	28.340	30169.646
3	3	3	3	479.920	1.553	1.532	1.517	1326.664	28.342	30182.676

Table 8.4

Total optimal cost for $\beta_i = 0.8$, $i = 1, 2, 3$

m	L_1	L_2	L_3	Q	k_1	k_2	k_3	p	$C(p)$	TEC
3	3	4	4	474.268	1.322	1.308	1.283	1327.729	28.340	30016.004
3	4	4	4	470.930	1.326	1.312	1.287	1328.357	28.339	29989.695
3	4	3	4	474.234	1.322	1.308	1.283	1327.736	28.340	30011.853
3	4	4	3	475.563	1.320	1.306	1.281	1327.485	28.340	30012.957
3	4	3	3	478.804	1.317	1.302	1.277	1326.875	28.341	30034.312
3	3	4	3	478.840	1.317	1.302	1.277	1326.868	28.341	30038.465
3	3	3	4	477.529	1.318	1.304	1.279	1327.115	28.341	30037.601
3	3	3	3	482.040	1.313	1.299	1.273	1326.265	28.342	30059.281

Table 8.5

Summarization of optimal values

β	m	L_1	L_2	L_3	Q	k_1	k_2	k_3	p	$C(p)$	TEC
0.0	3	4	4	4	470.930	1.326	1.312	1.287	1328.357	28.339	29989.695
0.5	3	4	4	4	468.487	1.565	1.544	1.529	1328.817	28.339	30131.424
0.8	3	4	4	4	470.930	1.326	1.312	1.287	1328.357	28.339	29989.695

0.8, respectively. We obtain many solutions in each table for different lead time. Final optimal decisions for optimal lead time are displayed by Table 8.5.

Therefore, we see that minimum cost is attend at 4 weeks of lead time for every buyer and the optimal shipment is 3 for each of the three backorder ratio values.

8.3.1 Sensitivity analysis

We change some key parameters by −50%, −25%, +25%, and +50%. Each parameter is changed one at a time keeping other parameters fixed. The effect of changes of the key parameters are illustrated in Table 8.6.

Variations of key parameters A_{bi}, h_{b1}, A_v, and h_v are considered. For the sake of simplicity, cost parameters of buyer 1 is taken under consideration. Observations made from sensitivity analysis are described as follows:

• Vendor's cost components are more sensitive than buyer's cost components.

• Holding cost of buyer is more sensitive that ordering cost, which is true for all buyers.

• Vendor's holding cost is also more sensitive than vendor's setup cost, but the rate of sensitivity is less than that of the buyer.

Table 8.6

Sensitivity analysis for different key parameters

Parameters	Changes(in %)	TEC^N	Parameters	Changes(in %)	TEC^N
A_{b1}	−50%	−0.41	A_v	−50%	−5.01
	−25%	−0.22		−25%	−2.65
	+25%	+0.19		+25%	+2.47
	+50%	+0.38		+50%	+4.75
h_{b1}	−50%	−1.23	h_v	−50%	−5.81
	−25%	−0.49		−25%	−3.00
	+25%	+0.57		+25%	+2.86
	+50%	+1.14		+50%	+5.70

8.4 Managerial insights

This chapter provides a two-echelon single-vendor multi-buyer supply chain model with variable production rate. The managerial insights of this chapter are as follows:

• The managerial decisions are made under variable production rate. This assumption is more realistic than a fixed production rate.

• Production cost is also considered as variable and a special type of cost function is assumed to obtain more generalized decisions than fixed production cost.

• Manager can reduce lead time and enhance the service level by incurring a lead time crashing cost.

8.5 Concluding remarks

This study proposed a single vendor and multiple buyers supply chain model. Variable lead time was considered at the buyer's end. The lead time demand was assumed to follow a normal distribution. The vendor's production rate was considered as variable rather than as a fixed entity. Moreover, the unit production cost was also considered as a variable (Khouja and Mehrez, 1994) that was dependent on the production rate, and a special type of function was considered to establish the relation between the production rate and the unit production cost. At the end of the production, the finished goods were delivered to a number of buyers through a multiple delivery policy.

Chapter 9
Relation between quality of products and production-rate in a single-vendor multi-retailer joint economic lot size model with variable production cost

9 Relation between quality of products and production-rate in a single-vendor multi-retailer joint economic lot size model with variable production cost

This chapter deals with a single-vendor multi-buyer supply chain model with imperfect quality and variable production rate. The vendor supplies the order quantity of the buyer over multiple shipments. The production rate of the vendor is considered as flexible because production rate may change for various situations. The quality of products is dependent on the production rate. Manufacturing quality deteriorates with an increasing rate of production. The relation between process quality and production rate is established in this context. Moreover, the unit production cost is also considered to be a function of the production rate. End products are delivered to satisfy the demand of buyers over multiple time segments. The lead time is variable and a lead time crashing cost is incorporated by buyers to reduce the lead time, whereas the lead time demand is considered to be stochastic and to follow a normal distribution. The target of this study is to examine how the flexibility of the production rate affects the entire supply chain cost under a single-setup multi-delivery policy. A supply chain consists of many facilities, which play very important role in establishing a perfect coordination among themselves in order to satisfy the end customer's demand. However, the situation, where parties concentrate on reducing their own cost is not suitable for modern supply chain management. Instead of having a single-sided optimal strategy, present-day manufacturers or vendors are interested for building their own set of retail outlets. Thus, obtaining optimal decisions to minimize the entire supply chain cost has recently been given a great deal of attention as opposed to minimizing individual costs of each player (vendor, buyer, etc.) separately. Moreover, vendors receive more profit if they deliver products to a large number buyers rather than to a single buyer.

9.1 Literature review

Sarmah et al. (2008) introduced a single-manufacturer multi-buyer coordinated supply chain policy with a trade-credit option. Jha and Shankar (2013) incorporated a service-level constraint in a multi-buyer integrated production inventory model. Guan and Zhao (2011) developed a multi-retailer inventory model with continuous review policy. The system optimizes the decisions of pricing and inventory management with the aim of maximizing profit.

In the classical supply chain model, the rate of production is assumed to be constant. However, in many cases the machine production rate may easily change (Khouja and Mehrez, 1994). Machine tool cost also increases with increasing production rate. According to the analysis of Conard and McClamrock (1987), a 10% change in the processing rate results in a 50% change in the machine tool cost. Moreover, as the production rate increases, the probability of failure to the process may gradually increase, which causes the product quality to deteriorate at some percentage. Thus, it is reasonable to assume the production rate as a decision variable rather than a constant parameter. The unit production cost also depends on the production rate and is treated as decision variable.

Porteus (1986) explained the gradual fall of the product quality with an increased amount of production. The process approaches to the *out-of-control* state from the *in-control* state as the number of produced units increases. Rosenblatt and Lee (1986) considered the elapsed time until the production process reaches the *out-of-control* state to be an exponentially distributed random variable.

9.2 Model formulation

This section consists of assumptions, formation of mathematical model with graphical representation, solution methodology with algorithm.

9.2.1 Assumptions

Following assumptions are considered to develop the model.

1. The model consists of a single-vendor and multi-buyer for single type of products.

2. To satisfy the demand of each buyer, the vendor supplies a total of Q items such that $Q = \sum_{i=1}^{n} q_i$.

3. When the i-th buyer orders the quantity q_i, the vendor manufactures mQ items but the shipment should be carried out m times with a quantity of Q such that $q_i = d_i \frac{Q}{D}$, i.e., $\frac{q_i}{d_i} = \frac{Q}{D}$.

4. Continuous review inventory policy is achieved by each buyer.

5. The production rate is a variable quantity which varies within the range P_{min} ($P_{min} > D = \sum_{i=1}^{n} d_i$) and P_{max}.

6. The unit production cost is dependent on the production rate P.

7. The quality of the product deteriorates with increasing production rate.

8. The elapsed time after the production system goes *out-of-control* is an exponentially distributed random variable and the mean of the exponential distribution is a decreasing function of the production rate (Khouja and Mehrez, 1994).

9. Shortages are allowed and are fully backordered.

10. For the i-th retailer, we assume $L_{i,0} \equiv \sum_{j=1}^{n_i} b_{i,j}$. $L_{i,r}$ is the length of the lead time with components $1, 2, ..., r$ crashed to their minimum duration. Thus, $L_{i,r}$ can be expressed as $L_{i,r} = L_{i,0} - \sum_{j=1}^{r}(b_{i,j} - a_{i,j})$, $r = 1, 2, ..., n$; and the lead time crashing cost/cycle $C_i(L_i)$ is expressed as $C_i(L_i) = c_{i,r}(L_{i,r-1} - L_i) + \sum_{j=1}^{r-1} c_{i,j}(b_{i,j} - a_{i,j})$, $L \in [L_{i,r}, L_{i,r-1}]$.

11. The lead time crashing cost belongs entirely to the buyer's cost component.

12. The time horizon is infinite.

Total expected cost for buyers

The expected cycle time for each buyer is $\frac{q_i}{d_i}$. Thus, the ordering cost for the i-th buyer is $\frac{A_{bi}d_i}{q_i}$. The i-th buyer places an order q_i only when the level of inventory reaches to the reorder point r_i. The net inventory levels for buyer i are $r_i - d_iL_i$ and $q_i + r_i - d_iL_i$ just before and after an order quantity is received, respectively. Therefore, the approximate average inventory for buyer i over this cycle is $\frac{q_i}{2} + r_i - d_iL_i$. The reorder point for buyer i is $r_i = D_iL_i + k_i\sigma_i\sqrt{L_i}$ Therefore, the average inventory for the i-th buyer becomes $\frac{q_i}{2} + k_i\sigma_i\sqrt{L_i}$. The holding cost for buyer i per unit time is $h_{bi}[\frac{q_i}{2}+r_i-d_iL_i]$. The shortage cost per unit for buyer i is $\pi_i\frac{d_i}{q_i}E(X_i-r_i)^+$ and the lead time crashing cost per unit for buyer i can be expressed as $R(L_i)\frac{d_i}{q_i}$.

Generally, the total expected cost for buyer i is

TEC_{bi} =ordering cost+holding cost+shortage cost+lead time crashing cost

Thus, TEC_{bi} can be written as

$$\begin{aligned} TEC_{bi}(q_i, k_i, L_i) &= \Big[\frac{A_{bi}d_i}{q_i} + h_{bi}\left\{\frac{q_i}{2} + k_i\sigma_i\sqrt{L_i}\right\} \\ &+ \frac{\pi_i d_i}{q_i}E(X_i - r_i)^+ + R(L_i)\frac{d_i}{q_i}\Big]. \end{aligned} \tag{119}$$

X_i is the lead time demand for buyer i having a normal distribution with d_iL_i and $\sigma_i\sqrt{L_i}$ as the mean and standard deviation, respectively. Shortages occur only when $X_i > r_i$ for each i-th buyer. The expected shortage at cycle end for the i-th buyer is $E(X_i - r_i)^+ = \int_{r_i}^{\infty}(x_i - r_i)dF(x) = \sigma_i\sqrt{L_i}\psi(k_i)$, where $\psi(k_i) = \phi(k_i) - k_i[1 - \Phi(k_i)]$, ϕ stands for the standard normal probability density function, and Φ = stands for the cumulative distribution function of the normal distribution.

According to assumption 3 and using $E(X_i - r_i)^+ = \sigma_i\sqrt{L_i}\psi(k_i)$, (119) becomes

$$\begin{aligned} TEC_{bi}(Q, k_i, L_i) &= \Big[\frac{A_{bi}D}{Q} + h_{bi}\left\{\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i}\right\} \\ &+ \pi_i\sigma_i\sqrt{L_i}\psi(k_i)\frac{D}{Q} + R(L_i)\frac{D}{Q}\Big] \end{aligned} \tag{120}$$

Total expected cost for the vendor

The setup cost for the vendor per unit time is $\frac{A_v D}{mQ}$.

