2015

NEW

Part II 3-Tier

MATHEMATICS

PAPER—II

(General)

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

[*Marks* : 45]

(*Differential Calculus*)

1. Answer any *one* question : 15×1

(a) (i) Show that the set of all irrational numbers is uncountable. 4

(ii) Prove that the sequence $\{2n^2 + 3n - 1\}$ is not a Cauchy sequence. 3

(Turn Over)

(iii) Prove that $\left\{\frac{3}{2n^2}\right\}$ is a null sequence. 3

(iv) State D'Alembert's ratio test for convergence of a series of positive terms. Use it to examine the convergence of : 2+3

$$\frac{1^2}{2}+\frac{2^2}{2^2}+\frac{3^2}{2^3}+\cdots+\frac{n^2}{2^n}+\cdots$$

(b) (i) If $f(x) = 0$ for $x = 0$

$$= 3x^2 \sin\frac{1}{2x}, \ x \neq 0$$

then show that f(x) is derivable but the derivative is not continuous at $x = 0$. 7

(ii) Find the minimum and maximum values of :

$f(x) = 3x + \frac{2}{3x}$ for all $x \in R - \{0\}$ 4

(iii) Evaluate : $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{2}{3x}}$. 4

2. Answer any *one* question : 8×1

(a) (i) If $\log y = \tan^{-1}x$, then show that $(1+x^2)y_2 + (2x-1)y_1 = 0$ and hence find a relation among y_{n+2}, y_{n+1} & y_n.

1+3

(ii) Determine the values of a and b such that

$$\lim_{x \to 0} \frac{x(1+2a\cos x)-3b\sin x}{x^3} = 1.$$ 4

(b) (i) State Langrange' Mean value theorem. In the Mean-value theorem $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$, find $\lim_{h \to 0+}$ when $f(x) = \cos\frac{3x}{2}$. 1+3

(ii) Differentiate : $\cot^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$

with respect to $\sin^{-1}x^2$. 4

3. Answer any *four* questions : 4×4

(a) Find the asymptotes of the curve :

$32x^3 - 6xy^2 + y^3 + 8x^2 + 2xy - y^2 - 1 = 0$.

(b) Show that the sum of the intercepts of any tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{2}$ is a constant.

(c) Find the radius of curvature, centre of curvature of $y^2 = 8y$ at (x, y).

(d) Verify Euler's theorem for the function :

$$f(x,y) = \cos^{-1}\left(\frac{x}{y}\right) + \cot^{-1}\left(\frac{x}{y}\right).$$

(e) Find the condition of orthogonality of the curves : $ax^2 + by^2 = 2$, $\alpha x^2 + \beta y^2 = 3$.

(f) Find the envelope of the family of lines $\frac{x}{a}+\frac{y}{b}=1$ where the parameters are connected by $a^2 + b^2 = 4$.

4. Answer any *three* questions : 2×3

(a) Find the first order partial derivatives of :

$$f(x, y) = \frac{2x+3y-4}{2x-3y+1} \text{ at } (4, 5).$$

(b) If $\sum_{n=1}^{\alpha} a_n$ is convergent, prove that $\lim_{n\to\alpha}(2a_n + 3) = 3$.

(c) State Leibnitz theorem on successive derivatives.

(d) State the Schwarz's theorem on the commutative property of mixed partial derivatives.

(e) The curves $y = x^2 + 1$, $(y - 1)^2 = x$ passes through the point (1, 2). Find the angle of intersection at this point.

Group—B

[Marks : 27]

5. Answer any *one* question : 15×1

A. (a) Evaluate any *two* : 4×2

(i) $\int\sqrt{\frac{a-x}{a+x}}\,dx$;

(ii) $\int\frac{dx}{4+5\sin x}$;

(iii) $\int\frac{dx}{2e^{2x}+5e^{x}+2}$.

(b) If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x\,dx$, then show that

$I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$, n being a positive integer greater than 1.

Hence evaluate I_6. 4+3

B. (a) Answer any *two* questions : 4×2

(i) Evaluate : $\int_0^{\pi}\frac{x\sin x}{1+\sin^2 x}dx$. 4

(ii) Evaluate : $\underset{n\to\infty}{\text{Lt}}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}$. 4

(iii) Evaluate :

$$\int_2^4 f(x)\,dx, \text{ where } f(x)=|x-2|+|x-3|.$$ 4

(b) (i) Define Gamma function and hence find the value of $\int_0^\infty e^{-x^2}dx$ 4

(ii) Assuming $\Gamma(m)\,\Gamma(1-m)=\pi\,\text{cosec}\,m\pi,\ 0<m<1$, evaluate $\Gamma(\frac{1}{9})\,F(\frac{2}{9})\,\Gamma(\frac{3}{9})\,\ldots\,\Gamma(\frac{8}{9})$ 3

6. Answer any *one* question : 8×1

(a) (i) Find the area enclosed by the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$ 4

(ii) Find the length of the arc of the parabola $y^2 = 12x$ measured from the vertex to erc extremity of the latus rectum. 4

(b) (i) Find the volume of the solid generated by the cycloid $x = a(1 + \cos\theta)$, $y = a(\theta + \sin\theta)$ about the y-axis. 4

(ii) Find the surface of a sphere generated by the circle $x^2 + y^2 = a^2$ about the y-axis. 4

7. Answer any *one* question : 4×1

(a) Evaluate $\iint\left(x^2 + y^2\right)dx\,dy$ over the region bounded by $x \geq 0$, $y \geq 0$ and $x + y \leq 2$ 4

(b) Evaluate $\iiint\left(x^2 + y^2 + z^2\right)dxdydz$ taken throughout the sphere $x^2 + y^2 + z^2 \leq 4$ 4

Group—C

[*Marks : 18*]

8. Answer any *two* questions : 8×2

(a) (i) Solve the differential equation : 4

$$\cos x\frac{dy}{dx} - y\sin x = y^2$$

(ii) Solve the differential equation : 4

$$y = 2px + p^2, \text{ where } p = \frac{dy}{dx}$$

(b) (i) Solve : $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = e^{-x}$ 4

(ii) Solve : $(D^2 + 9D + 18)y = \cos x,\ D \equiv \frac{d}{dx}$ 4

(c) (i) Solve the simultaneous differential equations :

$$\frac{dx}{dt} + 5x + 2y = e^t,\ \frac{dy}{dt} + 3x + 4y = t$$ 4

(ii) Find the eigen values of the differential equation $\frac{d^2y}{dx^2} + \lambda y = 0,\ (\lambda > 0)$, satisfying the boundary conditions $y(0) = 0 = y(1)$. 4

9. Answer any *one* question : 2×1

(a) Find the curve whose cartesian sub-tangent is constant. 2

(b) Find the orthogonal trajectories of the system of straight lines $y = mx$, m being the parameter. 2
