OLD

2015

Part-I 3-Tier

MATHEMATICS

(General)

PAPER—I

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(*Classical Algebra*)

[Marks : 27]

1. Answer any *one* question : 1×15

(a) (i) State De Moivre's theorem. 2

(ii) Show that $|Z_1+Z_2|^2+|Z_1-Z_2|^2=2\left(|Z_1|^2+|Z_2|^2\right)$, where Z_1 and Z_2 are two complex numbers. 3

(iii) If $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ and $A^2 - 5A + 7I = 0$, where I is the 2×2 unit matrix and 0 is the null matrix, then show that

$A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. 5

(iv) Show that the roots of the equation

$\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = \frac{1}{x}$, where a, b, c > 0 are all real.

5

(b) (i) If the roots of the equation $x^3 + 3px^2 + 3qx + r = 0$ be in harmonic progression then show that $2q^3 = r(3pq - r)$. 5

(ii) Show that the equation $3x^5 - 4x^2 + 8 = 0$ has at least two imaginary roots. 3

(iii) Show that $\frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is orthogonal. 3

(iv) Solve the equations by Cramer's rule

$$\begin{aligned} -x+3z+1&=0 \\ 2x-y-4z-2&=0 \\ y+2z-4&=0 \end{aligned}$$ 4

2. Answer any *one* question : 1×8

(a) (i) Prove that $\log(1+i)=\frac{1}{2}\log 2+i\left(2n+\frac{1}{4}\right)\pi$. 4

(ii) Express the matrix $\begin{pmatrix} 2 & -1 & 5 \\ 7 & 3 & 0 \\ 3 & -4 & 8 \end{pmatrix}$ as the sum of two matrices of which one is symetric and other is skew symetric. 4

(b) (i) If α, β, γ be the roots of the equation

$px^3 + 3qx^2 + 3rx + s = 0$ then find the value of

$\sum(\beta - \gamma)^2$. 4

(ii) Find the row-reduced echelon form of

$A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & -4 & 5 \end{bmatrix}$ and hence find the rank of it. 4

3. Answer any *two* questions : 2×2

(a) Show that $x^3 - 7x + 7 = 0$ has two roots between 1 and 2 and the other root between -3 and -4. 2

(b) Show that a Skew symmetric determinant of odd order is zero. 2

(c) Find the values of $(-i)^{\frac{3}{4}}$. 2

(d) If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, find $(A - 3I)(A - 2I)$, where I is 2×2 identity matrix. 2

Group B

(*Modern Algebra*)

[Marks : 18]

4. Answer any *two* questions : 2×8

(a) (i) Show that the n-th roots of unity form an abelian group under ordinary multiplication. 4

(ii) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 9 & 4 \end{bmatrix}$. 4

(b) (i) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then verify that A satisfies its own characteristics equation. 4

(ii) Prove that every field is an integral domain. Is the converse true ? Justify. 3+1

(c) (i) Examine whether the quadratic form $5x^2 + y^2 + 5z^2 + 4xy - 8xz - 4yz$ is positive definite or not.

(ii) If R be a ring such that $a^2 = a, \forall a \in R$, then Prove that i) $a + a = 0, \forall a \in R$ ii) $a + b = 0 \Rightarrow a = b$. 2+2

5. Answer any *one* question : 1×2

(a) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 2 & 6 & 1 & 3 \end{pmatrix}$ as a product of transposition. 2

(b) Give an example to show that union of two subgroups may not be a subgroup. 2

Group C

(Analytical Geometry)

[Marks : 32]

6. Answer any *one* question : 1×15

(a) (i) Reduce the equation $4x^2 + 4xy + y^2 - 4x - 2y + 4 = 0$ to its canonical form and determine the nature of the conic. 5+1

(ii) If, by a rotation of rectangular axes about the origin, (ax + by) and (cx + dy) be changed to $(a'x' + b'y')$ and $(c'x' + d'y')$ respectively, then show that $ab - bc = a'd' - b'c'$. 4

(iii) If $ax^2 + 2hxy + by^2 = 0$ be the equation of two adjacent sides of a parallelogram and $lx + my = 1$ be the equation of one of its diagonals, then show that the equation of its another diagonal is $y\,(bl - hm) = x\,(am - hl)$. 5

(b) (i) Show that the area of the triangle formed by the straight lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$

is $\dfrac{\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}$. 5

(ii) Show that the condition for which the straight line $\dfrac{l}{r} = a\,\mathrm{Cos}\theta + b\,\mathrm{Sin}\theta$ may touch the circle $r = 2k\,\mathrm{Cos}\theta$

is $b^2k^2 + 2ak = 1$. 5

(iii) The tangents at two points P and Q of a parabola, whose focus is S, meet at T. Show that $SP.SQ = ST^2$. 5

7. Answer any *three* questions : 3×5

(a) Show that the straight lines whose direction cosines are given by $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$, are perpendicular if $a^2(v + w) + b^2(w + u) + c^2(u + v) = 0$. 5

(b) Show that the length of the shortest distance between the straight lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and

$$\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$ is $4\sqrt{3}$ units. 5

(c) Find the equation of the sphere which touches the two planes $3x + 2y - 6z + 7 = 0$ and $3x + 2y - 6z + 35 = 0$ and whose centre lies on the straight line $x = 0, 2y + z = 0$. 5

(d) Find the equation of the right circular cone whose vertex is the point (1, 2, 3) and base is the curve $x^2 + y^2 = 25$, $z = 0$. 5

(e) A variable plane which is at a constant distance 3p from the origin O cuts the axes in A, B, C. Show that the locus of the centroid of the triangle ABC is

$$x^{-2}+y^{-2}+z^{-2}=p^{-2}.$$ 5

8. Answer any *one* question : 2×1

(a) Find the direction cosines of the normal to the plane : $x + 2y - 2z = 6$.

(b) Find the equation of the straight line through the point (1, 2, 3) and which is equally inclined to the axes.

Group — D

(Vector Algebra)

[Marks : 13]

9. Answer any *two* questions : 2×4

(a) Show that the three vectors

$\bar{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\bar{b} = \hat{i} - 3\hat{j} - 5\hat{k}$, $\bar{c} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ from the sides of a right-angled triangle. 4

(b) Show that $\left[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}\right] = 2\left[\vec{a}\;\vec{b}\;\vec{c}\right]$. 4

(c) Find the torque about the point $2\vec{i}+\vec{j}-3\vec{k}$ of a force represented by $\left(\vec{i}+2\vec{j}+\vec{k}\right)$ passing through the point $\left(3\vec{i}+4\vec{j}-\vec{k}\right)$. 4

10. Answer any *one* question : 5×1

(a) Prove that, in general $\left(\bar{a}\times\bar{b}\right)\times\bar{c} \neq \bar{a}\times\left(\bar{b}\times\bar{c}\right)$; but if the equality holds, then either $\vec{b}$ is parallel to $(\bar{a}\times\bar{c})$ or $\bar{a}$ and $\bar{c}$ are collinear. 5

(b) Show by vector method that in a triangle the perpendiculars drawn from the vertices to the opposite sides are concurrent. 5