The average inventory of the vendor is

$$\left[\left\{mQ\left(\frac{Q}{P}+(m-1)\frac{Q}{D}\right)-\frac{m^2Q^2}{2P}\right\}-\left\{\frac{Q^2}{D}(1+2+...+(m-1))\right\}\right]\frac{D}{mQ}$$
$$= \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] \ (Figure 7)$$

Therefore, the holding cost per unit time for the vendor is

$$h_v\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]$$

In order to establish a relation between the production rate and the process quality, we assume $f(P)$ as an increasing function of the production rate P which defines the number of failure of production process with an increased production rate. Accordingly, $1/f(P)$ becomes a decreasing function, which denotes the mean time to failure (Khouja and Mehrez, 1994). Therefore, it can be implied from the above discussion that when the production rate is increased, the mean time to failure decreases.

The number of defective units in a production cycle is given by

$$N = \begin{cases} 0 & \text{if } \eta \geq t \\ \alpha P(t-\eta(P)) & \text{if } \eta \leq t \text{ (Rosenblatt and Lee, 1986).} \end{cases}$$

The expected units of defective items in a lot size Q is given by

$$E(N) = \alpha P\left[\frac{Q}{P}+\frac{1}{f(P)}e^{(-\frac{Qf(P)}{P})}-\frac{1}{f(P)}\right].$$

For small $f(P)$, Maclaurin series is a valid approximation for $e^{(-\frac{Qf(P)}{P})}$, which yields

$$e^{(-\frac{Qf(P)}{P})} = 1-\frac{Qf(P)}{P}+\frac{(Qf(P))^2}{2P^2}.$$

From the above two equations, we obtain $E(N)=\alpha f(P)\frac{Q^2}{2P}$ and thus, the expected rework cost becomes $\frac{D}{Q}E(N) = RD\alpha f(P)\frac{Q}{2P}$.

The following observations are needed to obtain the unit production cost function.

The labour and energy costs rise to a large value as the production rate increases. The unit tool cost is a minimum at a particular production rate, after which the tool cost increases. Therefore, a 'U-shaped' convex cost function can be inferred at this stage (Hax and Candea, 1984).

According to the above discussion, the expression of the expected total cost for the vendor becomes

$$TEC_v(m,Q,P) = \frac{A_v D}{mQ} + \frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] + RD\alpha f(P)\frac{Q}{2P} + DC(P). \quad (121)$$

The objective of this article is to obtain centralized decisions for both the vendor and the buyers to minimize the joint total supply chain cost. Therefore, the expected joint total cost for both the vendor and buyers ($JTEC$) can be expressed as

$$\begin{aligned} JTEC(Q,k_i,L_i,P,m) &= \sum_{i=1}^{n}\left[\frac{A_{bi}D}{Q} + h_{bi}\left\{\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i}\right\}\right. \\ &+ \left.\frac{D}{Q}\pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i)\frac{D}{Q}\right] \\ &+ \frac{A_v D}{mQ} + \frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] \\ &+ RD\alpha f(P)\frac{Q}{2P} + DC(P). \end{aligned} \quad (122)$$

The necessary condition for Q, k_i, L_i, P, and m to be optimal is that the partial derivatives of $JTEC$ with respect to the above decision variables vanish separately, and the global minimum of the objective cost function exists if the second order partial derivatives are all positive. According to our assumption, m, the number of shipment, is a positive integer. Thus, it is not reasonable to take the derivative of $JTEC$ with respect to m. Besides this, the second order partial derivative of $JTEC$ with respect to L_i is negative:

$$\frac{\partial^2 JTEC(Q,k_i,L_i,P,m)}{\partial L^2} = -\frac{D}{4Q}\pi_i\sigma_i\psi(k_i)L_i^{-3/2} - \frac{1}{4}h_{bi}k_i\sigma_i L_i^{-3/2} < 0.$$

Therefore, for fixed $Q,\ k_i,\ P$, and m, the function $JTEC(Q,k_i,L_i,P,m)$ is concave in L_i, which yields the conclusion that the minimum value of $JTEC(Q,k_i,L_i,P,m)$ is attained at an endpoint of the interval $[L_{i,j},L_{i,j-1}]$ for fixed $Q,\ k_i,\ P$, and m. On the other hand, the positive integer property of m resists to obtain the derivative of $JTEC$ with respect to itself. Therefore, for fixed $Q,\ k_i,\ P$, and L_i, there exists a positive integer m such that the following inequality always holds true.

$$JTEC(Q, k_i, L_i, P, m-1) \geq JTEC(Q, k_i, L_i, P, m), \tag{123}$$

$$JTEC(Q, k_i, L_i, P, m) \leq JTEC(Q, k_i, L_i, P, m+1). \tag{124}$$

Now, after obtaining the partial derivatives with respect to $Q,\ k_i$, and P, we have

$$\begin{aligned}\frac{\partial JTEC}{\partial Q} &= \sum_{i=1}^{n}\Big[\frac{h_{bi}}{2D}d_i - \frac{A_{bi}D}{Q^2} - \frac{\pi_i\sigma_i\sqrt{L_i}\psi(k_i)D}{Q^2} - R(L_i)\frac{D}{Q^2}\Big] \\ &- \frac{A_vD}{mQ^2} + \frac{h_v}{2}\Big[m(1-D/P) - 1 + 2D/P\Big] \\ &+ \frac{RD\alpha f(P)}{2P},\end{aligned} \tag{125}$$

$$\frac{\partial JTEC}{\partial k_i} = \frac{D}{Q}\pi_i\sigma_i\sqrt{L_i}[\Phi(k_i) - 1] + h_{bi}\sigma_i\sqrt{L_i}, \tag{126}$$

$$\frac{\partial JTEC}{\partial P} = (m-2)h_v\frac{QD}{2P^2} + DC'(P) + \frac{R\alpha DQ}{2P^2}(Pf'(P) - f(P)). \tag{127}$$

Satisfying the necessary condition for the optimality of $JTEC$, the following expressions are obtained:

$$Q = \left\{\frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + \pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i + h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + \frac{RD\alpha f(P)}{P}}\right\}^{1/2}, \tag{128}$$

$$\Phi(k_i) = 1 - \frac{h_{bi}Q}{D\pi_i}, \tag{129}$$

$$\frac{1}{P^2} = \frac{2h_vDC(P)}{2QD(2-m) + h_vR\alpha DQ(f(P) - Pf'(P))}. \tag{130}$$

Here, we use a special type of unit production cost function as introduced by Khouja and Mehrez (1994):

$$C(P) = \left(\frac{a_1}{P} + a_2P\right), \tag{131}$$

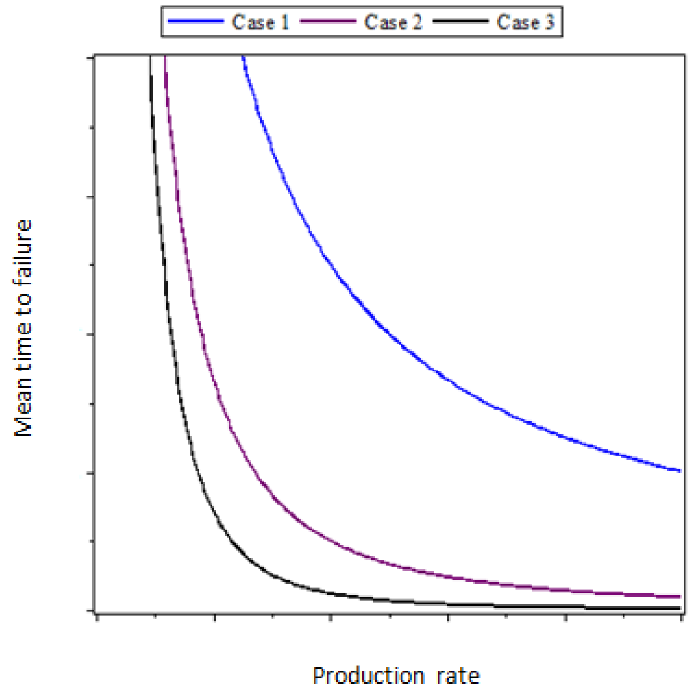

Figure 17 Production rate versus mean time to failure

where a_1 and a_2 are positive real numbers which provides the best fit of the production cost function (see Figure 17).

When the machines are inoperative i.e., the production process ceases, there is no chance of any defective products to be created or the probability of the process going *out-of-control* is zero. As the machines change into operation mode, chances of the arrival of defective goods appear. We consider an increasing function $f(P)$ of production rate P such that the mean time to failure $\frac{1}{f(P)}$ becomes a decreasing function of P (See Figure 18). We introduce three different cases with three different functions to define the mean time to failure.

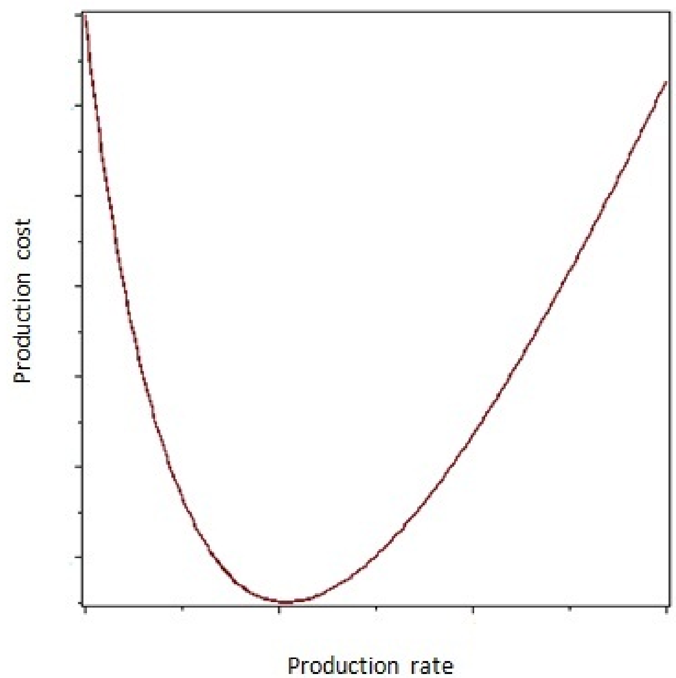

Figure 18 Production rate versus production cost

9.2.2 Special functions for mean time to failure

$$\text{Case 1: } \frac{1}{f(P)} = \frac{1}{b_1 P} \text{ (The quality function } f(P) \text{ is linear in } P\text{)}, \tag{132}$$

$$\text{Case 2: } \frac{1}{f(P)} = \frac{1}{b_2 P + c_2 P^2} \text{ (The quality function } f(P) \text{ is quadratic in } P\text{)}, \tag{133}$$

$$\text{Case 3: } \frac{1}{f(P)} = \frac{1}{b_3 P + c_3 P^2 + d_3 P^3} \text{ (The quality function } f(P) \text{ is cubic in } P\text{)}. \tag{134}$$

where $b_1,\ b_2,\ c_2,\ b_3,\ c_3$ and d_3 are non-negative real numbers that provide the best fit for the function $f(P)$ as well as $\frac{1}{f(P)}$. Figure 18 separately describes the reduction of the mean time of system failure with an increasing production rate for the above three cases. As we gradually shift from Case 1 to Case 3, the decrement of the function $\frac{1}{f(P)}$ becomes larger and larger.

The expressions for the total cost function and the decision variables as obtained from (122), (128), and (130) for the three different cases are given. We denote Q_p, k_i^p, and P_p for Case p; $p = 1, 2,$ and 3, denote the three cases described above, respectively.

