2015

MATHEMATICS

[Honours]

PAPER – IV

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

[*Marks* : 40]

(*Analytical Dynamics*)

1. Answer any *one* question : 15 × 1

(*a*) (*i*) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the are ; Show that, the time of reaching the vertex is

$$2\sqrt{a/g}\tan^{-1}\left[\sqrt{\frac{4ag}{V}}\right].$$ 8

(Turn Over)

(*ii*) Find the tangential and normal components of velocity and acceleration of a particle which describes a plane curve. 7

(*b*) (*i*) A heavy particle is attached to the lower end of an elastic string, the upper end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natural length and is then let go. Show that, the particle will return to this point in time

$\sqrt{a/g}\ (2\sqrt{3}+4\pi/3)$, where a is the unstreched length of the string. 8

(*ii*) A particle describes a path which is nearly a circle about a centre of force $f(u)$ at its centre where $u=\frac{1}{r}$. Find the condition under which this may be a stable motion. 7

2. Answer any *two* questions : 8×2

(*a*) If a planet were suddenly stopped in its orbit, supposed circular, show that, it would fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. 8

(*b*) A spherical rain drop, falling freely receives in each instant an increase of volume equal to λ times its surface at that instant. Show that the velocity at the end of time t is given by

$$\frac{g}{4\lambda}\left\{(a+\lambda t)-a^4/(a+\lambda t)^3\right\}$$

and the distance fallen through in that time is $\frac{gt^2}{8}\left(\frac{2a+\lambda t}{a+\lambda t}\right)^2$. 8

(*c*) One end of an elastic string, whose modulus of elasticity is λ and whose unstretched length is '*a*' which is fixed to a point on a smooth horizontal table and the other end is tied to a particle of mass m which is lying on the table. The particle is pulled to a distance, where the extension of the string is 'b' and then let go ; show that the time of complete oscillation is 8

$$2\left(\pi+\frac{2a}{b}\right)\sqrt{\frac{am}{\lambda}}$$

3. Answer any *three* questions : 3×3

(*a*) If an engine of horse-power H draws a train of W tons up an inclination α with a uniform velocity v ft./sec. against a resistance R lbs wt. per ton, then prove that

$$H = \frac{Hv}{550}(R + 2240 \sin \alpha)$$ 3

(*b*) Establish the principle of conservation of linear momentum form Newton's third law. 3

(*c*) Find the law of force to the pole when the path is the cardioide $r = a(1 - \cos \theta)$. 3

(*d*) If the angular velocity about origin be a constant ω, deduce the cross-radial component of rate of change of acceleration of the particle and show that if this rate of change of acceleration is zero, then, $\frac{d^2r}{dt^2} = \frac{1}{3}\omega^2 r$. 3

(*e*) Show that $\frac{v_e}{v} = \sqrt{2}$, where v_e is the escape velocity of the earth and v is the minimum velocity with which a particle is projected horizontally so that the particle will circulate around the earth. 3

GROUP – B

[*Marks* : 36]

(*Linear Programming and Game Theory*)

4. Answer any *one* question : 15 × 1

(*a*) (*i*) A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. Because of the need to ensure certain nutrient constituents, it is necessary to buy additionally one or two products which we shall call *A* and B. The nutrient constituents (vitamins and proteins) in each unit of the product are given below :

Product A cost Rs. 20 per unit and product B costs Rs. 40 per unit. Formulate the LP model for products A and B to be purchased at the

Nutients	Nutrients in the A	Contents product B	Mimimum amount of nutrients
1	36	6	108
2	3	12	36
3	20	20	100

lowest possible costs so as to provide the pigs with nutrients not less than given in the table. 4

(*ii*) If the objective function assumes its optimal value at more than one extreme points, then prove that every convex combination of these extreme points is also the optimal value of the objective function. 5

(*iii*) Solve the following LPP by graphical method :

