NEW

2015

BCA

2nd Semester Examination

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

PAPER—1203

Full Marks : 100

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer *Q. No. 1* and any *six* from the rest taking at least *one* from each group.

1. Answer any *five* questions : 5×2

(i) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $-\frac{1}{\alpha}, -\frac{1}{\beta}, -\frac{1}{\gamma}$.

(ii) Find the inverse of the matrix $A = \begin{bmatrix} 5 & 3 \\ -2 & 2 \end{bmatrix}$.

(iii) Prove that for any two events A and B,

$P(A + B) \leq P(A) + P(B)$.

(iv) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$.

(v) Solve the equation $x^3 - 3x^2 + 4 = 0$ where two roots are equal.

(vi) Show that for two independent variates, correlation coefficient is zero.

(vii) If X is a discrete random variable having the following probability mass functions :

x :	0	1	2	3	4	5	6
P(X = x) :	0	K	2K	3K	4K	5K	6K

Determine the constant K.

(viii) The mean and variance of the Binomial (n,p) distribution are respectively 20 and 16. Find the value of n and p.

(ix) Two coins are tossed at a time. Write down the sample space. Find the probality of getting one head and one tail.

(x) Prove that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$.

Group—A

(Algebra)

2. (a) State Descarte's rule of signs regarding the existance of roots of polynomial equation with real coefficients. Show that the equation $x^4 + 16x^2 + 7x - 10 = 0$ has one positive, one negative and two imaginary roots. 5

(b) If one of the roots of the equation $x^3 + px^2 + qx + r = 0$, equals the sum of the other two, then prove that $p^3 + 8r = 4pq$. 5

3. (a) Verify Cayby Hamilton theorem for the matrix :

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}.$$ Hence find A^{-1}. 5

(b) Solve the equation : $27x^3 + 42x^2 - 28x - 8 = 0$ whose roots are in G.P. 5

4. (a) Solve by Cardan's method : $x^3 - 24x + 72 = 0$ 5

(b) If α, β, ν be the roots of $x^3 - 3x^2 + 8x - 5 = 0$, find the equation whose roots are $2\alpha + 3, 2\beta + 3, 2\nu + 3$. 5

Group—B

(Calculus)

5. (a) State Rolle's theorem and Lagrange's Mean Value theorem. 5

(b) If $y = (\sin^{-1} x)^2$, prove that

$$(1 - x^2)\, y_{n+2} - (2n + 1)xy_{n+1} - n^2yn = 0.$$ 5

6. (a) If $u(x, y) = f\,(x^2 + 2yz,\ y^2 + 2zx)$, prove that

$$(y^2 - zx)\ \frac{\partial u}{\partial x} + (x^2 - yz)\ \frac{\partial u}{\partial y} + (z^2 - xy)\ \frac{\partial u}{\partial z} = 0.$$ 5

(b) If $u = \frac{x^2 + y^2}{\sqrt{x + y}}$, $(x,y) \neq (0,0)$, what should be the value of K so that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = Ku$.? 2

(c) If $u = x^y$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. 3

7. (a) Find :

$$\lim_{n \to \infty} \left[\frac{n}{n^2} + \frac{n}{1^2 + n^2} + \frac{n}{2^2 + n^2} + \dots + \frac{n}{(n-1)^2 + n^2} \right].$$

5

(b) Evaluate : $\int \frac{dx}{(1-x)\sqrt{1-x^2}}$ 5

Group—C

(Probabilities)

8. (a) State classical definition of probability. Deduce that :

(i) $0 \le P(A) \le 1$;

(ii) $P(S) = 1$, where A is any arbitrary event in the event space S. 2 + 3

(b) In an examination 30% of the students failed in Mathematics, 15% of the students failed in Chemistry and 10% of the students failed in both Chemistry and Mathematics. A student is selected at random. If he failed in Chemistry, then what is probability that the student passed in Mathematics ?

9. (a) Fit a suitable straight line to the following data by the method of least squares : 5

Year	: 1959	1960	1961	1962	1963
% of insured people	: 11·3	13·0	9·7	10·6	10·7

(b) Two regression lines of a sample are $x + 6y = 6$ and $3x + 2y = 10$. Find the correlation coefficient. 5

10. (a) Define the expectation of a random variable x. Prove that $E(ax + b) = 9E(x) + b$. 4

(b) Find the mean and variance of continous distribution given by the following density function :

$$f(x) = \frac{1}{2} - ax, \text{ if } 0 < x < 4$$
$$= 0, \quad \text{elsewhere}$$ 6

[Internal Assessment — 30]