Case 1: $f(P)$ is linear in P

$$Q_1 = \left[\frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + \pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i + h_v\left[m\left(1-\frac{D}{P_1}\right) - 1 + \frac{2D}{P_1}\right] + \frac{RD\alpha b_1 P_1}{P_1}}\right]^{1/2}, \tag{135}$$

$$\Phi(k_i^1) = 1 - \frac{h_{bi}Q_1}{D\pi_i}, \tag{136}$$

$$P_1 = \left[\frac{2a_1D - Q_1h_vD(m-2)}{2Da_2}\right]^{1/2}, \tag{137}$$

the joint total is given by

$$\begin{aligned} JTEC_1(Q_1, k_i, L_i, P_1, m) &= \sum_{i=1}^{n}\left[\frac{A_{bi}D}{Q_1} + h_{bi}\left\{\frac{Q_1}{2D}d_i + k_i\sigma_i\sqrt{L_i}\right\}\right. \\ &+ \left.\frac{D}{Q_1}\pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i)\frac{D}{Q_1}\right] \\ &+ \frac{A_vD}{mQ_1} + \frac{Q_1}{2}h_v\left[m\left(1-\frac{D}{P_1}\right) - 1 + \frac{2D}{P_1}\right] \\ &+ RD\alpha b_1P_1\frac{Q_1}{2P_1} + D\left(\frac{a_1}{P_1} + a_2P_1\right). \end{aligned} \tag{138}$$

Case 2: $f(P)$ is quadratic in P

$$Q_2 = \left[\frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + \pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i + h_v\left[m\left(1-\frac{D}{P_2}\right) - 1 + \frac{2D}{P_2}\right] + \frac{RD\alpha(b_2P_2 + c_2P_2^2)}{P_2}}\right]^{1/2}, \tag{139}$$

$$\Phi(k_i^2) = 1 - \frac{h_{bi}Q_2}{D\pi_i}, \tag{140}$$

$$P_2 = \left[\frac{2a_1D - Q_2h_vD(m-2)}{2Da_2 + R\alpha DQ_2b}\right]^{1/2}, \tag{141}$$

the joint total is given by

$$\begin{aligned} JTEC_2(Q_2, k_i, L_i, P_2, m) &= \sum_{i=1}^{n}\left[\frac{A_{bi}D}{Q_2} + h_{bi}\left\{\frac{Q_2}{2D}d_i + k_i\sigma_i\sqrt{L_i}\right\}\right. \\ &+ \left.\frac{D}{Q_2}\pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i)\frac{D}{Q_2}\right] \\ &+ \frac{A_vD}{mQ_2} + \frac{Q_2}{2}h_v\left[m\left(1-\frac{D}{P_2}\right) - 1 + \frac{2D}{P_2}\right] \\ &+ RD\alpha(b_2P_2 + c_2P_2^2)\frac{Q_2}{2P_2} + D\left(\frac{a_1}{P_2} + a_2P_2\right). \end{aligned} \tag{142}$$

Case 3: $f(P)$ **is cubic in** P

$$Q_3 = \left[\frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + \pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n} \frac{h_{bi}}{D} d_i + h_v \left[m\left(1 - \frac{D}{P_3}\right) - 1 + \frac{2D}{P_3} \right] + \frac{RD\alpha(b_3P_3 + c_3P_3^2 + d_3P_3^3)}{P_3}} \right]^{1/2}, \tag{143}$$

$$\Phi(k_i^3) = 1 - \frac{h_{bi}Q_3}{D\pi_i}, \tag{144}$$

$$P_3 = \left[\frac{2Da_1 - Q_3h_vD(m-2)}{2(d_3R\alpha DQ_3P_3 + Da_2) + R\alpha DQ_3c_2} \right]^{1/2}, \tag{145}$$

the joint total is given by

$$\begin{aligned} JTEC_3(Q_3, k_i, L_i, P_3, m) &= \sum_{i=1}^{n} \left[\frac{A_{bi}D}{Q_3} + h_{bi}\left\{ \frac{Q_3}{2D} d_i + k_i\sigma_i\sqrt{L_i} \right\} \right. \\ &+ \left. \frac{D}{Q_3}\pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i)\frac{D}{Q_3} \right] \\ &+ \frac{A_vD}{mQ_3} + \frac{Q_3}{2}h_v\left[m\left(1 - \frac{D}{P_3}\right) - 1 + \frac{2D}{P_3} \right] \\ &+ RD\alpha(b_3P_3 + c_3P_3^2 + d_3P_3^3)\frac{Q_3}{2P_3} + D\left(\frac{a_1}{P_3} + a_2P_3 \right). \end{aligned} \tag{146}$$

Now, a suitable procedure is established to optimize the above models. The explicit form of k_i is difficult to obtain from (129). Moreover, (145) shows that the function contains the variable P_3 in both the right-hand and left-hand side and so an explicit expression of P_3 cannot be obtained. In these circumstances, a closed form solution is very difficult to obtain. We introduce an efficient algorithm to obtain the optimal solutions of the decision variables as well as the expected joint total cost function. The following algorithm elaborately describes the procedure for obtaining the optimal solution of the model for all cases.

9.2.3 Solution algorithm

Step 1 Set $m = 1$ and for all buyers $i = 1, 2, ..., n$; assign values of all parameters and perform the following steps.

Step 2 For every combination of $L_{i,r}, r = 1, 2, ..., N_i$, $i = 1, 2, ..., n$ perform Steps 3a−3e.

Step 2a Set $k_{ij}^{p1} = 0$ for each buyer $i(p = 1, 2, 3$ represents cases 1,2, and 3, respectively).

Step 2b Substitute k_{ij}^{p1}, (i=1,2,...,n) into (135), (139) and (143) to evaluate Q_1^{j1}, Q_2^{j1}, and Q_3^{j1}.

Step 2c Utilize Q_p^{j1} to determine values of $\Phi(k_{ij}^{p2})$ for each i from (136), (140), and (144).

Step 2d Using values of $\Phi(k_{ij}^{p2})$, obtain values of k_{ij}^{p2} from the normal table for each i and each p.

Step 2e Repeat steps 2b to 2d until no changes occur in the values of Q_p^j and k_{ij}^p and denote these values by Q_p^{j*} and k_i^{jp*}, respectively.

Step 3 Evaluate P_p^{j*} from (137), (141), and (145) using the Q_p^{j*} for each p.

Step 4 Denote the latest updated values of Q_p^j, k_i^{pj}, and P_p^j by Q_p^{j**}, k_i^{pj**}, and P_p^{j**} for $p = 1, 2, 3$.

Step 5 Obtain $JTEC(Q_p^{j**}, k_i^{pj**}, P_p^{j**}, L_{i,r}^p, m_p)$ and $\text{Min}_{j=1,2,...,N_i} JATC(Q_p^{j**}, k_i^{pj**}, P_p^{j**}, L_{i,r}^p, m_p)$ for all i and $p = 1, 2, 3$.

Step 6 Set $m_p = m_p + 1$.

If $JATC(Q_{p,m_p}^{**}, k_{i,m_p}^{p**}, P_{p,m_p}^{**}, L_{i,m_p}^p, m_p) \leq JATC(Q_{p,m_p-1}^{**}, k_{m_p-1}^{p**}, P_{p,m_p-1}^{**}, L_{i,m_p-1}, m_p - 1)$, repeat Steps 2 to 6. Otherwise, go to Step 7.

Step 7 Set $JATC(Q_{p,m_p}^{**}, k_{i,m_p}^{p**}, P_{p,m_p}^{**}, L_{i,m_p}^p, m_p) = JATC(Q_{p,m_p-1}^{**}, k_{m_p-1}^{p**}, P_{p,m_p-1}^{**}, L_{i,m-1}^p, m_p - 1)$.

Then, $(Q_p^{**}, k_i^{p**}, P_p^{**}, L_i^{p**}, m_p^{**})$ is the optimal solution and the optimal reorder point can be obtained from $R_i^{p**} = d_i L_i^{p**} + k_i^{p**} \sigma \sqrt{L^{p**}}$, where R_i^{p**} denotes the optimal reorder point for buyer i and case p, $p = 1, 2, 3$.

9.3 Numerical experiments

Example 1

Following input values are used to illustrate the model numerically:

$d_1 = 200$ units/year, $d_2 = 100$ units/year, $d_3 = 100$ units/year, $A_v = \$4000$/setup, $A_{b1} = \$100$/setup, $A_{b2} = \$150$/setup, $A_{b3} = \$100$/setup, $h_v = \$10$/unit/week, $h_{b1} = \$11$/unit/week, $h_{b2} = \$11$/unit/week, $h_{b3} = \$12$/unit/week, $\sigma_1 = 9$, $\sigma_2 = 10$, $\sigma_3 = 15$, $\pi_1 = \$50$/unit, $\pi_2 = \$50$/unit, $\pi_3 = \$51$/unit, $a_1 = 35000$, and $a_2 = 0.1$. Three quality functions for three cases described above are as follows:

$$\text{Case 1} \; \frac{1}{f(P)} = \frac{1}{10^{-4}P}, \tag{147}$$

$$\text{Case 2} \; \frac{1}{f(P)} = \frac{1}{10^{-4}P + 10^{-6}P^2}, \tag{148}$$

$$\text{Case 3} \; \frac{1}{f(P)} = \frac{1}{10^{-4}P + 10^{-6}P^2 + 10^{-9}P^3}. \tag{149}$$

The lead time data is given in Table 9.1.

Table 9.1

Lead time data

Buyer i	Lead time component	Normal duration ($b_{i,r}$)	Minimum duration ($a_{i,r}$)	Unit crashing cost ($c_{i,r}$)
1	1	20	6	0.1
	2	20	6	1.2
	3	16	9	5.0
2	1	20	6	0.5
	2	16	9	1.3
	3	13	6	5.1
3	1	25	11	0.4
	2	20	6	2.5
	3	18	11	5.0

Using the above data, we obtain optimal values of the decision variables at which the joint total supply chain cost is minimized. Results are shown in Table 9.2.

Table 9.2

Optimal values of decision variables

	m	L_1	L_2	L_3	k_1	k_2	k_3	$C(P)$	Q	P	$JTEC$
Case 1	7	4	4	4	1.3158	1.3158	1.2778	118.57	171.10	554.27	54579.68
Case 2	8	4	4	4	1.3845	1.3845	1.3477	118.66	151.08	548.26	54896.47
Case 3	9	4	4	4	1.4289	1.4289	1.3928	118.78	139.11	541.89	55052.57

Table 9.3

Optimal values of reorder points and mean time to failure

	r_1	r_2	r_3	$1/f(P)$
Case 1	39	34	46	18
Case 2	40	35	48	3
Case 3	41	36	49	2

Example 2

This example illustrates a case, where the mean time to failure is independent of P. We use the same data as it is used in Example 1, except for $\frac{1}{f(P)}$. As the mean time until the machine shifts to the *out-of-control* mode is independent of the production rate, the expression $\frac{1}{f(P)}$ can be considered as $\frac{1}{f(P)} = \frac{1}{\beta}$, where β is a constant. Conversely, we can say that the quality of the product deteriorates at a constant rate β, which is uninfluenced by the rate of production. The optimal results are shown in Table 9.4.

9.3.1 Numerical discussion

Table 9.2 shows optimal results of decision variables for the three cases separately. The lead times, at which the total supply chain cost is at a minimum, is four weeks for all buyers for

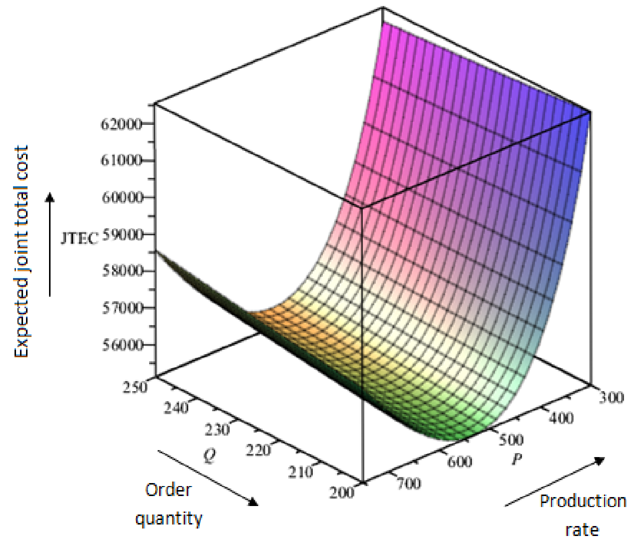

Figure 19 Order quantity versus production rate versus expected joint total cost

Table 9.4

Optimal values of decision variables for independent mean time to failure

β	m	L_1	L_2	L_3	k_1	k_2	k_3	$C(P)$	Q	P	$JTEC$
0.25	8	4	4	4	1.3781	1.3781	1.3411	118.56	152.87	554.59	54788.14
0.50	8	4	4	4	1.4021	1.4021	1.3655	118.52	146.27	559.20	55029.71
0.75	9	4	4	4	1.4552	1.4552	1.4195	118.51	132.37	559.11	55251.83
0.10	9	4	4	4	1.4735	1.4735	1.4382	118.46	127.81	562.82	55460.46

each case. Table 9.3 shows optimal reorder points of buyers and the actual mean time to failure for all cases. The expected joint total cost for Cases 1,2, and 3 are \$54579.68, \$54896.47, and \$55052.57, respectively. An increased total cost can be observed for Case 2 compared to Case 1 and the similar phenomenon happens for Case 3 compared to Case 2. Whereas, the situation is reversed for the mean time to system failure. This phenomenon is quite obvious because a larger time after when the production process shifts to the *out-of-control* stage implies a lower chance of defective items to being produced, which reduces the total supply chain cost. Now, if we look at the quality function ($f(P)$) as demonstrated in three different cases, we can observe that small changes in the production rate may result in larger deviations in the quality function of Case 3 than those of Case 1 and Case 2. Moreover, Cases 2 and 3 can be reduced to Case 1 by assigning c_2, c_3, and d_3 to be zero. Similarly, Case 3 coincides with Case 2 if d_3 vanishes. Thus, if same values of the coefficients of the linear and quadratic terms are used in quality functions for all cases, we observe the total cost from Case 1 to Case 3 increases.

