2017

M.Sc. Part-II Examination

PHYSICS

PAPER—VIII

Full Marks : 75

Time : 3 Hours

The figures in the right-hand *margin indicate full marks.*

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(Use separate Scripts for Group–A and Group–B)

Group—A

Answer Q.No. *1, 2, 3* and any *two* from the rest

1. Answer any *five* bits : 5×2

(a) For a spin S particle, in the eigen basis of S^2, S_z the expectation value $\langle Sm/S_x^2/Sm\rangle$ is ___ (show with calculation).

(b) For a spin $\frac{1}{2}$ particle, the expectation value of $S_xS_yS_z$ is ____ (show with calculation)

(c) Show that $\gamma_\mu\gamma^\mu = 4$ where γ_μ are Dirac γ-matrices.

(d) Prove that $\gamma_\mu^+ = \gamma^0\gamma_\mu\gamma^0$

(e) If $\psi_\alpha(x) = e^{-ip.x}u_\alpha(p)$ where $\alpha = 1, 2, 3, 4$

Show that Dirac equation in covariant form

$(\not{p} - m)u(p) = 0$

(f) If $V(\bar{r}) = g\delta(\bar{r})$ Prove that total scattering cross-section.

$\sigma = \frac{m^2g^2}{\pi\hbar^4}$ for a particle of mass 'm'.

(g) Define scattering length. What are the physical significance of +ve and –ve scattering length?

(h) Explain Thomas-Fermi statistical model.

2. Answer any *two* bits : 2×3

(a) If Projection operators

$\wedge_+(p) = \frac{\not{p} + m}{2m}$ and $\wedge_-(p) = \frac{-\not{p} + m}{2m}$

Calculate $\wedge_+(p)\wedge_+(p)$

and $\wedge_-(p)\wedge_-(p)$

(b) If $\tilde{\alpha}_i$ be double Pauli matrix then prove that

$\left[\tilde{\alpha}_i, \alpha_j\right] = 2i\,\epsilon_{ijk}\,\alpha_k$.

(c) If $V(r) = \beta e^{-\mu r}$, when β and μ are positive constants, then prove that scattering amplitude $= \frac{-4m\mu\beta}{\hbar^2\left(b^2+\mu^2\right)^2}$

where $b = 2k\sin\frac{\theta}{2}$ and $E = \frac{\hbar^2k^2}{2m}$

3. Answer any *one* bit : 1×4

(a) For a Dirac Hamiltonian

$$H = c\alpha_i p_i + \beta mc^2$$

Prove that - $\left[L_i + \frac{1}{2}\tilde{\alpha}_i, H\right] = 0$

where α_i is the double Pauli matrices and L_i is the component of orbital angular momentum.

(b) Prove that :

$$\left[\vec{a}\,.\,\vec{\sigma},\ \vec{b}\,.\,\vec{\sigma}\right] = 2i\left(\vec{a}\times\vec{b}\right).\vec{\sigma}$$

where $\vec{a}$, $\vec{b}$ are two arbitrary constant vectors in 3 dimensions.

4. (a) Consider an eigenstate of $\vec{L}^2$ and L_z operator denoted by $|l, m\rangle$. Let $A = \hat{n} \cdot \vec{L}$ denote an operator, where $\hat{n}$ is an unit Vector. Prove that ΔA in the state $|l, m\rangle$ is $\sqrt{l(l+1) - m^2}\ \hbar \sin\theta$.

(b) Calculate the reflection and the transmission coefficients of a Klein-Gordon particle with energy E, at the potential

$$A^0 = \begin{cases} 0 & z<0 \\ U_0, & z>0 \end{cases}$$

where U_0 is a positive constant. 5+5

5. (a) If radial momentum p_r and radial velocity α_r for an electron in a central potential are defined by

$$p_r = \frac{\vec{r}.\vec{p} - i\hbar}{r}, \quad \alpha_r = \frac{\vec{\alpha}.\vec{r}}{r}$$

show that $\vec{\alpha}.\vec{p} = \alpha_r p_r + \dfrac{i\hbar k \beta \alpha_r}{r}$

where $k = \dfrac{\beta\left(\vec{\alpha}'.\vec{L} + \hbar\right)}{\hbar}$, $\vec{\alpha}'$ = double Pauli-matrix.

