NEW

2016

BCA

2nd Semester Examination

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

PAPER—1203

Full Marks : 100

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer *Q. No. 1* and any *six* from the rest taking at least *one* from each group.

1. Answer any *five* questions : 5×2

(i) Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, if one root is $\sqrt{3} - 2$.

(ii) Find the value of c so that the set of vectors $\{(c, 2, 3), (2, 1, 3), (1, c, 0)\}$ is linearly dependent.

(iii) Find the value of $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

(iv) State Lagrange's Mean Value Theorem.

(v) Define Normal distribution.

(vi) Test whethere the events A and B are in dependent if $P(A) = 0{\cdot}6$, $P(B) = 0{\cdot}2$ and $P(A \cup B) = 0{\cdot}68$.

(vii) Find the mean of the random variable X whose density function

$$f(x) = e^{-x}, \quad 0 < x < \infty$$
$$= 0 \quad \text{elsewhere}$$

(viii) State geometrical interpretation of $\frac{dy}{dx}$.

(ix) Evaluate $\int \sqrt{1 + \sin x}\, dx$.

Group—A

(Algebra)

2. (a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ 5

(b) If the roots of $x^4 + ax^3 + bx^2 + ex + d = 0$ are in G.P. then prove that $c^2 = ad$. 5

3. (a) Solve the equation by matrix method. 5

$2x + 3y + z = 11,\ x + y + z = 6,\ 5x - y + 10z = 34.$

(b) Show that the roots of the equation. 5

$$\frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3} + \frac{4}{x-4} + \frac{5}{x-5} = 6 \text{ are all real.}$$

4. (a) If $\alpha_1, \alpha_2, \alpha_3$ be the roots of the equation $x^3 + x + 1 = 0$. then prove that $(\alpha_1^2 + 1)(\alpha_2^2 + 1)(\alpha_3^2 + 1) = 1$. 5

(b) Use cayley Hamilton theorem to computes A^{-1} where $A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$.

Group—B

(Calculus)

5. (a) If $y = (x^2 - 1)^n$ thin show that

$$\left(x^2 - 1\right) y_{n+2} + 2xy_{n+1} - n\left(n+1\right) y_n = 0.$$ 5

(b) State Euler's theorem on homogeneous functions and verify it for the function $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$.

5

6. (a) Evaluate any one of the integral 5

(i) $\int x\sqrt{\frac{a-x}{a+x}}\,dx$

(ii) $\int_0^{\pi/2} \frac{dx}{3 + 5\cos x}$

(b) Prove that $0 < \frac{1}{x} \log \frac{e^x - 1}{x} < 1$ using M.V.T. 5

7. (a) Evaluate $\underset{n \to \infty}{\text{Lt}} \left\{\left(2 + \frac{1}{n}\right)\left(2 + \frac{2}{n}\right) \ldots\ldots \left(2 + \frac{n}{n}\right)\right\}^{1/n}$ 5

(b) Evaluate : $\int_0^1 e^{mx}\,dx$ from the difinition. 5

Group—C

(Probabilities)

8. (a) Explain Kurtosis with geometrical interpretation. 5

(b) If A, B are any two events then show that :

$$P(A+B) = P(A) + P(B) - P(AB)$$ 5

9. (a) A random variable x has the probability density function $f(x) = 4(x - x^3)$, $0 \leq x < 1$. 5

$= 0$ elsewhere

Find mean and variance of the random variable X.

(b) Give the following totals for 10 pairs of observations on two characters x and y obtain the two regression equations and correlation coefficients :

$$\sum x = 12, \sum y = 4, \sum x^2 = 16 \cdot 20, \sum y^2 = 1 \cdot 96, \sum xy = 5 \cdot 2.$$

5

10. (a) A random variable x has the density function $f(x) =$

$$\begin{cases} \frac{1}{4} & \text{When } -2 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Obtain (i) $P(X < 1)$, (ii) $P(|X| > 1)$

(iii) $P(2X + 3 > 5)$. 5

(b) Fit a straight line to the following points :

x	1	2	3	4	5
y	5	7	9	10	11

[Internal Assessment — 30]