Table 9.4 illustrates the case of independency of the total cost with the production rate. The constant β is varied as 0.25, 0.50, 0.75, and 1.00, which implies that the mean time to failure $\frac{1}{\beta}$ gives values as 4 weeks, 2 weeks, 1.33 weeks, and 1 week, respectively. We observe an increased total cost with increasing β. This is because when β increases, the mean time to failure decreases. As a result chances of defective goods to be produced by the machine becomes large, resulting an increment of the total cost.

9.3.2 Sensitivity analysis

In this section, the deviation of the expected joint total cost with the change of all cost parameters present in the supply chain system is studied. Cost parameters are gradually increased from 10% up to 50% from their actual values as used in above examples, with a step length of 10%. Changes in the total cost with varying parameters are presented in Figure 20 to Figure 24.

The variation of total cost with vendor's cost components (setup cost, holding cost, and

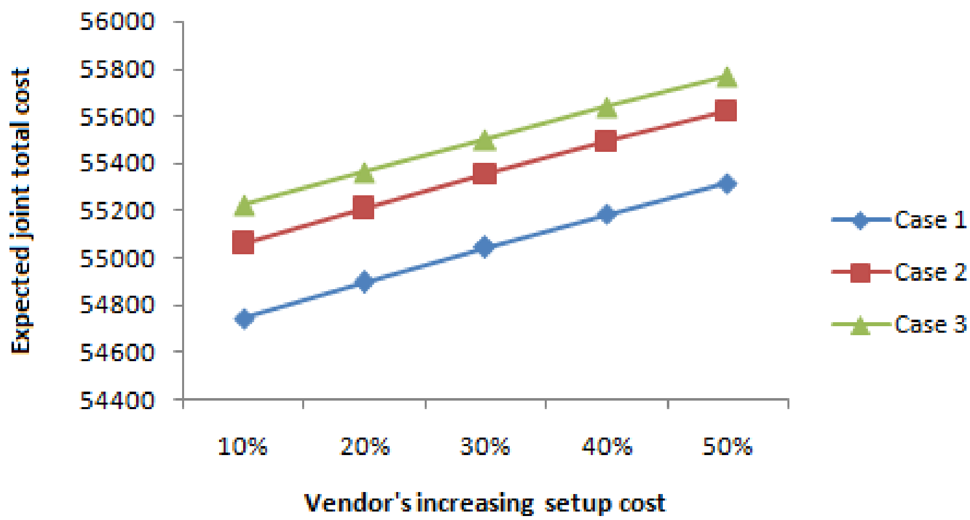

Figure 20 Setup cost versus expected joint total cost

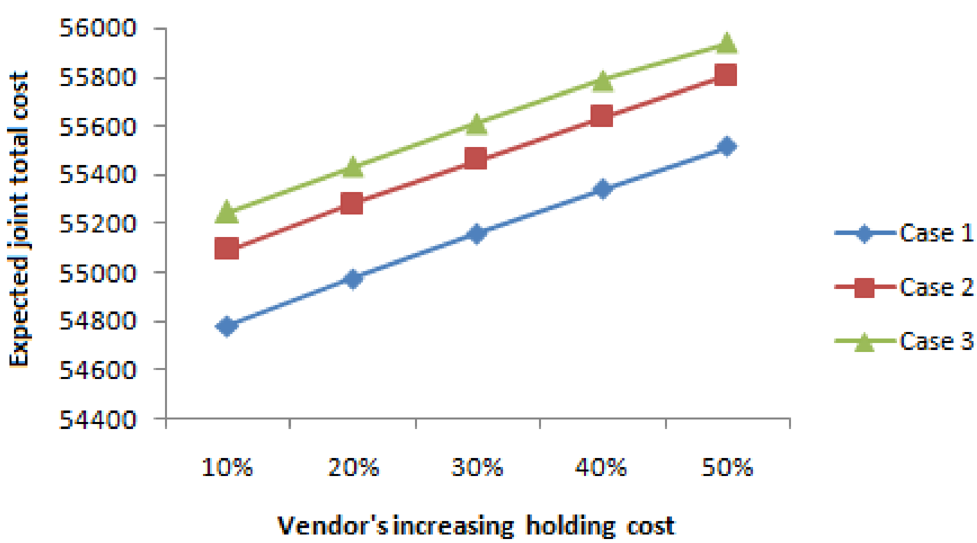

Figure 21 Vendor's holding cost versus expected joint total cost

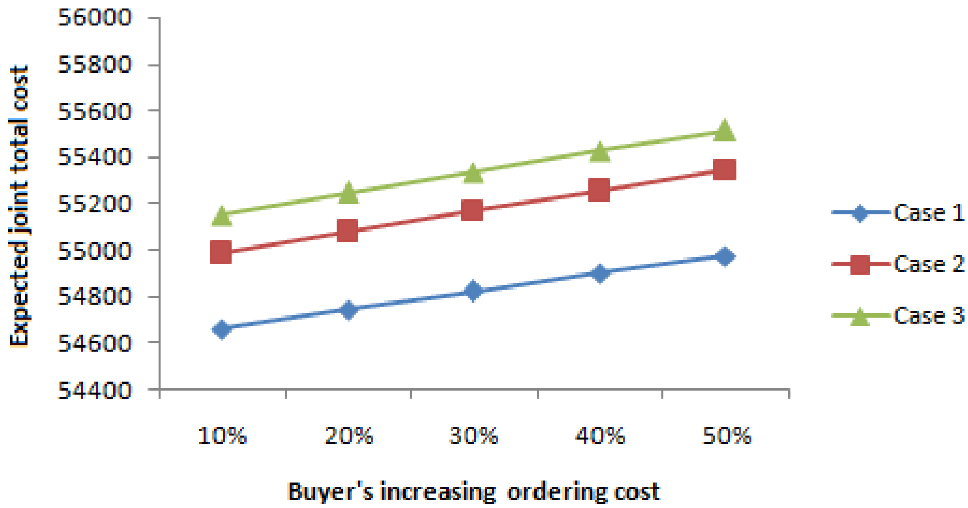

Figure 22 Ordering cost versus joint total expected cost

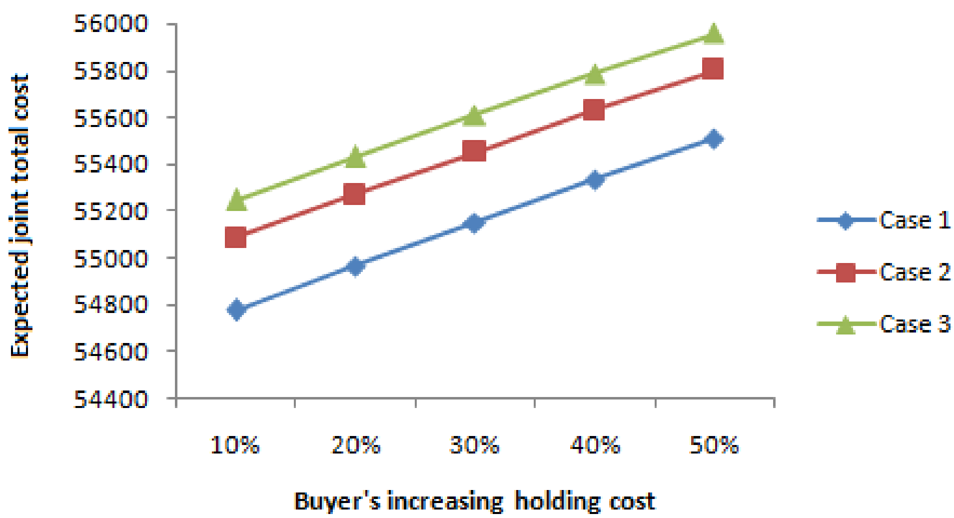

Figure 23 Buyer's holding cost versus expected joint total cost

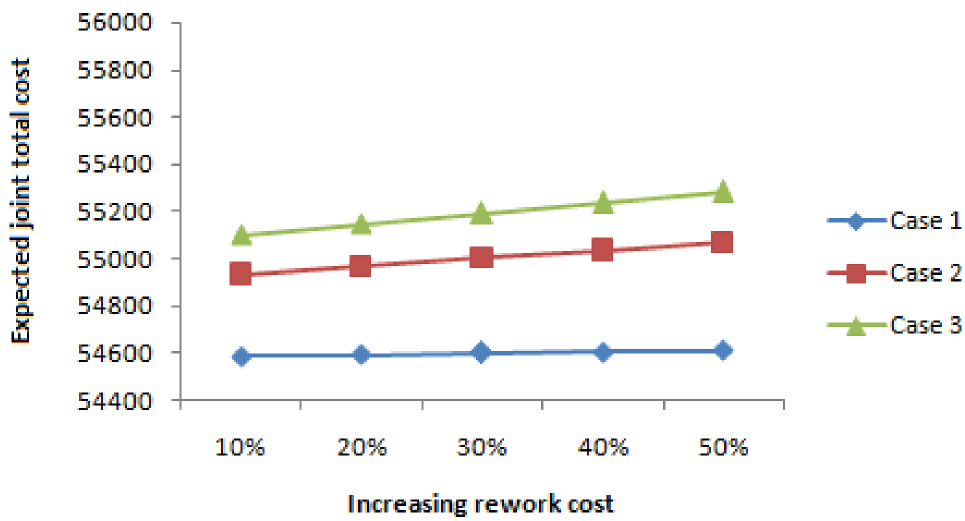

Figure 24 Rework cost versus expected joint total cost

rework cost) are represented in Figure 20, 21, and 24. Similarly, the buyer's cost components (ordering and holding cost) are illustrated in Figure 22 and 23. Observations made from these figures are described as follows.

The rate of change in total cost is linear with respect to all cost parameters of the vendors used to develop this model. The total cost is more sensitive for holding cost than that of the setup cost. The rate of change of $JTEC$ with respect to the setup and holding costs is the same for all of the cases. Therefore, a set of three parallel straight lines can be observed for Cases 1, 2, and 3 in Figure 20 and 21, separately. However, the deviation rate of the total cost for the three different cases is different regarding the rework cost. From Figure 24, we observe that $JTEC$ for Cases 2 and 3 are more sensitive than Case 1. On the other hand, figure 22 and 23 demonstrate the changes in the total cost with increasing ordering and holding cost of the buyers, respectively. As, the three buyers are used to obtain the optimal decisions of the model, we increase the cost components of the three buyers simultaneously, which means Figure 22 denotes the change in $JTEC$ with the percentage increase in ordering costs for three buyers at a time. A similar phenomenon is shown in Figure 23 for the percentage increase in holding cost.

Again, we observe a linear rate of change in $JTEC$ for both the ordering and holding costs at the buyer's end. The total cost is more sensitive towards the holding cost than the ordering cost.

9.4 Managerial insights

A relation production rate and product quality is established under single-vendor and multi-buyer supply chain model model. The managerial insights of this chapter are stated as follows:

• A significant managerial insight lies behind the effect of increasing production rate on quality of product. An increasing rate of production can damage the product quality which results a hike in total supply chain cost.

• Three different types of quality function is considered such that manager can take necessary actions to restrict the quality deterioration under different scenario.

• Optimal decisions is made by assuming variable production cost for more realistic solution.

