2018

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-105

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Classical Mechanics and Non-linear Dynamics

1. Answer any *four* questions : 4×2

(a) State basic postulates of special theory of relativity.

(b) Prove that, if the transformation does not depend explicitly on time then the Hamiltonian represents the total energy.

(c) Prove that the work done by the force in displacing a particle from the position P_1 to P_2 is equal to the difference between the potential energies of the particle at those two positions.

(Turn Over)

(d) Suppose that a rigid body is rotating about a fixed point. Prove that the kinetic energy is conserved throughout the motion.

(e) Is Poisson bracket commutative ? Justify your answer.

(f) Write a brief note on phase portrait.

(g) What do you mean by canonical transformation ? Explain.

(h) Define eigen frequencies and normal modes of small oscillation of a dynamical system.

2. Answer any *four* questions : 4×4

(a) Construct the Routhian for the two-body problem, for which

$$L = \frac{\mu}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r).$$

(b) A double pendulum consisting of two massess m_1 and m_2 oscillates in a vertical plane through small angles. m_1 is suspended from a fixed point by light inextensible string of length l_1 and m_2 is suspended from m_1 by a similar string of length l_2.
State the number of degrees of freedom of the system and find the equations of motion by using Lagrange's formulation.

(c) Prove that the Poisson bracket of two constants of motion is itself a constant even though the constants depend on time explicitly.

(d) Consider the following nonlinear dynamical system,

$$\dot{x} = x^2y - x^5, \ \dot{y} = -y + x^2.$$

Study the stability at the origin.

(e) Prove that the phase volume is invarient under canonical transformation.

(f) State and prove Jacobi identity related to Poisson bracket.

(g) State Hamilton-Jacobi function. Write its significance.

(h) Show that the transformation

$$Q = \log\left(1 + \sqrt{q}\cos p\right), P = 2\sqrt{q}\left(1 + \sqrt{q}\cos p\right)\sin p$$

is canonical. Find the generating function $G(q, Q)$.

3. Answer any *two* questions : 2×8

(a) (i) A body moves about a point Q under no forces. The principal moments of inertia at O being 3A, 5A and 6A. Initially, the angular velocity has components $w_1 = n$, $w_2 = 0$, $w_3 = n$ about the corresponding principal axes. Show that at any time t,

$$w_2 = \frac{3n}{\sqrt{5}} \tan h\left(\frac{nt}{\sqrt{5}}\right).$$

and that the body ultimately rotates about the mean axis.

(ii) If a transformation from q, p to Q, P be canonical then bilinear form $\sum_i (\delta p_i dq_i - \delta q_i dp_i)$ remains invariant. 5+3

(b) (i) Let $J = \int_{x_0}^{x_1} F(y, y', x)\, dx$, where y is an unknown function to be depended on x. Derive a differential equation to find the curve $y = y(x)$ which will