2015

NEW

Part II 3-Tier

MATHEMATICS

PAPER—III

(General)

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

(*Linear Programming*)

[*Marks : 36*]

1. Answer any *one* question : 15×1

(a) (i) What is B.F.S. (Basic feasible solution) in a L.P.P. ? Show that $x_1 = 5$, $x_2 = 0$, $x_3 = -1$ is a basic solution of the system of equations :

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

Find other basic solutions, if there be any. 2+3

(ii) A factory is engaged in manufacturing two products A and B which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively and for one unit of B are 3, 1 and 3 hours respectively. The profits on each unit of A and B are Rs. 2·00 and Rs. 3·00 respectively. Pose a L.P.P. in terms of maximizing the profit on the items manufactured while 300 hours of lathe time, 300 hours of grinding time and 240 hours of assembling time are available. 5

(iii) Find the dual of the following L.P.P. :

Maximize $Z = 6x_1 + 5x_2 + 10x_3$

Subject to
$$4x_1 + 5x_2 + 7x_3 \leq 5$$
$$3x_1 + 7x_3 \leq 10$$
$$2x_1 + x_2 + 8x_3 = 20$$
$$2x_2 + 9x_3 \geq 5$$

$x_j \geq 0$, $j = 1, 2$ and x_3 is unrestricted in sign. 5

(b) (i) What is convex set ? If x_1, x_2 be real, show that the set given by $X = \{(x_1, x_2) \mid 4x_1^2 + 9x_2^2 \leq 36\}$ is a convex set. 2+3

(ii) Show that the set of all convex combinations of a finite number of points is a convex set. 5

(iii) Solve the following LPP by graphical method :

$$\text{Maximize } Z = 3x_1 - x_2$$

Subject to

$$2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 2$$
$$x_1 \leq 4$$

where $x_1, x_2 \geq 0$ 5

2. Answer any *two* questions : 8×2

(a) Obtain an optimal basic feasible solution to the following transportation problem : 8

	M_1	M_2	M_3	M_4	
W_1	2	2	2	1	3
W_2	10	8	5	4	7
W_3	7	6	6	8	5
	4	3	4	4	

(b) Solve the assignment problem and obtain optimum solution for the cost matrix : 8

	a	b	c	d	e
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	15

(c) Apply the principle of duality to solve the L.P.P. :

Maximize $Z = 4x_1 + 3x_2$

subject to

$$x_1 + x_2 \leq 7$$
$$3x_1 + x_2 \leq 15$$
$$0 \leq x_1 \leq 6$$
$$0 \leq x_2 \leq 8$$
$$-x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

8

3. Answer any *one* question : 3×1

(i) Graph the convex hull of the points :

(0, 0), (0, 1), (1, 2), (1, 1), (4, 0)

which of these points is an interior point of the convex hull? Express it as a convex combination of the extreme points. 3

(ii) Rewrite the following L.P.P. in its standard form :

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to

$$2x_1 - 3x_2 \leq 3$$
$$4x_1 + 2x_2 - 4x_3 \geq 5$$
$$2x_1 + 3x_3 \leq 2$$

where $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

3

4. Answer any *one* question : 2×1

(a) Show that the vectors *(1, 2, 4), (2, −1, 3), (0, 1, 2)* and (−3, 7, 2) are linearly dependent and find the relation among them. 2

(b) Find the basic solutions of the system :

$$x_1 + 2x_3 = 1$$
$$x_2 + x_3 = 4$$

which of them are feasible ? 2

Group—B

(Numerical Analysis)

[Marks : 18]

5. Answer any *two* questions : 8×2

(a) Given the following table, calculate f(10·7) by Lagrange's interpolation formula : 8

x	10·5	10·6	10·8	10·9	11·1	11·4
f(x)	0·26969	0·33839	0·39544	0·40022	0·38332	0·32257

(b) Solve the following system of equation by Gauss elimination method : 8

$$5x_1 - x_2 = 9$$
$$-x_1 + 5x_2 - x_3 = 4$$
$$5x_3 - x_2 = -6$$

(c) Obtain an approximate value of $\int_0^1 \frac{dx}{1+x^2}$ upto four places of decimal by using Simpson's $\frac{1}{3}$rd rule taking four equal subintervals. Hence obtain the approximate value of π correct to four decimal places. 8

6. Answer any *one* question : 1×2

(a) Give the geometrical interpretation of Trapezoidal rule.

(b) Prove that $\Delta . \nabla = \Delta - \nabla$.

(c) What are the third order and fourth order differences of $f(x) = x^3 - 4x^2 + 3x + 1$? 1+1

Group—C

(*Analytical Dynamics*)

[*Marks : 36*]

7. Answer any *one* question : 15×1

(a) (i) To prove that for a Projectile the sum of the Kinetic and Potential energies is constant throughout its motion.

8

(ii) A particle starts from rest at a distance 'a' from a fixed point O, and moves with an acceleration *proportional to its distance* from O, away from it. Find out the velocity and position at any time t. 7

(b) (i) Find the radial and cross-radial acceleration components of a particle moving in a plane. 8

(ii) A particle falls to the ground from a height h. If e be the coefficient of restitution, then show that the whole distance described by the particle before it has finished rebounding is $h.\frac{1+e^2}{1-e^2}$ and the whole time taken is $\sqrt{\frac{2h}{g}}.\frac{1+e}{1-e}$. 7

8. Answer any *two* questions : 8×2

(a) Show that a particle moves in a circle of radius r with a uniform speed V, its acceleration is directed towards the centre and is of magnitude $\frac{v^2}{r}$.

(b) An engine is pulling a train and works at a constant power doing H units of work per second. If M be the mass of the whole train and F the resistance supposed to be constant, show that the time of generating the velocity V from rest is :

$$\left[\frac{MH}{F^2}\log\frac{H}{H-FV}-\frac{MV}{F}\right].$$ 8

(c) If a particle moves in a central orbit describes a path $r^n = a^n \cos n\theta$, then find the law of force. 8

9. Answer any *one* question : 3×1

(a) In the radial velocity of a particle be four times of its transverse velocity, find the equation of the path of the particle. 3

(b) State Kepler's laws for Planetary motion. 3

10. Answer any *one* question : 2×1

(a) If radial velocity is proportional to the transverse velocity, find the path in polar co-ordinate. 2

(b) The velocity V of a particle moving along a straight line is given by the relation $V^2 = aS^2 + b$ where S is the distance travelled from a fixed point and a, b are constants. Prove that its acceleration varies as the distance from a fixed point in the line. 2

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