9.5 Concluding remarks

This research proposed a supply chain model with a single-vendor and multi-buyer. The lead time was variable and the lead time demand was considered as stochastic following a normal distribution. The vendor's production rate was considered as variable rather than as a fixed entity. The relation between the production rate and the mean time to failure of the production process was studied. In this context, three cases using three different types of functions containing the relation between the production rate and mean time to machine failure were established. The effects of mean time to failure for the three cases stated above on the entire supply chain cost was examined, which provides a tremendous managerial insight for the industry. Again, the model was also studied when the mean time to failure was independent of the production rate. Moreover, the unit production cost was also considered as a variable that was dependent on the production rate, and a special type of function was considered to establish the relation between the production rate and the unit production cost (Khouja and

Mehrez ,1994). At the end of the production, the finished goods were delivered to a number of buyers through a multiple delivery policy.

Chapter 10
A study on three different dimensional facility location problems

10 A study on three different dimensional facility location problems

In supply chain strategy, designing a network is one of the most important part. This model deals with various dimensional facility location models. Initially, this study begins with two echelon facility location model of dimension two. Then, it is extended to three dimensional model by adding commodity type and then, different types of transportation modes are added to make it four dimensional model. Delivery lead time and outside suppliers are assumed to meet the retailer's demand too. Some lemmas are constructed to compare the optimal solution for each of the problem. The procedure of reducing the total cost of the supply chain network by applying a small change in constraint set is studied. This is described by another lemma. Some numerical examples are allowed to illustrate the models. The meaning of dimension of a facility location model is discussed. Two echelon supply chain model has been developed in this chapter. In first step, commodities are transported between manufacturing plants and warehouses. In the second step, same for warehouses and retailers. But, no item or commodity type or type of transportation mode has been considered in problem P1. Thus, only commodities of single product type along with single type of transportation mode have been used. Hence, the costs of transportation and continuous decision variables can be represented by a two dimensional array which makes the continuous decision variables, transportation costs as well as the entire problem as dimension two. But, one thing is to be remembered that all variables, costs and demands, do not lie in this category because capacities of plants or warehouses, inventory costs are fixed. They do not depend on product type, type of transportation mode or locations of retailers. Therefore, they always posses dimension one. Same situation happens with the binary variables too, as they only confirm that a manufacturing plant or a warehouse is opened or not at a particular site. Demand of the retailers depends on retailer's locations and product types. But since, in two dimensional problem, no product types have been assumed, so the demand becomes dimension one. Therefore, dimension two means, the highest dimension that the problem preserves. In the similar way, from two dimension to three dimension is extended by adding type of products. Problem P2 represents three dimensional model. Problem P3 is of dimension four where the type of transportation mode is set to extend the dimension of the model. One important thing is to be noted that two types of dimension is used to extend the problem P1. The first one is product type which is dependant on demands of the retailer. The second one is transportation mode which no longer depends on the demand. The aim of this study is to compare these three models to examine how they differ. Two lemmas have been described to compare them. Then, a small change has been applied on a particular constraint set and the difference between the previous and new values of the objective functions of the described models have been studied. Another lemma is described for this too.

10.1 Literature review

The facility location problem plays an important role in supply chain strategy. It has been studied for a long time ago. The first research work was done by Weber (1909) in his industrial location theory. It was extended by Hakimi (1964). The concept of supply chain management (SCM) was established by Weber and Oliver (1982). Since 1970, the global competition level among various companies throughout the world increased by many folds (for instance Erengüc *et al.*, 1999). SCM was introduced independent of OR (Operations Research). Then, it was gradually appeared to be the combination of OR and SCM. In the same way, facility location problem also entered into SCM after its independent appearance. Facility location models are used to design various distribution networks along with facilities which have a great importance in strategic supply chain. Chopra *et al.* (2006) showed an excel based solution of facility location model. According to ReVelle *et al.* (2008), future studies led to different location models such as analytic model, continuous model, discrete location model and network model. Sana (2012) introduced an inventory model in supply chain environment. Teng *et al.* (2012) developed a supply chain model where the optimal economic order quantity for buyer-distributor-vendor

B. Sarkar, A. Majumder, A study on three different dimensional facility location problems, Economic Modelling. 2013, 30, 879-887.

was derived without derivative. Sarkar (2012a) considered a two-echelon supply chain model with probabilistic deterioration.

This model deals with discrete location policy as it is more convenient for designing distribution networks. Melo *et al.* (2009) mentioned, in his review article that, six different groups of discrete facility location problem entitled as median problems, center problems, covering problems, uncapacitated facility location problems (UFLP), capacitated facility location problems (CFLP) and supply chain network design (SCND) problems. The first three problems were well discussed in Owen and Daskin (1998)'s study. Further extension of the above first five groups involves multi products, multi echelon networks, stochastic or dynamic costs, demands etc. in a facility location model. The combination of these extensions of those five models formed the SCND group. The two-echelon, multi-commodity, capacitated facility location problem was introduced by Pirkul and Jayaraman (1998), aiming to locate different facilities in a supply chain so that the total network cost was minimized. This model was again extended by them by assuming raw material vendors for supplying goods to plants. It was a mixed integer programming problem and also a lagrangian relaxation based heuristic procedure was proposed to solve this model. A supply chain model was considered by Wu *et al.* (2006) with facility setup cost function. The aim of that model was to determine the location along with number of facilities. Two-echelon supply chain network was introduced by Amiri (2006). That model was based on heuristic approach along with lagrangian relaxation. He assumed multi capacity level of each facility apart from the single capacity level used in the previous studies. A multi-stage multi-customer supply chain with optimizing inventory decision was introduced by Cárdenas-Barrón (2007). Hinojosa *et al.* (2008) studied a dynamic supply chain with inventory. A simple derivation for optimal manufacturing batch size with rework was developed by Cárdenas-Barrón (2008). An economic production quantity model with inflation in the imperfect production was found out by Sarkar and Moon (2011). Sarkar *et al.* (2011) considered an economic manufacturing quantity model for imperfect production and inflation and time varying demand. Cárdenas-Barrón (2011) studied the vendor-buyer integrated inventory system with arithmetic and geometric inequality. Chen *et al.* (2011) developed a joint inventory location problem. They considered the risk of probabilistic facility disruption. Teng *et al.* (2011) did a simple derivation for an economic lot size in an integrated vendor-buyer system. Cárdenas-Barrón *et al.* (2012) developed a manufacturing inventory model in a supply chain model with three layers. There was an improved algorithm to show the optimal solution of the model. Roy *et al.* (2012) obtained optimal replenishment order in a three layer supply chain with uncertain demand. Pal *et al.* (2012) assumed a multi item economic order quantity model where the demand rate decreases quadratically with increasing sales price and increases exponentially increasing level of price breaks. Some inventory models were developed which deal with variable demand, imperfect production, delay in payments and variable deterioration rate. The reliability in an imperfect production process was also included (for instances Sarkar, 2012b; Sarkar 2012c; Sarkar 2012d). An alternative heuristic algorithm to solve a vendor managed inventory system was proposed by Cárdenas-Barrón *et al.* (2012). They used multi-product and multi-constraint in that model. Pal *et al.* (2012) considered a multi-echelon supply chain for reworkable items in multiple markets with supply disruption. Farahani *et al.* (2012), in their review article, studied the covering problems in facility location model. Kucukdeniz *et al.* (2012) assumed the integrated use of fuzzy for convex programming in capacitated multi-facility location model. Sadjady and Davoudpour (2012) discussed two-echelon multi-commodity supply chain network design with mode selection. Also, a Lagrangian relaxation based heuristic solution procedure was implemented by them. The solving procedures are used to solve the facility location problem as branch and bound algorithm, plant growth simulation algorithm, combination of lagrangian-heuristic and ant colony algorithm (for instances Chen and Ting, 2008; Tong and Zhong-tuo, 2008, Dupont, 2008).

10.2 Model formulation

The description of mixed integer linear programming (MILP) model with some lemmas are illustrated.

10.2.1 Assumptions

Some assumptions are considered to develop this model.

1. The model deals with two-echelon supply chain network.
2. All plants and warehouses have with fixed capacities.
3. Delivery lead time is considered here.
4. The demand of each retailer is satisfied.
5. Outsider suppliers are considered to fulfill the demands of the retailers too.
6. An annual fixed cost is needed for each warehouse and plant to be opened.
7. Plant and warehouse at each site have a fixed inventory holding.

10.2.2 Problem P1

Here, a capacitated facility location problem in dimension two is assumed. The dimensions are considered as two locations in between which the commodity are to be shifted. This model deals with two-echelon supply chain. i.e., the commodities are to be delivered from plants to warehouses and from warehouses to retailers.

Objective function

$$\begin{aligned} Minf &= \sum_{i\in I}\sum_{j\in J} TC_{ij}x_{ij}D_i + \sum_{k\in K}\sum_{j\in J} PTC_{jk}y_{jk}WC_j \\ &+ \sum_{i\in I} OSC_iD_is_i + \sum_{j\in J} IC_jI_j + \sum_{k\in K} JC_kJ_k + \sum_{j\in J} TCW_jz_j \\ &+ \sum_{k\in K} TCP_kc_k + \sum_{i\in I}\sum_{j\in J} MD_iTWR_{ij}x_{ij} + \sum_{j\in J}\sum_{k\in K} MWC_jTPR_{jk}y_{jk} \end{aligned}$$

subject to the constraints

$$\sum_{j\in J} x_{ij} \geq 1 \tag{150}$$
$$s_i \geq 1 \tag{151}$$
$$\sum_{k\in K} y_{jk} \geq 1 \tag{152}$$
$$\sum_{i\in I} D_i x_{ij} + I_j \leq WC_j z_j \tag{153}$$
$$\sum_{j\in J} WC_j y_{jk} + J_k \leq PC_k c_k \tag{154}$$
$$z_j, c_k \in \{0,1\} \forall j \in J, k \in K \tag{155}$$
$$0 \leq x_{ij}, y_{jk}, s_i \leq 1 \tag{156}$$

10.2.3 Problem P2

Now, product type is added as another new dimension.

Objective function

$$\begin{aligned} Minf &= \sum_{i\in I}\sum_{j\in J}\sum_{p\in P} TC_{ijp}x_{ijp}D_{ip} + \sum_{k\in K}\sum_{j\in J}\sum_{p\in P} PTC_{jkp}y_{jkp}WC_j \\ &+ \sum_{i\in I}\sum_{p\in P} OSC_{ip}D_{ip}s_{ip} + \sum_{j\in J}\sum_{p\in P} IC_{jp}I_{jp} + \sum_{k\in K}\sum_{p\in P} JC_{kp}J_{kp} + \sum_{j\in J} TCW_j z_j \\ &+ \sum_{k\in K} TCP_k c_k + \sum_{i\in I}\sum_{j\in J}\sum_{p\in P} M_p D_{ip} TWR_{ijp} x_{ijp} + \sum_{j\in J}\sum_{k\in K}\sum_{p\in P} M_p WC_j TPR_{jkp} y_{jkp} \end{aligned}$$

subject to the constraints

$$\sum_{j \in J} x_{ijp} \geq 1 \tag{157}$$

$$s_{ip} \geq 1 \tag{158}$$

$$\sum_{k \in K} y_{jkp} \geq 1 \tag{159}$$

$$\sum_{i \in I} \sum_{p \in P} D_{ip} x_{ijp} + \sum_{p} I_{jp} \leq WC_j z_j \tag{160}$$

$$\sum_{j \in J} \sum_{p \in P} WC_j y_{jkp} + \sum_{p} J_{kp} \leq PC_k c_k \tag{161}$$

$$z_j, c_k \in \{0, 1\} \forall j \in J, k \in K \tag{162}$$

$$0 \leq x_{ijp}, y_{jkp}, s_{ip} \leq 1 \tag{163}$$

10.2.4 Problem P3

Again, another dimension is added here. Now, the type of transportation mode is set as new additional dimension.

