2018

M.Sc. 2nd Semester Examination

PHYSICS

PAPER—PHS-201

Subject Code—33

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B

Group—A

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 2×5

(a) Prove that $e^{i\alpha\sigma_j} = I\cos\alpha + i\sigma_j \sin\alpha$

where α is an arbitrary real constant.

(Turn Over)

(b) $\hat{H} = \varepsilon\, \vec{\sigma}, \hat{n}$ where $\hat{n}$ is an arbitrary unit vector.

Find a transformation matrix that diagonalizes $\hat{H}$.

(c) If S_e and S_p be the spin of electron and proton.

Find $\left\langle \hat{S}_e, \hat{S}_p \right\rangle$ for Hydrogen atom.

(d) If $\hat{H}' = \dfrac{p^4}{8m_0^3c^2}$

Evaluate $\left\langle 0 \middle| \hat{H}' \middle| 0 \right\rangle$.

(e) Prove that $\gamma_\mu \gamma^\mu = 4$.

(f) If $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$

Show that $\left\{\gamma_5, \gamma^\mu\right\} = 0$.

(g) Prove that $\mathrm{tr}\left(\gamma_5\gamma_\mu\right) = 0$.

(h) Derive the validity condition of WKB method.

2. (a) For hydrogen atom, $\psi_{100} = \frac{1}{\left(\pi a_0^3\right)^{1/2}} e^{-\frac{r}{a_0}}$ and $\hat{H}' = eEz$.

Prove that $\Delta E_1^{(2)} = -\frac{9}{4} a_0^3 E^2$.

(b) Prove that radial equation for an electron in central potential

$$H = c\,\alpha_r p_r + \frac{ic\hbar\alpha_r k\beta}{r} + \beta mc^2 + V(r)$$

where $\hat{k}\hbar = \beta\left(\bar{\sigma}'.\,\bar{L} + \hbar\right)$

evaluate the eigen values of k^2. 5+5

3. (a) Estimate the energy levels of a particle moving in a potential

$$V(x) = \infty\ ; \quad x < 0$$
$$= Ax\ ; \quad x > 0$$

A being a constant.

(b) If $H = \begin{pmatrix} 1 & 2\epsilon & 0 \\ 2\epsilon & 2+\epsilon & 3\epsilon \\ 0 & 3\epsilon & 3+\epsilon \end{pmatrix}$.

Find the energy levels corrected to second order for orthonormal basis. 4+6

Group—B

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 2×5

(a) Find the Fourier transform of f(x) defined by

$$\begin{aligned} f(x) &= 1 \quad \text{if } -a \le x < a \\ &= 0 \quad \text{if } |x| > a \end{aligned}$$

What happens to f(k)) at k = 0 ?

(b) Solve $\left(D^2 - 4DD' + 5D'^2\right)z = 0$

where $D \dfrac{\partial}{\partial x}$; $D' = \dfrac{\partial}{\partial y}$.

(c) Verify convolution theorem for $f(x) = g(x) = e^{-x^2}$.

(d) Find the Laplace transform of $f(x) = \dfrac{\delta(x-2)}{x}$.

(e) Fourier cosine transform

$$F_c(s) = \frac{1}{\sqrt{2\pi}}\left(a - \frac{s}{2}\right) \quad \text{if } 0 < s < 2a$$

$$= \quad 0 \qquad \text{if } s \geq 2a$$

Find f(x).

(f) Find the Green's function in terms of eigen values and eigen function of operator $\hat{L}$ for the differential equation

$$\frac{d^2\psi}{dx^2} + \omega_0^2 x = f(x)\ ; \quad 0 \leq x \leq 1$$

with the boundary conditions

$$\psi(0) = 0 = \psi(1).$$

(g) Prove that the identity element in a group is unique.

(h) Prove that a group of order three is always a cyclic group.

2. (a) Find Fourier cosine transform of $\frac{1}{1+x^2}$ and sine transform of $\frac{x}{1+x^2}$.

(b) Solve the integral equation :

$$y(x) = \cos x + 3\int_0^{\pi} \sin(x-t)y(t)dt.$$ 5+5

3. (a) Find the invariant subgroup and factor group of group D_3.

(b) Character-table

O_h	E	$6C_4$	$3C_2$	$6S_4$	$8C_3$	$8S_5$	$3\sigma_h$	i	$6\sigma_d$	$6C_2'$
T_{1g}	3	1	−1	1	0	0	−1	3	−1	−1
T_{2g}	3	−1	−1	−1	0	0	−1	3	1	1
E_g	2	0	2	0	−1	−1	2	2	0	0
T_{Ag_1}	1	1	1	1	1	1	1	1	1	1
$T_{2g} \otimes T_{2g}$	9	1	1	1	0	0	1	9	1	1

Prove that $T_{2g \times 2g} = A_{1g} + E_g + T_{1g} + T_{2g}$. 5+5
