2017

M.Sc. Part–I Examination

PHYSICS

PAPER—II

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answerscripts for Gr. A & Gr. B.

Group—A

[Marks—50]

Answer Q. No. *1* and *2* and any *two* from the rest.

1. Answer any *six* of the following : 6×2

(a) If $(x) = Ae^{-x^2/2}, g(x) = Bxe^{-x^2/2}$, then prove that $f(x)$ and $g(x)$ are orthogonal as well as linerly independent.

(b Prove that $p_\psi = |\psi\rangle\langle\psi|$ is projection operator.

(c) At time $t = 0$, the state vector

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|\phi_1\rangle + |\phi_2\rangle\right)$$

It is given as Hamiltonian is defined as $H|\phi_n\rangle = n^2 \epsilon_0 |\phi_n\rangle$. What is the wave function $|\psi(t)\rangle$ at latter time t.

(d) Write down the expression of evolution of $|\psi(x,t)|^2$ in question (c).

(e) Prove that for any normalized wave function of particle of mass 'm' in one dimension.

$$\int_{-\infty}^{+\infty} J(x)dx = \frac{\langle p_x \rangle}{m}$$

(f) If the potential of two dimensional harmonic oscillator is $V(x,y) = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m4\omega^2 y^2$, then find energy eigen value for second excited state.

(g) Consider a particle of mass 'm' is confined to a finite potential well

$$\begin{aligned} V(x) &= 0 \quad \text{if} \quad |x| < L \\ &= V_0 \quad \text{if} \quad |x| \geq L \end{aligned}$$

The particle is found to be in a bound state with energy $\frac{\pi^2 \hbar^2}{32mL^2}$. Find the depth of the well.

(h) An alpha particle is trapped in a nucleus of radius 1.4 fm. What is the probability it will escape from the nucleus if its energy is 2 MeV? The potential barrier at the surface of the nucleus 4 Mev.

(i) If the ground state wave function for the hydrogen atom is

$$\psi = \frac{1}{\sqrt{\pi}} \frac{1}{a_0^{3/2}} e^{-r/a_0}$$

Find the average distance of the electron from the nucleus.

2. Answer any *three* bits : 3×4

(a) The wave function of a particle at a certain time $\psi(x) = A\bar{e}^{\mu/x}$. Show tht momentum wave function is

$$\frac{1}{\sqrt{2\pi\hbar}} \frac{2\mu^{3/2}}{\mu^2 + k^2}; \quad (k = \frac{p}{\hbar})$$

(b) An operator representating an observable for a two-state system is given by

$$\hat{A} = \alpha\left(|\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2| + |\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)$$

where
$$|\phi_1\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$
$$|\phi_2\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

Find its eigen value and eigen vectors.

(c) Consider a particle trapped in two dimensional rigid box potential is given by $V = 0,\ 0 < x < a\ ;\ 0 < y < a$. Find $\langle xy \rangle$. $= \infty$ otherwise.

(d) If $\hat{A}$ is an operator and $\langle \hat{A} \rangle$ is expectation value of $\hat{A}$ and if $\hat{H}(t)$ is Hamiltonian operator, then $\left[\hat{A}, \hat{H}(t)\right]$ is equal to $i\hbar\left[\frac{d}{dt}\langle A\rangle - \left\langle\frac{\partial A}{\partial t}\right\rangle\right]$.

3. (a) Find the ground state energy of a particle in two dimensional potential given by

$$V(x,y) = \frac{1}{2}mw^2(x^2 + y^2) - qE_0 x$$

(b) If $\langle 0 \rangle$ and $\langle 1 \rangle$ denote the normalized eigen states correstponding to the ground and the first excited states of a one-dimensional harmonic oscillator, then prove that Δx in the state $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$ is $\sqrt{\frac{\hbar}{2mw}}$.

(b) Prove that the parity of spherical harmonics is $(-1)^l$.

5+4+4

4. (a) A rigid rotator in a plane in the first excited state perturbation represented by

$H' = \frac{V_0}{2}(3\cos^2\phi - 1)$, V_0 = constant. Calculate the energies to first order in H'.

(b) Consider a particle of mass '*m*' in one-dimensional short range potential $V(x) = -V_0\delta(x)$, $(V_0 > 0)$. Find the energy of the system. 7+5

5. (a) If A and B two operators which both commute with their commutor, then prove that

$$e^{\hat{A}}e^{\hat{B}} = \exp\left\{\hat{A} + \hat{B} + \frac{1}{2}[\hat{A}, \hat{B}]\right\}$$

(b) Given $\hat{p}_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$

Find the uncertainty Δp_r in the ground state

$$\psi_o(r) = \frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_0}$$

(c) If parity $\hat{\pi}$ operator is defined as $\hat{\pi}\left|\vec{r}\right\rangle = \left|-\vec{r}\right\rangle$.

Prove that $\hat{\pi}p_- = (I - \hat{\pi})$, where $\hat{p}_- = \frac{1}{2}(I - \hat{\pi})$ 5+5+4

Group—B

[Marks—25]

Answer Q. No. 1 and any *two* from the rest.

1. Answer any *three* : 3×3

(a) Explain what is meant by Hot Bond. 3

(b) The 2886 cm^{-1} fundamental bond of HCl molecule can be shown to fit in empirical relation δ = 2885.90 + 20.577 m – 0.3034 m^2. Calculate the value of Be(rotational constant in the vibratioanl state υ), given α_e = 0.3312 cm^{-1}. 3

(c) What is vibrational frequency corresponding to a thermal energy of kT at 298 K. What is the wavelength of this radiation ? 3

(d) The average spacing between successive rotation line of Carbon monoxide molecule is 3.8626 cm^{-1}. Determine the transition which gives the most intense line at room temperature 300 K.

(e) The rotational spectrum of $^{79}Br^{79}F$ shows a series of equidistant lines 0.71433 cm^{-1} apart. Calculate the rotational constant B, moment of inertia and the bond length of the molecule. Determine the wave number of the $J = 9 \rightarrow J = 10$ transition.

Assuming the molar mass

^{79}Br = 131.03 × 10^{-27} kg

^{19}F = 31.55 × 10^{-27} kg 3

(f) Convert the following spectroscopic quantities as indicated

2000 cm^{-1} to μm

0.3 cm^{-1} tp GHz

0.15 nm to Hz 3

2. (a) Find the rotational fine structure of a vibration-electronic transition for a diatermic molecule.

(b) Show how an electronically excited molecule can loose energy through phosphorescence. 6+2

3. (a) Assuming molecule anharmonic oscillator find the fundamental absroption, first harmonics & second harmonics.

(b) Explain what is meant by progression with reference to vibrational coarse structure. 6+2

4. (a) A space probe was desigened to seek CO in the atmosphere of Saturn by looking for lines in its rotational spectrum. If the bond length of CO is 112.8 pm, at what wave numbers do the first three rotatioanl transition appear ? What resolution would be needed to determine the isotopic ratio of ^{13}C to ^{12}C on saturn by observing the first there ^{13}CO rotational lines as well ?

Assuming the molar mass

$$^{12}C = 19.93 \times 10^{-27} \text{ kg}$$

$$^{13}C = 21.59 \times 10^{-27} \text{ kg}$$

$$^{16}O = 26.56 \times 10^{-27} \text{ kg}$$

(b) HCl has a B value of $10.593\ cm^{-1}$ and a centrifugal distortion constant D of $5.3 \times 10^{-4}\ cm^{-1}$. Estimate the vibrational frequency and force constant of the molecule. 5+3