Objective function

$$\begin{aligned} Minf &= \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} TC_{ijp}^{t} x_{ijp}^{t} D_{ip} + \sum_{t \in T} \sum_{k \in K} \sum_{j \in J} \sum_{p \in P} PTC_{jkp}^{t} y_{jkp}^{t} WC_j \\ &+ \sum_{i \in I} \sum_{p \in P} \sum_{t \in T} OSC_{ip}^{t} D_{ip} s_{ip}^{t} + \sum_{j \in J} \sum_{p \in P} IC_{jp} I_{jp} + \sum_{k \in K} \sum_{p \in P} JC_{kp} J_{kp} \\ &+ \sum_{j \in J} TCW_j z_j + \sum_{k \in K} TCP_k c_k + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} M_p D_{ip} TWR_{ijp}^{t} x_{ijp}^{t} \\ &+ \sum_{t \in T} \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} M_p WC_j TPR_{jkp}^{t} y_{jkp}^{t} \end{aligned}$$

subject to the constraints

$$\sum_{t\in T}\sum_{j\in J} x_{ijp}^{t} \geq 1 \tag{164}$$

$$\sum_{t\in T} s_{ip}^{t} \geq 1 \tag{165}$$

$$\sum_{t\in T}\sum_{k\in K} y_{jkp}^{t} \geq 1 \tag{166}$$

$$\sum_{t\in T}\sum_{i\in I}\sum_{p\in P} D_{ip}x_{ijp}^{t} + \sum_{p} I_{jp} \leq WC_j z_j \tag{167}$$

$$\sum_{t\in T}\sum_{j\in J}\sum_{p\in P} WC_j y_{jkp}^{t} + \sum_{p} J_{kp} \leq PC_k c_k \tag{168}$$

$$z_j, c_k \in \{0,1\} \forall j \in J, k \in K \tag{169}$$

$$0 \leq x_{ijp}^{t}, y_{jkp}^{t}, s_{ip}^{t} \leq 1 \tag{170}$$

Now, we discuss about the objective functions of the above three problems. The objective functions of problem P1, P2, and P3 minimize the total cost of the supply chain network. The first term represents variable cost of transportation for the fulfilment of retailer's demand. The second term gives the variable manufacturing and transportation cost for the goods to be produced as well as transported from plants to warehouses. The third term shows the variable transportation cost for shifting of product from outside suppliers to retailers. The forth and fifth terms indicate the variable inventory cost for holding of any item in the warehouses and plants, respectively. The sixth and seventh terms represent the fixed annual cost for opening and maintaining the warehouses and plants, respectively. The last two terms show the variable lead time cost for the transportation of goods between warehouses to retailers and plants to warehouses, respectively.

Now, we discuss about the constraint sets contained by the above stated three problems. Constraint set (150) states that all demands of each retailer has to be met. In the same way, constraint sets (157) and (164) are same as (150) but for (157), demands are met for each product type also, and (164) is quite identical to (157), the summation varies over $j \in J$ and the type transportation mode $t \in T$. Thus, these constraint sets satisfy assumption (153). Constraint sets (151), (158) and (165) fulfill the criteria of assumption (154) i.e., demand of each retailer is to be fulfilled by outside suppliers. Constraint sets (152), (159), and (166) are exactly same as the constraint sets (150), (157), and (164) respectively which state that atleast one plant must be open to supply products to the warehouses. Constraint sets (153), (160), and (167) indicate that demands of the retailers and the inventory holding of each warehouse must not exceed the capacity level of that warehouse. Constraint sets (154), (161), and (168) are identical to constraint sets (153), (160), and (167). The only difference is that these are for each plants i.e., the warehouse capacity and inventory holding of each plant can not exceed the limit of capacity level of that plant. Finally, constraint sets (155), (162), and (169) indicate the binary property of decision variables. Constraint sets (156), (163), and (170) are the non-negativity constraints.

10.2.5 Comparison of Problem P1 and Problem P2

We construct a lemma to compare the problems.

Lemma 1

If the commodity type is added to extend the dimension of problem P1, forming problem P2, and if all the parameters of P1 (costs, demands of the retailers, capacities of plants and warehouses, inventory holding) and their values are unchanged, then, the optimal cost of P2 exceeds the optimal cost of P1.

Proof

Let only one product p_1 be used to define the problem P1. If we consider P^1 as the set of product type for problem P1, then,

$P^1 = \{p_1\}$.

Similarly, if P^2 be the set of all products of P2, then,

$P^2 = \{p_1, p_2, ...p_n\} = \{p_1\} \cup \{p_2, p_3, ...p_n\}$.

This is because, all the parameters and their values corresponding to the product p_1 of problem P1 are unchanged. Since, the dimension is increased, thus, we add the products $p_2, p_3, ...p_n$ to problem P2 other than p_1 to extend the dimension.

Thus, the objective function of P1 can be written as,

$$
\begin{aligned}
F = Minf_{P1} &= \sum_{i\in I}\sum_{j\in J} TC_{ijp_1} x_{ijp_1} D_{ip_1} + \sum_{k\in K}\sum_{j\in J} PTC_{jkp_1} y_{jkp_1} WC_j \\
&+ \sum_{i\in I} OSC_{ip_1} D_{ip_1} s_{ip_1} + \sum_{j\in J} IC_{jp_1} I_{jp_1} + \sum_{k\in K} JC_{kp_1} J_{kp_1} + \sum_{j\in J} TCW_j z_j \\
&+ \sum_{k\in K} TCP_k c_k + \sum_{i\in I}\sum_{j\in J} M_{p_1} D_{ip_1} TWR_{ijp_1} x_{ijp_1} \\
&+ \sum_{j\in J}\sum_{k\in K} M_{p_1} WC_j TPR_{jkp_1} y_{jkp_1}
\end{aligned}
$$

The objective function of P2 can be written as,

$G = Minf_{P2} = F + Q$

where,

$$
\begin{aligned}
Q &= \sum_{i\in I}\sum_{j\in J}\sum_{p_r\in P^2} TC_{ijp_r} x_{ijp_r} D_{ip_r} \\
&+ \sum_{k\in K}\sum_{j\in J}\sum_{p_r\in P^2} PTC_{jkp_r} y_{jkp_r} WC_j + \sum_{i\in I}\sum_{p_r\in P^2} OSC_{ip_r} D_{ip_r} s_{ip_r} \\
&+ \sum_{j\in J}\sum_{p_r\in P^2} IC_{jp_r} I_{jp_r} \\
&+ \sum_{k\in K}\sum_{p_r\in P^2} JC_{kp_r} J_{kp_r} + \sum_{i\in I}\sum_{j\in J}\sum_{p_r\in P^2} M_{p_r} D_{ip_r} TWR_{ijp_r} x_{ijp_r} \\
&+ \sum_{j\in J}\sum_{k\in K}\sum_{p_r\in P^2} M_{p_r} WC_j TPR_{jkp_r} y_{jkp_r}
\end{aligned}
$$

for $r = 2, 3, ..., n$.

The constraint set (150), (151), and (152) of problem P1 become

$$\sum_{j\in J} x_{ijp_1} \geq 1 \tag{171}$$

$$s_{ip_1} \geq 1 \tag{172}$$

$$\sum_{k\in K} y_{jkp_1} \geq 1 \tag{173}$$

Again, the set of constraints (157), (158), and (159) of problem P2 transform into

$$\sum_{j \in J} x_{ijp_1} \geq 1 \tag{174}$$

$$\sum_{j \in J} x_{ijp_r} \geq 1 \tag{175}$$

$$s_{ip_1} \geq 1 \tag{176}$$

$$s_{ip_r} \geq 1 \tag{177}$$

$$\sum_{k \in K} y_{jkp_1} \geq 1 \tag{178}$$

$$\sum_{k \in K} y_{jkp_r} \geq 1 \tag{179}$$

for $r = 2, 3, ..., n$.

Clearly, the constraints (171), (172), and (173) are identical to (174), (176), and (178). The constraint sets (175), (177), and (179) are added for problem P2. As, all demands are to be fulfilled for each product for each retailer, hence, at least one of $x_{ijp_r} > 0$ such that $0 < x_{ijp_r} \leq 1$, for $r = 1, 2, ..., n$.

In the same way, from (172), (173), and from (176) to (179), we can say that at least one of $s_{ip_r} > 0$ such that $0 < s_{ip_r} \leq 1$ and that of y_{jkp_r} also for $r = 1, 2, ...n$.

Thus, we can easily say that, $F > 0$ and $Q > 0$ and
$G = F + Q > F$.
Therefore, $G > F$.

10.2.6 Comparison of Problem P2 and Problem P3

We construct another lemma to compare the problems.

Lemma 2

If the type of transportation mode is considered to extend the dimension of problem P2 to form a new problem P3, and if all the parameters of P2 and their values are unchanged, then, the value of the objective function of the present problem P3 is less than or equal to problem P2.

Proof

As, problem P2 contains no transportation mode as its dimension, thus we can consider only one transportation mode t_1 to transport goods from one node to another.
If T^1 is the set of all transportation mode for P2, then, $T^1 = \{t_1\}$. Let T^2 be the set of all transportation mode for P3. As, mode of transportation is added to P3 to make this problem dimension four, we consider ,
$T^2 = \{t_1, t_2, ..., t_n\}$ i.e., $T^2 = \{t_1\} \cup \{t_2, t_3, ..., t_n\} = T^1 \cup \{t_2, t_3, ..., t_n\}$.
The objective function of P2 can be written as,

$$\begin{aligned}
F_1 = Minf_{P2} &= \sum_{i \in I}\sum_{j \in J}\sum_{p \in P} TC^{t_1}_{ijp} x^{t_1}_{ijp} D_{ip} + \sum_{k \in K}\sum_{j \in J}\sum_{p \in P} PTC^{t_1}_{jkp} y^{t_1}_{jkp} WC_j \\
&+ \sum_{i \in I}\sum_{p \in P} OSC^{t_1}_{ip} D_{ip} s^{t_1}_{ip} + \sum_{j \in J}\sum_{p \in P} IC_{jp} I_{jp} + \sum_{k \in K}\sum_{p \in P} JC_{kp} J_{kp} + \sum_{j \in J} TCW_j z_j \\
&+ \sum_{k \in K} TCP_k c_k + \sum_{i \in I}\sum_{j \in J}\sum_{p \in P} M_p D_{ip} TWR^{t_1}_{ijp} x^{t_1}_{ijp} \\
&+ \sum_{j \in J}\sum_{k \in K}\sum_{p \in P} M_p WC_j TPR^{t_1}_{jkp} y^{t_1}_{jkp}
\end{aligned}$$

The objective function of P3 can be written as,

$$\begin{aligned}
Minf &= F_1 + \sum_{t \in T^2}\sum_{i \in I}\sum_{j \in J}\sum_{p \in P} TC^{t_r}_{ijp} x^{t_r}_{ijp} D_{ip} + \sum_{t \in T^2}\sum_{k \in K}\sum_{j \in J}\sum_{p \in P} PTC^{t_2}_{jkp} y^{t_r}_{jkp} WC_j \\
&+ \sum_{i \in I}\sum_{p \in P}\sum_{t \in T^2} OSC^{t_r}_{ip} D_{ip} s^{t_r}_{ip} + \sum_{t \in T^2}\sum_{i \in I}\sum_{j \in J}\sum_{p \in P} M_p D_{ip} TWR^{t_r}_{ijp} x^{t_r}_{ijp} \\
&+ \sum_{t \in T^2}\sum_{j \in J}\sum_{k \in K}\sum_{p \in P} M_p WC_j TPR^{t_r}_{jkp} y^{t_r}_{jkp}
\end{aligned}$$

Constraint sets (157), (158), and (159) becomes

$$\sum_{j \in J} x^{t_1}_{ijp} \geq 1 \tag{180}$$

$$s^{t_1}_{ip} \geq 1 \tag{181}$$

$$\sum_{k \in K} y^{t_1}_{jkp} \geq 1 \tag{182}$$

Constraint sets (164), (165), and (166) becomes

$$\sum_{j\in J} x_{ijp}^{t_1} + \sum_{t\in T^2}\sum_{j\in J} x_{ijp}^{t_r} \geq 1 \tag{183}$$

$$s_{ip}^{t_1} + \sum_{t\in T^2} s_{ip}^{t_r} \geq 1 \tag{184}$$

$$\sum_{k\in K} y_{jkp}^{t_1} + \sum_{t\in T^2}\sum_{k\in K} y_{jkp}^{t_r} \geq 1 \tag{185}$$

for $r = 2, 3, ..., n$.

