UG/I/STAT/H/I/15

2015

STATISTICS

[Honours]

PAPER – I

Full Marks : 100

Time : 4 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP – A

(*Descriptive Statistics*)

[*Marks* : 45]

1. Answer any *five* questions : 5 × 5

(*a*) What do you mean by primary data ? Discuss mail questionnaire method and interview method for collecting primary data. Also discuss their relative merits and demerits.

(*b*) Distinguish (*i*) between an attribute and a variable, and (*ii*) between time series data and cross-sectional data.

(*c*) What is tabulation ? Describe the different parts of a standard statistical table.

(*d*) What is a histogram ? Describe the construction of a histogram.

(*e*) Let X be a variable assuming positive values only. Show that the arithmetic mean of the reciprocal of X cannot be smaller than the reciprocal of its arithmetic mean.

(*f*) Show that the quartile deviation is independent of change of origin but dependent on scale.

(*g*) Given the observations 32, 67, 43, 12, 91 on a variable X, find out the values of 'a' and 'b' such that

$$(i)\ \sum_{i=1}^{n} |x_i - a| \text{ and } (ii)\ \sum_{i=1}^{n} (x_i - b)^2$$

are minimum, specify the results clearly, if you use any.

(*h*) Write a short note on stem and leaf display.

2. Answer any *two* questions : 10 × 2

(*a*) (*i*) Suppose the variable X takes positive values $x_1, x_2, ..., x_n$ only and their deviations from arithmetic mean($\overline{x}$) are small compared to $\overline{x}$ itself. Show that

$$\overline{x}_g = \overline{x}\left(1 - \frac{s^2}{2\overline{x}^2}\right) \text{ approximately,}$$

where $\overline{x}_g$ is the geometric mean and s is the standard deviation of X.

(*ii*) Show that

$$\frac{R^2}{2n} \leq s^2 \leq \frac{R^2}{4},$$

where R and s are the range and standard deviation of n values of a variable. 5 + 5

(*b*) (*i*) Define skewness of a frequency distribution. Mention three measures of skewness and deduce their limits.

(*ii*) In case of a study of bivariate data define the two regression coefficients. Show that the correlation coefficient cannot exceed the average of the two regression coefficients in magnitude. 5 + 5

(*c*) (*i*) Define correlation ratio (e_{yx}) of y on x. If r is the product-moment correlation coefficient between x and y then prove that $0 \leq r^2 \leq e^2_{yx} \leq 1$.

(*ii*) Define correlation index. Show that,

$$r_p^2 \geq r_{p-1}^2$$

where r_p is correlation index of order p. 5 + 5

(*d*) (*i*) Define the notion of independence and association for a 2×2 contingency table. Define odds ratio. How do you assess the nature of association between two categorical variables on the basis of odds ratio ?

(*ii*) What do you mean by intra-class correlation ? Derive intra-class correlation coefficient when a characteristic X is observed for 'p' families each having 'k' members. 5 + 5

GROUP – B

(*Matrix Algebra*)

[*Marks* : 20]

3. Answer any *two* questions : 5 × 2

(*a*) Define a vector space. Show that the number of vectors in the basis of a vector space is constant.

(*b*) If A' denotes the transpose of a matrix A, then show that rank A = rank AA'.

(*c*) Show that a non-singular matrix A is positive-definite iff $A = PP'$ for some non-singular matrix P.

(*d*) If any two rows of the determinant $|A|$ are interchanged, prove that the new determinant equals $-|A|$.

4. Answer any *one* question : 10 × 1

(*a*) (*i*) What do you mean by dimension of a vector space ? Suppose v is a vector space and v_1 and v_2 are the two vector subspaces of v. Show that,

$$\dim(v_1 + v_2) = \dim(v_1) + \dim(v_2) - \dim(v_1 \cap v_2)$$

(*ii*) Suppose v_1 and v_2 are the two vector subspaces of R^3 as

$$v_1 = \{(x_1, x_2, x_3)' : x_1 + x_2 + x_3 = 0\}$$

$$v_2 = \{(x_1, x_2, x_3)' : x_1 + 2x_2 - x_3 = 0\}$$

Find $\dim(v_1)$, $\dim(v_2)$, $\dim(v_1 \cap v_2)$, $\dim(v_1 + v_2)$ 10 (*b*) Investigate for what values if α and β the system of simultaneous equations given by

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \alpha z = \beta$$

has (*i*) no solution, (*ii*) a unique solution, (*iii*) an infinite member of solutions. 10

GROUP – C

(*Mathematical Analysis*)

[*Marks* : 25]

5. Answer any *three* questions : 5 × 3

(*a*) Show that the sequence $\{u_n\}$ where

$$u_n = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots + \frac{1}{\lfloor n}$$

is convergent and that $2 < \lim_{n \to \infty} u_n < 3$

(*b*) Show that $\sqrt[n]{n} \to 1$ as $n \to \infty$.

(*c*) Test the convergence of the series

$$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\cdots\cdots$$

(*d*) Prove that an absolutely convergent series is convergent while the converse of this result is not necessarily true.

(*e*) Show that

$$\Gamma(x) > \frac{1}{e}\int_0^1 t^{x-1}dx,\ \ x>0$$

and hence show that $\lim_{x\to 0+}\Gamma(x)=\infty$.

6. Answer any *one* question : 10 × 1

(*a*) (*i*) What is a sequence of real numbers ? When do you call a sequence to be convergent ?

(*ii*) Prove that every convergent sequence is bounded.

(*iii*) If $\{a_n\}$ and $\{b_n\}$ be two convergent sequences such that

$$\lim_{n\to\infty} a_n = A \quad \text{and} \quad \lim_{n\to\infty} b_n = B,$$

prove that the sequence $\{a_n b_n\}$ is also convergent and converges to AB. 4 + 3 + 3

(*b*) (*i*) If $f''(x) \geq 0$ on $[a, b]$, prove that,

$$f\left(\frac{x_1 + x_2}{2}\right) \leq \frac{1}{2}[f(x_1) + f(x_2)]$$

for any two points $x_1, x_2 \in [a, b]$.

(*ii*) Examine the convergence of the integral. 10

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx, \; m > 0, \; n > 0.$$

[*Internal Assessment* : 10 Marks]