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M.Sc. 3rd Semester Examination, 2018

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Numerical Method and Computer Programming*)

PAPER – MTM- 304 (CBCS)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

1. Answer any *four* questions out of eight questions : 2 × 4

   (*a*) How many type of error occurred in Numerical Analysis ?

   (*b*) Write the formula for 4th order Runge-Kutta method to solve

   $$\frac{dy}{dx} = f(x, y), y(x_0) = y_0.$$

(*c*) Write the error term for Trapezoidal and Simpson's 1/3rd rule to evaluate

$$\int_a^b f(x)dx \text{ with spacing } h.$$

(*d*) Find the iteration formula to find the solution of $x^3 + x - 1 = 0$ using Newton-Raphson method.

(*e*) Form the divided difference table to approximate $f(x) = x^3 - 1$ based on the four points $x = -1$, 0, 1, and 3.

(*f*) Write the all the Lagrange polynomials based on points $x = x_0, x_1, x_2$ and x_3.

(*g*) Write the central difference formula to approximate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_i$.

(*h*) Write the iteration formula of Gauss-Seidal method to solve $3x + 4y + 8z = 3$, $5x + y + z = 5$ and $3x + 8y - z = 3$ so that the iteration converges to the true solution.

2. Answer any *four* questions out of eight questions : 4×4

(*a*) State and prove the existence and uniqueness property of interpolation.

(*b*) Derive the Newton's divided difference interpolation formula to approximate the function $f(x)$ based on $x = x_0, x_1, x_2 \ldots x_n$.

(*c*) Derive the Simpson's 1/3rd formula to evaluate the integration

$$\int_a^b f(x)dx$$

by taking three points with spacing h.

(*d*) Using composite Trapezoidal rule, approximate $\int_{-4}^{4} |x+1|\, dx$ with $h = 1$, hence compare this computed value with the exact value.

(*e*) Approximate $5^{\frac{1}{4}}$ using Newton-Raphson method of a non-linear equation with initial approximation as 1. Do five iterations.

(*f*) Do two iterations of Gauss-Seidal method, so that the sequence converges to the true solution, to find the solution of $x - 2y + 4z = 10$, $8x - 3y + 2z = 2$ and $-x + 5y + 2z = 4$ with initial guess (0, 1, 1).

(*g*) Jacobi's method $X^{(m+1)} = BX^{(m)} + C$ is applied to the system of equations written in 2 (*f*) so that the sequence $\{X^{(m)}\}$, $m = 1, 2, 3$ ... converges to the true solution. Then find two matrices B and C.

(*h*) Find the value of $y(2{\cdot}1)$ as solution of $(x^3 + y^3)\,dx = xy^2 dy$, $y(2) = 2{\cdot}55$ using Euler's method with $h = 0{\cdot}1$.

3. Answer any *two* questions out of four questions : 8 × 2

(*a*) (*i*) $P_2(x) = x^2 + x + 1$ interpolates $f(x)$ at $x = -3, -1$ and 0. Prepare the divided difference table. Now $f(2) = 15$ is added to the above data, find $P_3(x)$ which interpolates $f(x)$ at $x = -3, -1, 0$ and 2 by adding a term to $P_2(x)$.

(*ii*) Write a program to solve the following differential equation by 4th order Runge-Kutta method

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0. \qquad 4 + 4$$

(*b*) (*i*) Find a root of $x^3 + x - 1 = 0$ in $[0, 1]$ using Bisection method. Do four iterations.

(*ii*) Write a program to solve a system of linear equations by Gauss-Seidal iteration method. 4 + 4

(*c*) (*i*) Find the value of $y(1{\cdot}2)$ as solution of

$$xdy = (1 + y^2)\, dx, \quad y(1) = 2$$

using Runge-Kutta method of 4th order with $h = 0.2$.

(*ii*) Write a program to implement Lagrange's interpolation method. 4 + 4 (*d*) (*i*) Consider BVP :

$$\frac{x^2 d^2 y}{dx^2} + \frac{4x dy}{dx} + 6y = 2x, y'(1) = 1 \text{ and } y'(2) = -1.$$

Applying the finite difference method of order 2 to the above differential equation, derive the linear system of equation with $h = 0.25$

(*ii*) Write a program to find a complex root of the equation $f(z) = 0$ by Newton-Raphson method. 4 + 4

[*Internal Assessment* : 10 Marks]