From the above constraint sets, it can be easily seen that transportation mode is independent of demand.

Let $C^1 = \{c_1^{t_1}, c_2^{t_1}, ..., c_m^{t_1}\}$ be the set of all costs for P2 and $C^2 = \{c_1^{t_r}, c_2^{t_r}, ..., c_s^{t_r}\}$ be the set of all costs for P3.

Now, if $c_i^{t_1} > c_j^{t_r}$, for $i = 1, 2, ..., m; j = 1, 2, ..., s$; and $r = 2, 3, ..., n$.
then, $min\{c_i^{t_1}, c_j^{t_r}\} = c_j^{t_r}$ and the values of the continuous variables multiplied with $c_i^{t_1}$ becomes zero. Thus, the value of F_1 for P3 becomes lower than that of P2.
Now, for $i = 1, 2, ..., m; j = 1, 2, ..., s$ and $r = 2, 3, ..., n$;

Case 1

If all $c_1^{t_1}, c_2^{t_1}, ..., c_m^{t_1} < c_j^{t_r} \forall c_j^{t_r}$.
Then, all the continuous variables multiplied with all $c_j^{t_r}$ $\forall j$ and r will be zero. We can say, $G_1 = F_1$.

Case 2

If $c_i^{t_1} > c_j^{t_r}$, for some $c_i^{t_1}, c_j^{t_r}$,

Then, the continuous variables, multiplied with those $c_i^{t_1}$ for some i, will be zero. From the set of constraints (183), (184), and (185), we can see that no particular transportation mode is needed to fulfill the demand, i.e., mode of transportation is independent of demand. So from Case 1, we can say,

$G_1 < F_1$.

Case 3

If $c_i^{t_1} > c_j^{t_r} \forall c_i^{t_1}$, then, it is obvious that,

$G_1 < F_1$.

From above three cases, we can say that $G_1 \leq F_1$.

10.2.7 Change in constraint set

Lemma 3

If the demand constraint sets for outside suppliers of the above three problems P1, P2, and P3 are changed from each retailer to for all retailers i.e., if the demand is divided into all retailers, then, the total cost will be minimized.

Proof

The demand constraint sets for outside suppliers of problem P1 is,

$$s_i \geq 1 \tag{186}$$

which is converted into,

$$\sum_{i \in I} s_i \geq 1 \tag{187}$$

for the sake of simplicity, we consider the above constraint sets as,

$$s_i = 1 \tag{188}$$

and

$$\sum_{i \in I} s_i = 1 \tag{189}$$

also let us consider the set $I = \{1, 2..., n\}$.

constraint set (188) can be written as

$$\left\{\begin{matrix} s_1 = 1 \\ s_2 = 1 \\ .... \\ s_n = 1 \end{matrix}\right\} \tag{190}$$

which shows $s_i = 1 \forall i \in I$.

But, from constraint sets (189), we get

$$s_1 + s_2 + ... + s_n = 1 \tag{191}$$

which implies that, $s_i < 1$ for each i or $s_i = 1$ for any i and the rest of s_i are zero. Both of them shows that the objective value of the original problem P1 exceeds the objective value of the modified P1. Similar proof may be allowed for problem P2 and P3.

10.3 Numerical experiments

Now, we show the numerical examples of the above problems in different dimensions using LINGO 13.0 optimization software. We first show this example for two dimensions. Then, we proceed to the next higher dimensions. We consider that there are two plants, two warehouses and two retailers. Moreover we consider one outside supplier who supplies commodities to the retailers. For the sake of simplicity, we assume the set of retailers, warehouses and plants as $I = \{A, B\}$, $J = \{A, B\}$, and $K = \{A, B\}$, where A and B are name of the locations of different facilities and retailers so that goods are shifted between A to A, A to B, B to A, and B to B. The transportation of products between A to A and B to B imply that products are shifted between any two locations of same region. If we suppose A as a particular country then, the shipment of goods may be considered as between any two states of the same country.

Now, we study for two dimensional problem i.e., problem P1. By our assumptions, the transportation and inventory costs, retailer's demand, lead times and capacity levels of plants and warehouses are given below.

Capacities of warehouses and plants

$WC_A = 120$ units	$WC_B = 120$ units
$PC_A = 400$ units	$PC_B = 400$ units

Demands of each location

$D_A = 12$ units	$D_B = 8$ units

Total cost for a warehouse and plant at each site to be opened

$TCW_A = \$\ 6000$	$TCW_B = \$\ 4500$
$TCP_A = \$\ 10000$	$TCP_B = \$\ 8000$

Production and transportation cost per unit of product from plants to warehouses

$PTC_{AA} = \$\ 81$	$PTC_{AB} = \$\ 97$
$PTC_{BA} = \$\ 110$	$PTC_{BB} = \$\ 77$

Transportation cost per unit of product from warehouses to retailers

$TC_{AA} = \$\ 90$	$TC_{AB} = \$\ 100$
$TC_{BA} = \$\ 120$	$TC_{BB} = \$\ 80$

Unit inventory holding cost for warehouses and plants

$IC_A = \$\ 11$	$IC_B = \$\ 15$
$JC_A = \$\ 8$	$JC_B = \$\ 11$

Transportation cost per unit of product to the retailers from outside suppliers

$OSC_A = \$\ 82$	$OSC_B = \$\ 90$

Unit lead time cost

$M = \$\ 10$

Delivery lead time per unit of product from warehouses to retailers and from plants to warehouses

$TWR_{AA} = 2$ days	$TWR_{AB} = 3$ days
$TWR_{BA} = 4$ days	$TWR_{BB} = 2$ days
$TPR_{AA} = 3$ days	$TPR_{AB} = 4$ days
$TPR_{BA} = 5$ days	$TPR_{BB} = 3$ days.

Now, we proceed to the next higher dimension i.e., dimension 3. Here, we add the commodity or product type as an extra dimension. So the model deals with two echelon, capacitated, multi commodity facility location problem. All the notation used in this model, are stated on problem P2. The set P is added here and we consider P=$\{p_1, p_2\}$. All the costs, capacity level of plants and warehouses, demands of the retailers and lead time used to solve the problem P1, are unchanged. All extra costs, demands and lead times added here, caused by the addition of the extra dimension only. Now, we put all values.

Capacity levels of warehouses and plants

$WC_A = 120$ units	WC_B= 120 units
$PC_A = 400$ units	$PC_B = 400$ units

Demands of the retailers for each product

$D_{Ap_1} = 12$ units	$D_{Bp_1} = 8$ units
$D_{Ap_2} = 10$ units	$D_{Bp_2} = 9$ units

Total cost for each warehouse and plant to be opened

$TCW_A = \$\ 6000$	$TCW_B = \$\ 4500$
$TCP_A = \$\ 10000$	$TCP_B = \$\ 8000$

Production and transportation cost per unit of each product from plants to warehouses

$PTC_{AAp_1} = \$\ 81$	$PTC_{ABp_1} = \$\ 97$
$PTC_{BAp_1} = \$\ 110$	$PTC_{BBp_1} = \$\ 77$
$PTC_{AAp_2} = \$\ 79$	$PTC_{ABp_2} = \$\ 98$
$PTC_{BAp_2} = \$\ 120$	$PTC_{BBp_2} = \$\ 70$

Transportation cost per unit of each product transported from warehouses to retailers

$TC_{AAp_1} = \$ \ 90$	$TC_{ABp_1} = \$ \ 100$
$TC_{BAp_1} = \$ \ 120$	$TC_{BBp_1} = \$ \ 80$
$TC_{AAp_2} = \$ \ 88$	$TC_{ABp_2} = \$ \ 115$
$TC_{BAp_2} = \$ \ 125$	$TC_{BBp_2} = \$ \ 75$

Inventory holding cost per unit of each product at each warehouse and plants

$IC_{Ap_1} = \$ \ 11$	$IC_{Bp_1} = \$ \ 15$
$IC_{Ap_2} = \$ \ 12$	$IC_{Ap_2} = \$ \ 17$
$JC_{Ap_1} = \$ \ 9$	$JC_{Bp_1} = \$ \ 12$
$JC_{Ap_2} = \$ \ 10$	$JC_{Bp_2} = \$ \ 14$

Transportation costs per unit of each product via outside supplier

$OSC_{Ap_1} = \$ \ 82$	$OSC_{Bp_1} = \$ \ 90$
$OSC_{Ap_2} = \$ \ 80$	$OSC_{Bp_2} = \$ \ 92$

Delivery lead time per unit of each product from warehouses to retailers and from plants to warehouses

$TWR_{AAp_1} = 2$ days	$TWR_{ABp_1} = 3$ days
$TWR_{BAp_1} = 4$ days	$TWR_{BBp_1} = 2$ days
$TWR_{AAp_2} = 2$ days	$TWR_{ABp_2} = 3$ days
$TWR_{BAp_2} = 5$ days	$TWR_{BBp_2} = 4$ days
$TPR_{AAp_1} = 3$ days	$TPR_{ABp_1} = 4$ days
$TPR_{BAp_1} = 5$ days	$TPR_{BBp_1} = 4$ days
$TPR_{AAp_2} = 4$ days	$TPR_{ABp_2} = 3$ days
$TPR_{BAp_2} = 6$ days	$TPR_{BBp_2} = 5$ days

Unit lead time cost for each product

$M_{p_1} = \$ \ 10$
$M_{p_2} = \$ \ 12$

Now, we extend the dimension again i.e., dimension 4. Problem P2 is converted into problem P3. Here, we add the mode of transportation as an extra dimension. We consider two types of transportation mode available here. Thus, we assume the set T of different transportation mode as $T = \{t_1, t_2\}$. Again, in the similar manner, we do not change the transportation and inventory costs, the capacity levels, demands of the retailers and lead times used in the previous three dimensional model. The extra costs, lead times are just due to the addition of new dimension. Now, according to our assumption, we consider as follows:

Capacity levels of warehouses and plants

$WC_A = 120$ units	$WC_B = 120$ units
$PC_A = 400$ units	$PC_B = 400$ units

Demands of each item to each retailers

$D_{Ap_1} = 12$ units	$D_{Bp_1} = 8$ units
$D_{Ap_2} = 10$ units	$D_{Bp_2} = 9$ units

Total cost for each warehouse and plant to be opened

$TCW_A = \$\ 6000$	$TCW_B = \$\ 4500$
$TCP_A = \$\ 10000$	$TCP_B = \$\ 8000$

Production and transportation costs per unit of each product delivered from plants to warehouses via each transportation mode

$PTC^{t_1}_{AAp_1} = \$\ 81$	$PTC^{t_1}_{ABp_1} = \$\ 97$	$PTC^{t_1}_{BAp_1} = \$\ 110$	$PTC^{t_1}_{BBp_1} = \$\ 77$
$PTC^{t_1}_{AAp_2} = \$\ 79$	$PTC^{t_1}_{ABp_2} = \$\ 98$	$PTC^{t_1}_{BAp_2} = \$\ 120$	$PTC^{t_1}_{BBp_2} = \$\ 70$
$PTC^{t_2}_{AAp_1} = \$\ 100$	$PTC^{t_2}_{ABp_1} = \$\ 110$	$PTC^{t_2}_{BAp_1} = \$\ 120$	$PTC^{t_2}_{BBp_1} = \$\ 90$
$PTC^{t_2}_{AAp_2} = \$\ 97$	$PTC^{t_2}_{ABp_2} = \$\ 115$	$PTC^{t_2}_{BAp_2} = \$\ 130$	$PTC^{t_2}_{BBp_2} = \$\ 82$

Transportation costs per unit of each product delivered from warehouses to retailers via each transportation mode

$TC^{t_1}_{AAp_1} = \$ \ 90$	$TC^{t_1}_{ABp_1} = \$ \ 100$	$TC^{t_1}_{BAp_1} = \$ \ 120$	$TC^{t_1}_{BBp_1} = \$ \ 80$
$TC^{t_1}_{AAp_2} = \$ \ 88$	$TC^{t_1}_{ABp_2} = \$ \ 115$	$TC^{t_1}_{BAp_2} = \$ \ 125$	$TC^{t_1}_{BBp_2} = \$ \ 75$
$TC^{t_2}_{AAp_1} = \$ \ 110$	$TC^{t_2}_{ABp_1} = \$ \ 120$	$TC^{t_2}_{BAp_1} = \$ \ 140$	$TC^{t_2}_{BBp_1} = \$ \ 100$
$TC^{t_2}_{AAp_2} = \$ \ 105$	$TC^{t_2}_{ABp_2} = \$ \ 140$	$TC^{t_2}_{BAp_2} = \$ \ 145$	$TC^{t_2}_{BBp_2} = \$ \ 92$

