M.Sc. 3rd Semester Examination, 2018

PHYSICS

PAPER —PHS-301

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Use separate scripts for each Group

GROUP – A

[*Marks* : 20]

Answer Q.Nos.1 & 2 and any one from the rest

1. Answer any *three* bits : 2×3

(*a*) A particle of mass *m* scattered from a hard sphere potential given as

$$V(r) = \infty \quad \text{for} \quad r < a$$
$$= 0 \quad \text{for} \quad r > a.$$

If $\vec{k}$ is wave vector associated with particle then find the scattering cross-section for low-energy limit.

(*b*) An exchange operator $\hat{P}_{12}$ acting on a state of two identical particles. Show that $\hat{P}_{12}$ is Hermitian.

(*c*) Consider two electrons confined in a potential well of size 'a'. Assume that electrons are in the same spin state, i.e. $\sigma_1 = \sigma_2$. Find the energy of the first excited state of this two electron system.

(*d*) Write down Lippmann-Schwinger equation and explain transition operator.

(*e*) Draw the hyperfine structure of Na(3 $^2P_{3/2}$) with nuclear spin I = 3/2.

2. Answer any *one* bit : 4 × 1

(*a*) For H-atom

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

If perturbation

$$V = \frac{e^2 \vec{A}^2}{2m_e c^2}$$

is applied then prove that shift in energy $\Delta = -\frac{1}{2}\chi \vec{B}^2$, where χ is the diamagnetic susceptibility.

(*b*) Consider a large number of Fermions, and of mass *m*, confined in a cubical box of size *L*. Find the number of Fermions with energy less than E_F.

3. (*a*) If the incident wave coming from left is

$\langle x | \phi \rangle = e^{ikx} / \sqrt{2\pi}$ and $V(x) \neq 0$ for $0 < |x| < a$.

Find an expression of Green's function for $x > x'$ and for $x < x'$ using Lippman-Schwinger equation. 10

(*b*) If

$$V(x) = -\gamma \frac{\hbar^2}{2m} \delta(x)$$

Then find an expression of reflection and transmission coefficient also discuss the poles of $R(k)$ and $T(k)$. 2 + 2 + 3 + 3

4. (*a*) N identical spin $\frac{1}{2}$ particle are subjected to a one-dimensional S.H.O. potential. What is the ground-state energy ? What is the Fermi energy ? 5

(*b*) What are the ground state and fermi energies if we ignore the mutual interactions and assume N to be very large. 3 + 2

(*c*) Discuss what would happen on the energy levels of a $_2He^4$ atom if the electron were a spinless boson. Explain Ramsaver Townsend effect. 3 + 2

GROUP – B

[*Marks* : 20]

Answer **Q.No.1** and any **one** from the rest

1. Answer any *five* bits : 2 × 5

(*a*) The equation of state of a real gas is given by $P(v-b) = RT$. Find the partition function of the system.

(*b*) For a diatomic molecule rotational energy levels are

$$\in(J) = \frac{\hbar^2}{2I} J(J+1).$$

Find the partition function.

(*c*) Prove that entropy of a canonical system

$$S = -k_B \sum_i \rho_i \ln \rho_i$$

where ρ_i is the probability of the system to be found in ith state.

(*d*) The Hamiltonian of a relativistic free gas with N particles is given by

$$H = \sum_{i=1}^{N} C\sqrt{p_i^2 + m^2c^2}$$

calculate $\left\langle p_i \frac{\partial H}{\partial p_i} \right\rangle$.

(*e*) $\rho = a_0 + \vec{\sigma} \cdot \vec{a}$ where ρ is the density matrix. Obtain a_0 and a_k in terms of the matrix elements ρ_{ij}.

(*f*) For a free particle evaluate

$$\left\langle \phi_k \,|\, \exp(-p\hat{H}) \,|\, \phi_{k'} \right\rangle.$$

(*g*) Write down the expression of quantum mechanical density distribution of an oscillator in the ground state at $T \to 0$. What is its value of $T \neq 0$?

2. (*a*) Does the equipartition theorem hold for the Hamiltonian

$$H = \sum_i A_i p_i^2 + \sum_i B_i q_i^2 + \sum_i C_i p_i q_i$$

where q_i and p_i are the canonical variables.

(*b*) Calculate the partition function for a three dimensional quantum mechanical oscillator.

5 + 5

3. (*a*) Deduce an expression for grand potential for Bose|Fermi gas.

(*b*) $\hat{H} = \hbar w\left(a^{+}a + \frac{1}{2}\right)$

show that $\hat{\rho}_{\text{thermal}} = \frac{1}{1+\bar{n}}\sum_{n=0}^{\infty}\left(\frac{\bar{n}}{1+\bar{n}}\right)^{n}|n\rangle\langle n|.$

5 + 5
