2016

STATISTICS

[Honours]

PAPER – II

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Notations and symbols bear their usual meanings

[NEW SYLLABUS]

GROUP – A

1. Answer any *five* questions :   5 × 5

(i) If three numbers are chooser at random from the first 30 natural numbers, what is the probability that they will be in geometric progression ?   5

(ii) Consider a sample space $\Omega = \{\omega_1, \omega_2, \ldots, \omega_r\}$, where the probability of $\{\omega_i\}$ is twice that of $\{\omega_{i-1}\}$, $i = 2, 3, \ldots, r$. If $A_k = \{\omega_1, \omega_2, \ldots, \omega_k\}$, find $P(A_k)$ and $P(A_{k-1} | A_k)$.   5

(iii) Using Komogorov's axiomatic definition of probability, establish that probability is finitely additive for mutually exclusive events. Find the value of the sum of the probabilities of these events when they are also mutually exclusive.   5

(iv) Define the cumulative distribution function (c.d.f.) of a random variable X. Show that it is monotone non-decreasing and also right-continuous.   5

(*Continued*)

(*v*) Find the coefficient of variation (c.v.) of a Poisson (λ)-distribution. 5

(*vi*) If the raw moment of the order r exists for a probability distribution, prove that all raw moments of order less than r must exist for the distribution. Also express the raw moment of the order r in terms of appropriate central moments. 5

(*vii*)If the probability density functon (p.d.f.) of a random variable X is given by

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \text{, if } \alpha < x < \beta \\ 0 & \text{, otherwise} \end{cases}$$

then find $P(c \leq X < \beta)$ when $\alpha \leq c < \beta$ and $P(\alpha \leq X < c \mid X \leq \dot{c})$. 5

(*viii*)If random variables X and Y are such that $P(X = +1) = p$ and $P(X = 0) = 1-p$ with $0 < p < 1$, $P(Y = +1) = q$ and $P(Y = 0) = 1-q$ with $0 < q < 1$, and $E(XY) = pq$ then show X and Y are independently distributed. 5

(*ix*) Check whether the two random variables X and Y having the joint cumulative distribution function (c.d.f.)

$$F(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y} & , x > 0, \quad y > 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

are independent or not.　　　　　　5

2. Answer any *two* questions :　　　　　10×2

(*i*) Establish an appropriate inequality concerning the mean, the median and the mode of a log-normal distribution with parameters μ and δ.　　　　　10

(*ii*) Find the covariance between the number of 1-dots and the number of 6-dots obtained from $n \ (> 1)$ throws of an unbiased die.　　10

(*iii*) Describe the procedures of Z-scaling of test-scores and scaling of rates.　　　10

(*iv*) (*a*) Derive the moment generation function (m.g.f.) of a bivariate normal distribution.

(*b*) If the correlation coefficient between two random variables X and Y jointly following a bivariate normal distribution is zero, judge whether you can say X and Y to be independently distributed. $7 + 3$

GROUP – B

3. Answer any *two* questions : 5×2

(*i*) Derive a relationship between the operators Δ and E on polynomials. Also show that the operator Δ reduces the degree of a polynomial by unity but E does not. 5

(*ii*) A polynomial $g(\cdot)$ is such that $g(0) = 1$, $g(1) + g(2) = 10$, $g(3) + g(4) = 65 = g(5)$. Find the form of $g(x)$. 5

(*iii*) Derive Newton's formula for forward interpolation. 5

(*iv*) Write a note on inverse interpolation. 5

4. Answer any *one* question : 10×1

(*i*) (*a*) What is numerical integration? Obtain the Trapezoidal rule of numerical integration.

(*b*) When and how will you use the method of false position ? $(1 + 5) + 4$

(*ii*) Establish the method of iteration for finding the root of a numerical equation in one unknown and derive the required condition for convergence of the method. Also give the geometrical interpretation of the method.   10

GROUP – C

5. Answer any *three* questions : 5×3

(*i*) Mention the main functions of the National Sample Survey Office (NSSO) of the Government of India.   5

(*ii*) What are price-relatives ? Examine the statement : "If we ignore the weights when we take the average of the price-relatives, we will get an inappropriately weighted index number."   5

(*iii*) Explain the uses of price index numbers.   5

(*iv*) What are the different tests for consistency that a formula for price index number should satisfy ?   5

(*v*) Give a brief account on each of 'choice of a base period' and 'family budget survey' in the context of construction of a cost of living index number.   5

(*vi*) Explain the meaning of secular trend in the context of analysis of time-series data with examples. State the other components of time-series.   5

(*vii*) Distinguish between fixed-base index numbers and chain-base index numbers.   5

6. Answer any *one* question : 10×1

(*i*) Mention the major steps to be considered in the construction of a wholesale price index number. Describe 'collection of data' and 'combining the data' I this context. 10

(*ii*) (*a*) How will you fit a linear trend equation to time series data ?

(*b*) Describe the method of exponential smoothing used in the analysis of time series data. 5 + 5

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