(b) Obtain the eigenvalues of the operator $\hat{k}$. 5+5

6. (a) If $V(r) = V_0\delta(r - a)$ using partial wave analysis prove that $\dfrac{d\sigma}{d\Omega} = \dfrac{4\pi a^2}{k^2a^2 + \left(1 + \dfrac{2mV_0 a}{\hbar^2}\right)^2}$, where $k^2 = \dfrac{2mE}{\hbar^2}$.

(b) Calculate fine structure energy levels for H-atom for n = 2. Use the formula you deduce. 5+5

Group—B

Answer Q.No. 1 and *two* from the rest

1. Answer any *five* bits : 5×3

(a) A linear simple harmonic oscillator of mass m and frequency ν. Calculate the number of microstates between the energy range E to E + δE.

(b) A one-dimensional random walker takes steps to left or right with equal probability. Find the probability that the random walker starting from origin is back to origin after N even number of steps.

(c) A two dimensional box in a uniform magnetic field B contains $\frac{N}{2}$ localised spin $\frac{1}{2}$ particles with magnetic moment μ and N/2 free spinless particles which do not intereact with each other. Find the average energy of the system at a temperature T.

(d) If for N localized distinguishable freely orientable dipoles $E = -\sum_{i=1}^{N} \vec{\mu}_i \cdot \vec{H}$

Find the cononical partition function.

(e) Calculate the density maxtrix for a particle of mass m in an infinite potential box of volume 'V' in co-ordinate representation.

(f) If energy level of 'e' in magnetic field B

$$\varepsilon = \frac{e\hbar B}{mc}\left(j + \frac{1}{2}\right) + \frac{p_z^2}{2m}$$

Calculate the degeneracy factor (where j = 0, 1, 2, 3, ...)

(g) In Bethe approximation of Ising model explain the condition of self consistency of

$$H_{q+1} = -\mu B\sigma_o - \mu(B + B')\sum_{j=1}^{q} \sigma_j - Je\sum_{j=1}^{q} \sigma_o\sigma_j$$

where q is the no. of nearest neighbours.

(h) Explain first order phase transition and second order phase transition in terms of order parameter with examples.

2. (a) Calculate the quantum mechanical partition function for a three deminsional Harmonic oscillator.

(b) Prove that $\mu(T, P) = K_B T \ln\left[\frac{h^3c^3P}{8\pi(K_B T)^4}\right]$ for photon gas.

5+5

3. (a) Deduce the equation of state for ideal Bose and Fermi gas.

(b) If the density matrix $\rho = \frac{1}{4}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ and $J_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

show that $\Delta J_z = 0.829$.

(c) If $\rho = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} \end{pmatrix}$. Is it acceptable density matrix?

5+3+2

4. (a) Calculate the Debye frequency for n-dimensional solids in terms of N, V, C_l, C_t. Where C_l and C_t are longituidinal and transverse speeds.

(b) Deduce total density of radiation in n-dimensional space using Planck's law.

5. (a) Using Bragg William appronination for Ising model, prove that long range order parameter

$$L(T) = \tanh \mu_o \beta (H + H_m).$$

(b) In Bethe Pearl's approximation, if

$$\frac{Q_+}{Q_-} = \frac{1 + \tanh(\alpha + \alpha^1 - J_e\beta)}{1 - \tanh(\alpha + \alpha^1 + J_e\beta)}$$

where $\alpha = \mu_o H\beta$; $\alpha^1 = \mu_o H_m \beta$;

Q+ is the probability that the spin in the upward direction and similarly for Q– then prove that for two dimensional latice $T_C = \frac{2.88\ Je}{K_B}$. 5
