

NEW
2015
Part II 3-Tier
STATISTICS
PAPER—II
(General)

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

1. Answer any four questions : 4×5
- (a) If $f(x)$ is a second degree polynomial in x and $f(0) = 1$, $f(1)+f(2) = 10$, $f(3)+f(4)+f(5) = 65$, Find $\Delta f(0)$ and $\Delta^2 f(0)$. Hence find $f(4)$.
- (b) Let t_n be a consistent estimator of θ . Now consider $t'_n = \frac{n-1}{n-2} t_n$. Is t'_n a consistent estimator of θ ?
- (c) A coin is to be tested for unbiasedness. The hypothesis that it is unbiased is rejected if 9 or more tosses of the coin out of 10 tosses result in heads. Can we take 1 % as the level of significance?
- (d) Write down the properties of Chi-square distribution with n degrees of freedom.

(Turn Over)

- (e) 'A maximum likelihood estimator is always unbiased'-- Is this statement true? Explain.
- (f) State the control limits of a no. of defects chart when the value of the parameter is not given.
- (g) Examine the statement : NRR is a suitable measure for forecasting future population growth.
- (h) What is the difference between the no. of defects chart and number of defectives chart?

2. Answer any five questions :

5×8

- (a) Derive Newton's forward interpolation formula.
- (b) Describe the construction of mean-chart when the standards are not given and when standards are given.
- (c) Describe, Shewhart's control chart technique in process control, explaining the principles behind it.
- (d) What do you mean by 'consistency' and 'efficiency' in the theory of estimation? Show that the sample mean is a consistent estimator of the population mean in the case of random sampling from a population having a finite variance.
- (e) If X follows a Poisson distribution, find the MLE of the population parameter.
- (f) Explain the concept of age-specific fertility rate (ASFR) and total fertility rate (TFR).
- (g) Derive a suitable test for testing $H_0 : \sigma_1^2 = \sigma_2^2$ against all possible alternatives for two independent normal populations with parameters (μ_1, σ_1^2) and (μ_2, σ_2^2) , respectively, when μ_1 and μ_2 are unknown.

- (h) How will you test the hypothesis for equality of the population means if (X, Y) follows a bivariate normal distribution? In this case, set up a confidence interval for the difference of the means.
- (i) Define Pearsonian frequency chi-square statistic and mention the conditions for the validity of a test on the basis of the statistic.
- (j) Using the method of separation of symbol to prove the following identities :

$$u_0 + u_1 + \dots + u_n = \binom{n+1}{1} u_0 + \binom{n+1}{2} \Delta u_0 + \binom{n+1}{3} \Delta^2 u_0 + \dots + \Delta^n u_0.$$

Answer any *two* questions from the question numbers
3 to 8.

3. (a) Describe the method of iteration to find the numerical solution of an equation in one unknown. Find the condition under which roots obtained by this method converge to its actual roots.
- (b) Derive the Simpson's 1/3 rule of numerical integration. 8+7
4. Show that \bar{x} and s^2 (where s^2 is an unbiased estimator of σ^2) in a random sample of size n from $N(\mu, \sigma^2)$ distribution are independently distributed. Find the distribution of \bar{x} and that of $(n-1)s^2/\sigma^2$. 10+5
5. (a) Define the following terms used in statistical quality control : rational subgroups, control limits and control charts.
- (b) Describe the construction of control Charts for no. of defectives, when standards are given and when these are not given. 6+9

6. (a) Write notes on i) Critical region ii) Type-I error and Type -II error in the context of testing of hypothesis.
- (b) For a normal distribution with unknown mean μ and known variance σ^2 , describe the testing procedure for $H_0 : \mu = \mu_0$ against all suitable alternatives. Also setup a $100(1 - \alpha)\%$ confidence interval for μ .

6+(6+3)

7. (a) Explain why CDR is not suitable for comparing the mortality situation of two countries. How can we compare the mortality situation of two countries?
- (b) Describe in detail the different columns of a complete life table.

8+7

8. (a) If $X_1, X_2, X_3, \dots, X_n$ are random observations on a variable X taking the value 1 with probability P and the value 0 with probability $(1-P)$ then show that $[T(n-T)/n(n-1)]$ is an unbiased estimator of $P(1-P)$,

$$\text{where } T = \sum_{i=1}^n X_i.$$

- (b) Suppose x_1, x_2, \dots, X_n are random samples from an infinite population with a finite variance σ^2 . Show

that $\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$ is a biased but is a consistent estimator of σ^2 . Hence, find out an unbiased estimator of σ^2 .

6+(5+3+1)