# 2017

## M.Sc. Part–I Examination

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

**PAPER—III**

*Full Marks* : 100

*Time* : 4 Hours

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

**Write the answer to questions of each group in Separate answer booklet.**

### Group—A

### (*Probability and Statistics*)

[Marks : 30]

Answer any *two* questions : 2×15

**1.** (a) Describe the Pure-Birth process and deduce the corresponding Yule-Furry process. What will be the probability generating function for this process? 8

(b) When do we say that a state $j$ is a accessible from a state $i$? When do we say that the two states $i$ and $j$ communicate ? 4

(c) Define periodicity property of a state. 3

**2.** (a) State and prove Chapman-Kolmogorov equation. 2+4

(b) What is meant by Galton-Watson Branching process? Prove that

$P_n(s) = P_{n-1}\{P(s)\}$ and

$P_n(s) = P\{P_{n-1}(s)\}$, the symbols have their usual meanings. 2+5

(c) Define the multiple correlation coefficient of a variable $x_1$ with $x_2, x_3, \ldots, x_p$. 2

**3.** (a) Prove that in an irreducible chain, all the states are the same type. They are either all transient, all persistent null, or all persistent non-null. All the states are aperiodic and in the latter case they all have the same period. 7

(b) What is stochastic process ? Define state and state space with examples. 4

(c) Write a short note on extinction probability ? 4





## Group—B

## (*Numerical Analysis*)

[Marks : 40]

Answer *Q. No. 4* and any *three* from the rest.

**4.** Answer any *one* question. 1×4

(a) Define average and central difference operators. Find a relation between them.

(b) Define Chebyshev polynomial. What is its advantages to approximate a function ?

**5.** (a) Deduce striling's interpolation formula starting from appropriate Gauss's General difference interpolation formula. 8

(b) Show that the function $f(x) = \begin{cases} x^3 + ax^2 + 3, & 0 \le x \le 1 \\ x^3 + x^2 + 4, & 1 \le x \le 2 \end{cases}$

is a cubic spline if $a = 1$. 4

**6.** (a) Deduce 3-point Gauss-Legendre quadrature formula. State its error term. Hence find the value of

$$\int_0^1 \frac{\sin(\pi x)}{\{x(1-x)\}^{3/2}} dx$$ 6

(b) Using Gauss-Legendre integration formula, find the value of $\int_0^1 \frac{\sin(\pi x)}{\{x(1-x)\}^{3/2}} dx$. 6

7. (a) Describe a method to approximate a function using orthogonal polynomial. 6

(b) Prove the sufficient condition for the convergence of Gauss-Seidal's iteration method. Also mention the rate of convergence of this method. 6

8. (a) Use finite difference method to find the values of $a$ and $b$ in the following tables :

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $f(x)$ | -5 | $a$ | 8 | $b$ | 20 | 32 |

4

(b) Describe Jacobi's method to find all eigenvalues and eigen vectors of a real symmetric matrix. 8

9. (a) Describe fourth order Runga-Kutta method to solve a pair of first order differential equations. 4

(b) Explain Crank-Nicolson implicit method to solve the founding PDE : 8

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial t^2}, t > 0, 0 < x < 1$$

and $u(x,0) = f(x),\ u(0,t) = \psi(t),\ u(1,t) = \phi(t), t > 0$





## Group—C

## (*Introduction to Computing*)

[Marks : 30]

10. Answer any *six* questions : 6×5

(a) Explain the parity method for error detection. Determine the single-error correlating code for BCD number 1010 (information bits) using odd parity.

(b) Write a program in C to find the product of two complex numbers defining a complex number using a structure.

(c) Explain 'if', 'if else' and nested 'if else' statements in C with examples.

(d) Using Karnaugh map, simplify the following Boolean function: $f(a,b,c,d) = \Sigma(0,2,5,9,15) + \sum_{\phi}(6,7,8,10,12,13)$.

(e) Write program segments in C that will read the value of $x$ and evaluate the following function: 3

$$y = \begin{cases} 5x^2 + 63 & \text{for } x < 5 \\ 105 & \text{for } x = 5 \\ 3x + 2e^{-x} & \text{for } x > 5 \end{cases}$$

using (i) else if ladder, (ii) conditional operator.

(f) Write a program in C to count number of vowels in a string using a function.

(g) Discuss any four string handling functions in C?

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