

2015

OLD

Part I 3-Tier

STATISTICS

PAPER—II

(Honours)

Full Marks : 90

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A (Marks : 30)

1. Answer *either* (a) and (b) or (c) and (d).

(a) Prove that for a normal (μ, σ^2) distribution, the mode is μ . Also show that Mean = Median = Mode.

4+4

(b) If two numbers p and q are chosen at random from the set $\{1, 2, 3, \dots, 10\}$ with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

7

(Turn Over)

- (c) Prove that the mode ($\tilde{\mu}$) for a Poisson distribution having parameter λ is given by :

$$\tilde{\mu} = \begin{cases} [\lambda], & \text{if } \lambda \text{ is not an integer} \\ \lambda - 1 \text{ and } \lambda, & \text{if } \lambda \text{ is an integer.} \end{cases} \quad 8$$

- (d) A total of n shells are fixed at a target. The probability of the j -th shell hitting the target is p_j , $j = 1, 2, \dots, n$. Assuming that the n firings are n mutually independent trials, find the probability that at least two shells out of n find the target. 7

2. Answer either (a) and (b) or (c) and (d).

- (a) Prove that the variance of the random variable x having p.d.f.

$$f(x) = \frac{1}{B(a, b)} x^{a-1} (1-x)^{b-1}, \quad 0 < x < 1, \quad a > 0, \quad b > 0$$

where $B(a, b)$ is the beta distribution is given by

$$\frac{ab}{(a+b+1)(a+b)^2} \quad 8$$

- (b) A problem in Mathematics is given to $(n-1)$ students whose chances of solving it are respectively

$$\frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}. \text{ What is the probability that the problem}$$

will be solved? 7

- (c) Let $\{A_n\}_n$ be a sequence of events of a random experiment E and is monotonic then prove that

$$P\left(\lim_{n \rightarrow \infty} A_n\right) = \lim_{n \rightarrow \infty} P(A_n). \quad 8$$

- (d) The Probability of detecting tuberculosis in X-ray examination of a person suffering from the disease is 1-b. The probability of diagnosing a healthy person as tubercular is 'a'. If the ratio of the tubercular patients to the whole population is C, find the probability that a person is healthy if after examination he is diagnosed as tubercular. 7

Group—B

3. Answer any five questions : 8×5

- (a) If the function $h: \mathbb{R} \rightarrow \mathbb{I}$ be continuous and $h(x) \geq 0 \forall x \in \mathbb{R}$ having mean $E\{h(x)\}$ of the random variable X then show that for each

$$\epsilon > 0, P\{h(x) \geq \epsilon\} \leq \frac{E\{h(x)\}}{\epsilon}$$

provided $E\{h(x)\}$ exists. 8

- (b) If X be a non-negative random variable with finite mean m , prove that $P(X \geq m\alpha) \leq \frac{1}{\alpha}$ for any arbitrary $\alpha > 0$. 8

(c) Central limit theorem for equal components implies the law of large numbers for equal components — Explain. 8

(d) If $\{X_n\}$ is a sequence of independent random variables such that each X_i has the same distribution with mean m and standard deviation σ , then prove that

$\frac{\bar{X} - m}{\sigma/\sqrt{n}}$ is asymptotically normal $(0, 1)$ i.e.

$$\bar{X} \sim N\left(m, \frac{\sigma}{\sqrt{n}}\right), \text{ where } \bar{X} = \frac{\sum X_n}{n}. \quad 8$$

(e) Find the characteristic function of $X_n(t)$ of the continuous random variable X with p.d.f.

$$f_x(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{elsewhere.} \end{cases} \quad 8$$

(f) What do you mean by interpolation? Establish Lagrange's interpolation formula. 2+6

(g) What do you mean by Numerical integration? Establish Trapezoidal rule for numerical integration. 2+6

(h) Derive the marginal distributions of a bivariate normal distribution. Also derive the moment generating function of the distribution. 3+5

- (i) What do you mean by time-series analysis? Give the additive and multiplicative model of the time series. 2+3+3
- (j) What do you mean by a 'Consumer Price Index'? Explain different types of price index numbers with applications. 2+6

Group—C

4. Answer any *five* questions : 4×5
- (a) What are the importance of Newton's forward and backward interpolation formula? 2+2
- (b) What are the geometrical significance of Simpson's $\frac{1}{3}$ rd rule of numerical integration? 4
- (c) Find the error terms of the numerical iteration method for finding the solution of a quadratic equation. 4
- (d) What is the errors of Index numbers? 4
- (e) Make a comparative study of constructing the Chain-base and Fixed-base method of pricing index. 4
- (f) Write a short note on the functions of CSO for Indian Statistical System. 4
- (g) Explain how the seasonal components of a time series are obtained by ratio-to moving average method. 4

- (h) What are the uses of Pareto and Log normal distributions in studying income distribution? 4
- (i) What do you mean by Periodogram Analysis? Explain how it is used to obtain cyclical components. 2+2
- (j) How are the operators Δ , E , and ∇ related? [The symbols keep their usual meanings.] 4
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