Maximize $Z = 2x_1 + 3x_2$

subject to $x_1 - x_2 \leq 5$

$2x_1 + 5x_2 \geq 6$

$x_1, x_2 \geq 0$ 3

(*iv*) Put the following problem in a standard maximization form :

Minimize $Z = 3x_1 - 4x_2 - 3x_3$

subjet to $x_1 + 3x_2 - 4x_3 \leq 12$

$x_1 + x_2 - 2x_3 \leq 20$

$x_1 - 4x_2 - 5x_3 \geq 5$

$x_1 \geq 0$, x_2 and x_3 unrestricted in sign. 3

(*b*) (*i*) If an L.P.P. has an optimal solution, then show that at least one B.F.S. must be optimal. 7

(*ii*) A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost (in Rs. '000) of each city form a particular city in given below :

To city

From city	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

What is the sequence of visit of the salesman, so that the cost is minimum? 8

5. Answer any *two* questions : 8 × 2

(*a*) (*i*) Use two-plase simplex method to solve the following L.P.P. :

Minimize $Z = 4x_1 + x_2$

subject to
$$x_1 + 2x_2 \leq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$3x_1 + x_2 = 3$$
and $x_1, x_2 \geq 0$

(*ii*) Compare two-phase method with Big-M method 6 + 2

(*b*) (*i*) Find the dual of the following L.P.P. and hence solve it.

Maximize $Z = 3x_1 - 2x_2$

subject to
$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$x_1 + x_2 \leq 5$$
$$-x_2 \leq -1$$
and $x_1, x_2 \geq 0.$

(*ii*) What is the advantages to use duality to solve an L.P.P ? 7 + 1 (*c*) For the following pay-off table transform the zero-sum game into equivalent L.P.P and hence solve it. 8

Player Q

		Q_1	Q_2	Q_3
Player P	P_1	9	1	4
	P_2	0	6	3
	P_3	5	2	8

6. Answer any *one* question : 3 × 1

(*a*) Define the following terms :

basic solutions, basic feasible solutions, degenerate solutions. 1 + 1 + 1

(*b*) What do you mean by cycling in L.P.P. ? How can it be overcome ? 1 + 2

7. Answer any *one* question : 2 × 1

(*a*) Define saddle point and optimal point of a game. 1 + 1

(*b*) Define convex set. Show that the plane $2x + 5y - 8z = 10$ is a convex set. 1 + 1

GROUP – C

[*Marks* : 14]

(*Tensor Calculus*)

8. Answer any *one* question : 8 × 1

(*a*) (*i*) If A^i is an arbitrary contravariant vector and $C_{ij} A^i A^j$ is an invariant, show that $C_{ij} + C_{ji}$ is a covariant tensor of order 2. 4

(*ii*) Define Ricci tensor. Prove that Ricci tensor is symmetric. Find an expression for div R^h_{ijk} 4

(*b*) (*i*) Prove that the covariant derivative of the tensors g_{ij}, g^{ij} and $\delta_j^{\ i}$ all vanish identically. 5

(*ii*) Calculate the Christoffel symbols $[12, 2]$ and $\left\{ \begin{matrix} 2 \\ 12 \end{matrix} \right\}$ in a 3-dimensional Riemannian space in which the line element ds is given by

$$ds^2 = (dx^1)^2 + (x^1)^2 (dx^2)^2 + (dx^3)^2$$ 3

9. Answer any *two* questions : 3 × 2

(*a*) If a_{ij} and b_{ij} are co-variant tensors, show that the roots of the determinant equation $|x a_{ij} - b_{ij}| = 0$ are invariant under co-ordinate transformation. 3

(*b*) If A^i is a contravariant vector, show that

$$\text{div } A^i = \sqrt{g}\frac{\partial}{\partial x^i}\left(A^i \sqrt{g}\right) \text{ where } g = | g_{ij} |.$$ 3

(*c*) Define covariant and contravariant vectors. Show that the Kroneeker delta δ_j^i is a mixed tensor. 3