Inventory holding cost per unit of each product at each warehouse and plant

$IC_{Ap_1} = \$ \ 11$	$IC_{Bp_1} = \$ \ 15$
$IC_{Ap_2} = \$ \ 12$	$IC_{Bp_2} = \$ \ 17$
$JC_{Ap_1} = \$ \ 9$	$JC_{Bp_1} = \$ \ 12$
$JC_{Ap_2} = \$ \ 10$	$JC_{Bp_2} = \$ \ 14$

Transportation costs per unit of each product delivered to each retailers via outside supplier through each transportation mode

$OSC^{t_1}_{Ap_1} = \$ \ 82$	$OSC^{t_1}_{Bp_1} = \$ \ 90$
$OSC^{t_1}_{Ap_2} = \$ \ 80$	$OSC^{t_1}_{Bp_2} = \$ \ 92$
$OSC^{t_2}_{Ap_1} = \$ \ 92$	$OSC^{t_1}_{Bp_1} = \$ \ 102$
$OSC^{t_2}_{Ap_2} = \$ \ 91$	$OSC^{t_2}_{Bp_2} = \$ \ 100$

Lead time per unit of each product for the shipment between warehouses to retailers

$TWR^{t_1}_{AAp_1} = 2$ days	$TWR^{t_1}_{ABp_1} = 3$ days	$TWR^{t_1}_{BAp_1} = 4$ days	$TWR^{t_1}_{BBp_1} = 2$ days
$TWR^{t_1}_{AAp_2} = 2$ days	$TWR^{t_1}_{ABp_2} = 3$ days	$TWR^{t_1}_{BAp_2} = 5$ days	$TWR^{t_1}_{BBp_2} = 4$ days
$TWR^{t_2}_{AAp_1} = 0.5$ days	$TWR^{t_2}_{ABp_1} = 1$ days	$TWR^{t_2}_{BAp_1} = 1.5$ days	$TWR^{t_2}_{BBp_1} = 0.5$ days
$TWR^{t_2}_{AAp_2} = 1$ days	$TWR^{t_2}_{ABp_2} = 1.5$ days	$TWR^{t_2}_{BAp_2} = 2$ days	$TWR^{t_2}_{BBp_2} = 1$ days

Lead time per unit of each product for the shipment between plants to warehouses

$TPR^{t_1}_{AAp_1} = 3$ days	$TPR^{t_1}_{ABp_1} = 5$ days	$TPR^{t_1}_{BAp_1} = 6$ days	$TPR^{t_1}_{BBp_1} = 3$ days
$TPR^{t_1}_{AAp_2} = 3$ days	$TPR^{t_1}_{ABp_2} = 4$ days	$TPR^{t_1}_{BAp_2} = 7$ days	$TPR^{t_1}_{BBp_2} = 5$ days
$TPR^{t_2}_{AAp_1} = 1$ days	$TPR^{t_2}_{ABp_1} = 2$ days	$TPR^{t_2}_{BAp_1} = 2.5$ days	$TPR^{t_2}_{BBp_1} = 1$ days
$TPR^{t_2}_{AAp_2} = 2$ days	$TPR^{t_2}_{ABp_2} = 2.5$ days	$TPR^{t_2}_{BAp_2} = 3$ days	$TPR^{t_2}_{BBp_2} = 2$ days

Unit lead time cost for each product

$M_{p_1} = \$ \ 10$
$M_{p_2} = \$ \ 12$

The solutions are described below.

Numerical results for dimension two are given in Table 10.1 and Table 10.2.

Case 1

Table 10.1

Result based on actual problem P1

Continuous variable	Value	Binary variables	value
x_{AB}	1.00000	z_B	1
x_{BB}	1.00000	c_B	1
y_{AB}	1.00000		
y_{BB}	1.00000		
s_A	1.00000		
s_B	1.00000		

Minimum cost= \$ 45844

Total variable= 18

Total constraint= 33

Case 2

Change of constraint set (151), demands are satisfied for all retailers instead of for each retailer.

Constraint set (151) become,

$$\sum_{i \in I} s_i \geq 1 \tag{192}$$

Table 10.2

Result based on changed constraint set of problem P1

Continuous variable	Value	Binary variables	value
x_{AB}	1.00000	z_B	1
x_{BB}	1.00000	c_B	1
y_{AB}	1.00000		
y_{BB}	1.00000		
s_B	1.00000		

Minimum cost= $ 44860

Total variable= 18

Total constraint= 32

Numerical results for dimension three are given in Table 10.3 and Table 10.4.

Case 1

Table 10.3

Result based on actual problem P2

Continuous variable	Value	Binary variables	value
x_{AAp_1}	1.00000	z_A	1
x_{BAp_1}	1.00000	c_A	1
x_{AAp_2}	1.00000	c_B	1
x_{BAp_2}	1.00000		
y_{AAp_1}	1.00000		
y_{BBp_1}	1.00000		
y_{ABp_2}	1.00000		
y_{BBp_2}	1.00000		
s_{Ap_1}	1.00000		
s_{Ap_2}	1.00000		
s_{Bp_1}	1.00000		
s_{Bp_2}	1.00000		

Minimum cost= \$ 78797

Total variable= 32

Total constraint= 65

Case 2

Change of constraint set (158), demands are met for all the retailers but for each product, and constraint set (158) becomes,

$$\sum_{i \in I} s_{ip} \geq 1 \tag{193}$$

Table 10.4

Result based on changed constraint set of problem P2

Continuous variable	Value	Binary variables	value
x_{AAp_1}	1.00000	z_A	1
x_{BAp_1}	1.00000	c_A	1
x_{AAp_2}	1.00000	c_B	1
x_{BAp_2}	1.00000		
y_{AAp_1}	1.00000		
y_{BBp_1}	1.00000		
y_{ABp_2}	1.00000		
y_{BBp_2}	1.00000		
s_{Ap_2}	1.00000		
s_{Bp_1}	1.00000		

Minimum cost= \$ 76987

Total variable= 32

Total constraint= 63

Numerical results for dimension three are given in Table 10.5 and Table 10.6.

Case 1

Table 10.5

Result based on actual problem P3

Continuous variable	Value	Binary variables	value
$x^{t_1}_{BBp_1}$	1.00000	z_B	1
$x^{t_2}_{ABp_1}$	1.00000	c_A	1
$x^{t_2}_{BBp_2}$	1.00000		
$y^{t_2}_{AAp_1}$	1.00000		
$y^{t_2}_{BBp_1}$	1.00000		
$y^{t_2}_{BBp_2}$	1.00000		
$s^{t_1}_{Ap_1}$	1.00000		
$s^{t_1}_{Bp_1}$	1.00000		
$s^{t_1}_{Ap_2}$	1.00000		
$s^{t_1}_{Bp_2}$	1.00000		

Minimum cost= \$ 59048

Total variable= 52

Total constraint= 105

Case 2

Change of constraint set (165), demands are met for all the retailers and for all transportation mode too but for each product, and constraint set (165) becomes,

$$\sum_{i \in I} \sum_{t \in T} s^t_{ip} \geq 1 \tag{194}$$

Table 10.6

Result based on changed constraint set of problem P3

Continuous variable	Value	Binary variables	value
$x^{t_1}_{BBp_1}$	1.00000	z_B	1
$x^{t_2}_{ABp_1}$	1.00000	c_A	1
$x^{t_2}_{BBp_2}$	1.00000		
$y^{t_2}_{AAp_1}$	1.00000		
$y^{t_2}_{BBp_1}$	1.00000		
$y^{t_2}_{BBp_2}$	1.00000		
$s^{t_1}_{Bp_1}$	1.00000		
$s^{t_1}_{Ap_2}$	1.00000		

Minimum cost= \$ 57236

Total variable= 52

Total constraint= 103

10.3.1 Numerical discussion

In the above tables, non-zero decision variables along with their values are allowed and rest of the decision variables are all zero. First, discuss about Case 1 of the above tables i.e., Table 10.1, Table 10.3 and Table 10.5. From the above numerical experiments, the cost \$ 45844 is found for problem P1 and \$ 78797 is found for problem P2 as minimum cost. The cost is increased due to the additional product. Problem P1 consists of single product only, also each cost for P1 has not been changed for problem P2, moreover the additional costs for the new product have been added too, resulting the increment of minimum cost. But in case of problem P3, the cost is decreased. It is really an interesting matter that using the type of transportation mode as an extra dimension causes reduction of cost. From Table 10.3 and Table 10.5, the first one has 14 non-zero decision variables but the second one has 13, which has made the objective function of problem P3 lower than that of problem P2. Thus, allotment of goods in cheaper transportation mode can dwindle the total minimum cost of the network. Now, from Table 10.2, Table 10.4 and Table 10.6, we see a small change, in constraint sets (151), (153), and (165), which has reduced the total network cost. We have just varied the summation over $i \in I$, the set of retailer location which indicates that the demand of one retailer is divided into all retailers. From the tables for Case 2 above, it can be seen that at least one supply of product via outside suppliers to the retailers is stopped with respect to the tables for Case 1, resulting reduction of total cost.

10.4 Managerial insights

A facility location model is considered in this chapter for three different dimensions. The managerial insights of this chapter are as follows:

• Three different dimensional facility location problems are compared in this chapter to suggest managers adopting proper action to reduce entire supply chain cost.

• An procedure is developed to reduce the system cost by applying a small change in constraint set, which would help the managers very easily.

10.5 Concluding remarks

Three different dimensions in the facility location problem was considered to match the real life situation of locating different facilities for industrialization. The variations between the objective functions of three different dimensional problems were investigated. This chapter concluded that the increment or reduction of cost depends on the type of the dimension used. Two separate type of dimensions were used such as product type and transportation mode. Types of products depend on the retailer's demand, hence, these create the increments of costs. Again, the mode of transportation is independent of the retailer's demand which indicates the reduction of the cost. Lastly, a small change in the constraint sets was considered which results the decrement of total cost.

Conclusions and future extensions

This dissertation covered many practical problems related to modern supply chain management. The research model in Chapter 3 developed the procedure to diminished the entire supply chain cost by an investment to reduce the vendor's setup cost. Chapter 4 used another investment function to reduce the probability of imperfect production of items along with reduction of vendor's setup cost. Chapter 5 optimized the joint supplier-buyer's cost under lot-splitting policy and deterioration of products. The investment functions are also incorporated to reduce the setup cost and imperfect production. Chapter 6 concluded that in decentralized supply chain model, manufacturer should determine the retailer's purchasing cost depending on retailer's decision to achieve a profitable business. Chapter 7 compared a tradition and consignment policy in a supply chain. This chapter observed that the joint profit for the consignment policy is greater than joint profit of the traditional policy. To reduce the royalty for the retailer a new method was provided. By using the proposed method the royalty for the retailer was reduced without affecting the joint profit of the supply chain. Chapter 8 minimized the cost of an one-vendor multi-retailer supply chain model with partial backorder. The model relaxed the classical assumption of fixed production rate with variable production rate and a 'U'-shaped function for variable production cost was also utilized to make the model a realistic one. Chapter 9 analyzed the relation between production rate and the mean time to failure of a production process. Three cases with three different types of functions were provided, which contains the relation between production rate and mean time to machine failure. Chapter 10 compared three facility location problems with three different dimensions. The study proved that the increment or reduction of cost depends on the type of the dimension used.

Future extensions

There are many possible extensions of the research models stated above. Some of them are pointed below.

- The single-setup multi-delivery policy can be extended by unequal shipment.

- The dimension of a facility location problem can be extended by generalized n dimension for $n \in N$ (Set of natural numbers).

- A fruitful research can be done by assuming a discrete investment to reduce setup cost and quality improvement of products instead of continuous investment.

- An integrated supply chain model can de extended with three or more echelon with multi-type of products.

- The consignment policy can be extended by variable selling-price and controllable lead time.

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